Exploring Discrete Dynamics

by *Andrew Wuensche*
(Luniver Press, 2011)
ISBN-10: 1905986319
ISBN-13: 978-1905986316
xxxvii + 498 pages, 290 figures, 31 tables
`http://www.sussex.ac.uk/Users/andywu/EDDreviews.html`

August 3, 2012

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Vol. 18, No. 3, Pages 325–328, 2012
DOI:10.1162/artl_r_00068
`http://www.mitpressjournals.org/doi/abs/10.1162/artl_r_00068`

Exploring Discrete Dynamics is a very extended computational and analytic exploration of discrete dynamical systems. The book makes a summary of more than 19 years of results, programming, and research by Andrew Wuensche.

In 1992, at the *Santa Fe Institute*, Wuensche together with Mike Lesser published the celebrated book in cellular automata theory *The Global Dynamics of Cellular Automata* [26]. This book introduced a reverse algorithm for cellular automata, and presented an Atlas of basin of attraction fields computed by means of the algorithm. Motivated by these results and Kauffman's model of genetic regulatory networks [12], Wuensche subsequently developed new algorithms for random Boolean networks and discrete dynamical networks in general. His achievements have had a great influence on outstanding researchers such as Stuart Kauffman [12], Harold V. McIntosh [18], Andrew Adamatzky [1], and Christopher Langton [26], among many others; and hundreds of references in books and research papers.

These results have been obtained mainly making use of his popular open source software DDLab (*Discrete Dynamics Lab*, `http://www.ddlab.org/`), which is widely used in the scientific community and with free access to software, code, and manual. Wuensche's latest book, *Exploring Discrete Dynamics*, presents a very extensive description of the current features of DDLab. Successive chapters describe, in detail and depth, every function of this tool, illustrated with numerous examples from his research.

Analyses concentrate mainly on four systems of increasing generality: cellular automata (CA), random Boolean networks (RBN), discrete dynamical networks (DDN), and random maps. Consequently, in this book we have a ramification that connects and relates concepts naturally derived from these main subjects: reverse algorithms, rule-space, state-space, basins of attraction, stability, order, chaos, complexity, networks, emergent structures, classes, filters, self-reproduction, reaction-diffusion, cryptography, and beyond [30, 32, 33, 35, 31, 36, 37, 4, 7, 9, 21, 11, 24, 28].

Without doubt, the DDLab software is unique in its ability to study and classify discrete dynamical systems, analyse and unravel networks with the "network-graph," create flexible simulations where parameters can be changed on-the-fly, and generate basins of attraction and sub-trees. In DDLab we can experiment with mutations, calculate pre-images (or ancestors [10, 23]), and analyse state-space configurations iterating for unlimited spans of time, including simulations in one, two, and three dimensions. In the state-space implementation, we can calculate the changing input-entropy and pattern-density, which helps us to understand the properties of dynamical systems—applied in particular to automatically categorise CA rule-space between order, complexity, and chaos. The static Z-parameter, based on just the rule-table, also categorises CA rule-space by predicting the in-degree in sub-trees to identify "maximum" chaos—this is applied for a method of encryption [34].

An interesting point in the book is the incorporation of a *jump-graph of the basin of attraction field* (see page 207). Thinking in terms of Edward Fredkin's work *Finite Nature hypothesis* [13], the most important implication of this hypothesis is "that every volume of space-time has a finite amount of information in it. Every small region of space-time (a cell) must be in one of the small number of states." Fredkin focuses this implication precisely in CA models because they relate naturally his idea. A CA is a discrete dynamical system with a finite alphabet evolving on an infinite or finite regular lattice. Its dynamics are simple; basically each (central) cell is affected by the values of its close neighbours and itself; thus, depending on this combination of values, each cell is tranformed at the next time-step. All the cells in the lattice are updated synchronously at each time-step, though DDLab also provides asynchronous and partial-order updating, as well as probabilistic (noisy) updating. In this way, the CA (or discrete dynamical network) evolves from some initial lattice configuration along a deterministic trajectory. Hence, its space-time dynamics corresponds to the system's "local" behaviour. These artificial universes are able to yield a number of different classes of behaviour: trivial evolutions, periodic, quasi-periodic, chaotic, and complex. All possible trajectories combined make up the basin of attraction field with a topology of trees rooted on attractor cycles – the global dynamics. This is represented by a diagram, (know as, state transition graph) which provides a global perspective on the dynamics. A specific initial condition now defines one path through the graph.

Therefore, we have basin of attraction fields, where nodes (representing strings) are connected by directed arcs. In CA, these also display equivalencies and symmetries. In natural phenomena and other complex processes, the paradigm of the basin of attraction field could be extended. With the "jump-graph", we can have sets of basin fields connected deterministically or probabilistically to others. As a consequence, other kinds of dynamics could be analysed in a more complete and realistic way, in the sense of Fredkin's hypothesis. Thus, as an Ackermann function [5], we could think about sets of jump-graphs interconnecting other jump-graphs and so on.

Filters in DDLab are another powerful characteristic to recognize information inside the noise or chaos of discrete dynamical systems. Some CA evolutions present dominant periodic backgrounds, some times called "ethers," as in the famous universal elementary CA (ECA) rule 110 [8, 29]. Other more exotic CA (by alphabet, neighbourhood, and dimensions) can be studied in DDLab; they may evolve with two, three, four or more periodic or choatic backgrounds at the same time, with gliders and other complex patterns emerging during their histories [30, 6, 22, 16]. Filters have been demonstrated to be useful to explore and clarify particles (gliders or mobile self-localizations) and collisions between them in unconventional computing models [15, 14, 16]. DDLab can easily manipulate filters in such systems (see pages 410–416) because it can identify the frequencies of sub-patterns (blocks of cells) and separate the higher frequency blocks on-the-fly in order to reveal just the important information by changing the colours of cells.

An improvement to DDLab would be to extend the maximum lattice size beyond the current limits of 65,025 cells. An update to do this is in the pipeline, and I have personally tested the advance version (ddlabX07), where the new maximum is 159,072,862 cells, allowing 2D lattices with sides of 12,612 cells, and a 3D cubes with sides of 541 cells. Large evolution sizes will permit a better view of the macroscopic world of such systems, and consequently facilitate the construction of large initial configuration, which will be useful to develop enhanced kinds of experiments. For example, to handle combinations of gliders and collisions to implement computable systems (see [1, 25, 4, 15, 17, 20, 27, 19]), periodic sequences, regular expressions, or tiles.

Exploring Discrete Dynamics provides an essential and detailed description of how to operate the DDLab software, which together make an important contribution to understand, construct, and play with networks, CA, RBN, and discrete dynamical systems in general. If you have no experience using DDLab or are unfamiliar with these kinds of concepts, I would suggest a quick introduction to play with some interesting and attractive simulations by checking `http://www.ddlab.org` and links, for example for a fast and short introduction look `http://www.cogs.susx.ac.uk/users/andywu/multi_value/dd_life.html`; you can also view a short tutorial video illustrating the basic DDLab functions in `http://www.youtube.com/watch?v=N2hEiKOYsKo`.

Finally, Wuensche has explored a huge number of complex evolution rules and architectures in CA, RBN, and discrete dynamical systems; with mutations, totalistic functions, majority rules, chain-rules, reaction-diffusion, Postfunctions, canalizing, small-world and hybrid networks, and many others functions that actually offer a vast number of unexplored universes to research in several scientific fields. There will always be extra functions in any wish list (and users can themselves revise the open source code). My suggestion is to consider another variation of CA, the *CA with memory* (see [2, 3]). This could be easily done by including information from historical time-steps (which DDLab already records for other functions) when calculating the next time-step. CA with memory have been demonstrated to be simple, interesting, and powerful to explore other new domains of complex evolution rules and computable systems [14, 16, 17].

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