Wuensche
Lesser

The Global Dynamics of
Cellular Automata

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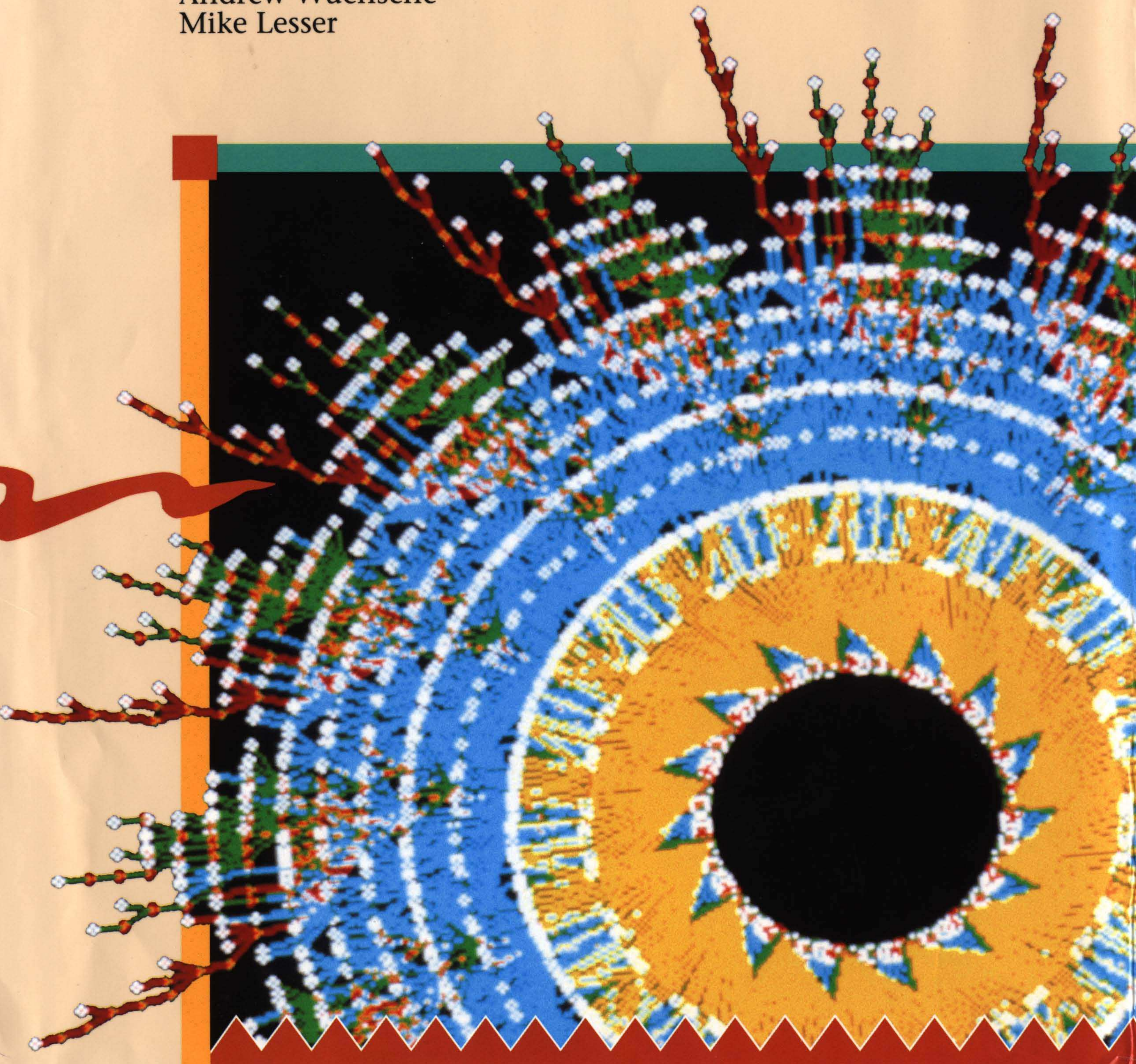
SANTA FE INSTITUTE STUDIES IN THE SCIENCES OF COMPLEXITY/REFERENCE VOLUME I

The Global Dynamics of Cellular Automata

An Atlas of Basin of Attraction Fields of One-Dimensional Cellular Automata

Diskette Included

Andrew Wuensche
Mike Lesser



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About the Authors

Andrew Wuensche was born in Warsaw, Poland in 1943. He studied physics briefly at the University of Geneva before switching to architecture. He has pursued an active career as an architect since graduating in 1968 from the Architectural Association School of Architecture in London. Several years ago he embarked on an independent research project into cellular automata, culminating in this book. He is currently working on disordered CA networks as brain-like models. He is based in London, and is a Member of the Santa Fe Institute.

Mike Lesser was born in London in 1943. He became friends with Andrew Wuensche in 1956. His background is in industrial automation, robotics, and the organization of engineering research. Some years ago he joined the staff of IERC, Cranfield Institute of Technology, where he has subsequently worked on the dynamics of evolution.

Jacket design by Hannus Design Associates

Jacket art was generated by Andrew Wuensche and Mike Lesser using the software included with this book.

The Global Dynamics of Cellular Automata introduces a powerful new global perspective for the study of discrete dynamical systems. After first looking at the unique trajectory of a system's future, an algorithm is also presented that directly computes the multiple merging trajectories that may have constituted the system's past. A given set of cellular automaton parameters will, in a sense, crystallize state space into a set of basins of attraction that will typically have the topology of branching trees rooted on attractor cycles. *The Global Dynamics of Cellular Automata* makes accessible the explicit portraits of these mathematical objects through computer-generated graphics. The atlas presents a complete class of such objects, and is intended, with the accompanying software, as an aid to navigation into the vast reaches of rule behavior space.

The Global Dynamics of Cellular Automata will appeal to students and researchers interested in cellular automata theory, complex systems, dynamical systems, computational theory, artificial life, neural networks, and aspects of genetics. The book contains work previously unpublished in scientific journals that may have profound significance in many areas of the sciences of complexity.

The accompanying diskette includes programs for drawing basin of attraction fields and space-time patterns. The software requires an 80286 (or higher) IBM® PC or compatible computer (math co-processor recommended) with 640K of memory, VGA graphics, and DOS 2.0 or higher.

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The Global Dynamics of Cellular Automata

AN ATLAS OF BASIN OF ATTRACTION FIELDS OF ONE-DIMENSIONAL CELLULAR AUTOMATA

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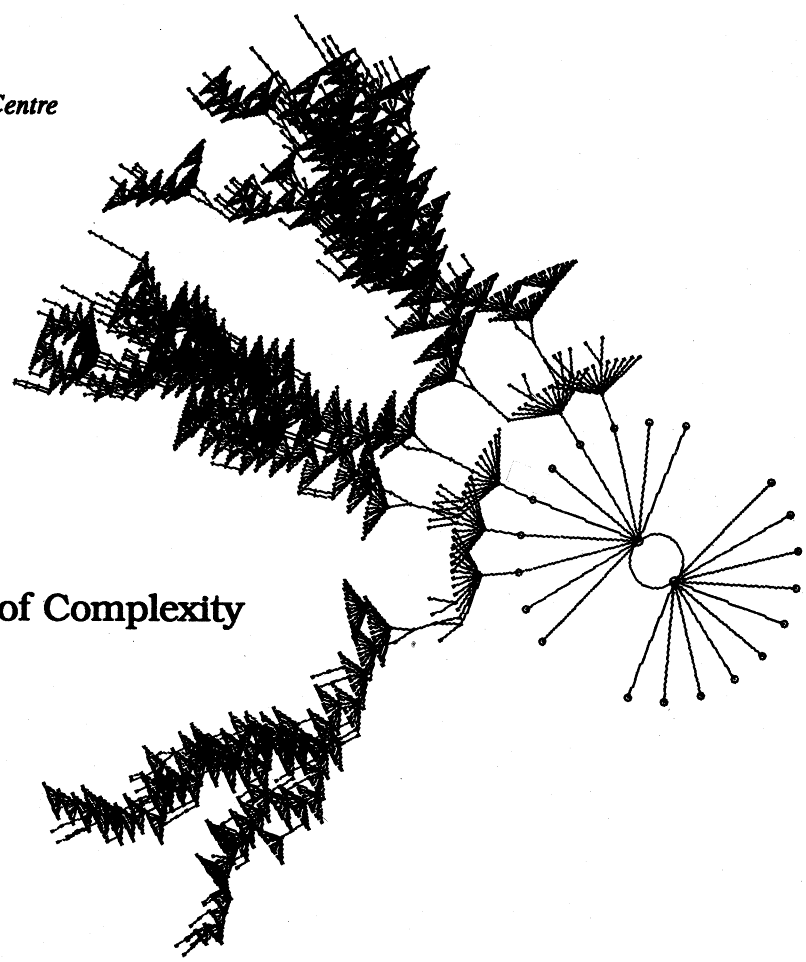

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The front cover shows a basin of attraction consisting of 8580 global states (about 26% of state space) converging onto an attractor cycle with period 120. The system parameters are $n = 5$, rule 54461424, $L = 15$, seed singleton.

The title page shows a basin of attraction which may be seen in the context of its basin of attraction field on page 203. The system parameters are $n = 5$, code 53, $L = 15$, seed singleton.

These and all similar graphics are examples of screen or printer output from the software included with the book.

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Santa Fe Institute Studies in the Sciences of Complexity

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To my wife Stephanie, and my daughters Silole and Alice.

— Andrew Wuensche

To the memory of the achievements and the tragic end of Alan Turing.

— Mike Lesser

Foreword

There are a wide variety of methods for representing the behavior of dynamical systems. Perhaps the most familiar representation method is the traditional time-series plot, in which some observable variable of the system (e.g., angular position) is plotted on the vertical axis, with time progressing to the right on the horizontal axis.

Such time-series plots trace the behavior of a system through time from a specific initial state. Thus, such plots represent the behavior of a system "localized" to a particular initial state, and are referred to as "local" representations of behavior. In order to get a feeling for the "global" behavior of a system, behavior independent of any particular initial state, one can collect an ensemble of such time-series plots, each rooted at a different initial state, and superimpose them together in the same plot. For certain systems, such ensembles of local representations can, in fact, lead to useful insights into the global dynamics of the system.

However, the "state-space" representation, introduced by Poincaré, provides a much clearer portrait of the global behavior of dynamical systems. In a state-space representation, the ensemble of all possible time series is captured in the notion of a vector-field on the state space: the "field of flow" imposed on the space of states by a particular dynamical rule. A great deal of insight can be gained into the behavior of dynamical systems by understanding specific behaviors in terms of the topological properties of their associated trajectories in state space.

Although much of the work in the state-space analysis of dynamical systems has been carried out in the context of continuous state spaces, many of the concepts and methods carry over to discrete state spaces. In a discrete state space, the flow field can be seen to be a graph, in which the states are the nodes and the "flow" is captured by the edges linking the nodes. Just as one may have fixed points, limit cycles, and chaotic attractors in continuous flow fields, one may have fixed points, cycles, and infinite chains in graphs (in the latter case, of course, the state space must be infinite). Concepts such as the degree of spreading of a local patch of the flow field in continuous state spaces have their analogs in the degree of convergence—or "in-degree"—of a node in the flow graph in discrete state spaces.

The study of Cellular Automata (CA) has proven to be a particularly rewarding vehicle for gaining insights into the behaviors and peculiarities of discrete dynamical systems. However, a good deal of the previous analysis of CA has been carried out via the equivalent of time-series perspective, in which various properties of the space-time diagrams of the evolution of CA's from specific initial states are investigated.

This Atlas presents a comprehensive overview and analysis of CA from the state-space perspective. Although explicitly treating CA, many of the observations and results derived here depend only on properties of the flow graphs themselves, and consequently should be equally valid when applied to the flow graphs for other discrete dynamical systems.

This Atlas, together with the associated program for generating and analyzing flow graphs, should prove to be an invaluable tool for pursuing, in the context of discrete dynamical systems, the kinds of insights that can only be obtained from a global perspective.

Christopher Langton

Santa Fe, New Mexico
November 21, 1991

Preface

The study of the dynamical behavior of cellular automata (CA) has become a significant area of experimental mathematics in recent years. CA provide a mathematically rigorous framework for a class of discrete dynamical systems that allow complex, unpredictable behaviour to emerge from the deterministic local interactions of many simple components acting in parallel.

Such *emergent* behavior in complex systems, relying on ***distributed*** rather than centralised control, has become the accepted paradigm in the attempt to understand biology in terms of physics (and vice versa?), encompassing such great enigmas as the phenomena of life and the functioning of the brain. Rather than confronting these questions head on, an alternative strategy is to pose the more modest question: how does emergent behaviour arise in CA, one of the simplest examples of a complex system.

In this book we examine CA behaviour in the context of the global dynamics of the system, not only the unique trajectory of the system's future, but also the multiple merging trajectories that could have constituted the system's past.

In a CA, discrete values assigned to an array of sites change synchronously in discrete steps over time by the application of simple local rules. ***Information structures***, consisting of propagating ensembles of values, may *emerge* within the array, and interact with each other and with other less active state configurations. Such emergent behaviour has lead to the notion of ***computation emerging spontaneously*** close to what may be a ***phase transition*** in CA rule space. Emergent behaviour in 2-D CA has given rise to the new field of ***artificial life***.

In the simpler case of 1-D CA, a trace through time may be made which completely describes the CA's evolution from a given initial configuration. This is portrayed as rows of successive ***global states*** of the array, the ***space-time pattern***. Space-time patterns represent a deterministic sequence of global states evolving along one particular path within a ***basin of attraction***, familiar from continuous dynamical systems. In a finite array, the path inevitably leads to a state cycle. Other sequences of global states typically exist leading to the same state cycle. The set of all possible paths make up the basin of attraction. CA basins of attraction are thus composed of global states linked according to their evolutionary relationship, and will typically have a topology of branching trees rooted on attractor cycles.

Other separate basins of attraction typically exist within the set of all possible array configurations (***state space***). A CA will, in a sense, crystallise state space into a set of basins of attraction, known as the ***basin of attraction field.*** The basin of attraction field is a mathematical object which, if represented as a graph, is an explicit global portrait of a CA's entire repertoire of behaviour. It includes all possible space-time patterns.

The study of basin of attraction fields as a function of CA rule systems, and how the topology of the fields unfold for increasing array size, may lead to insights into CA behaviour, and thus to emergent behaviour in general. This book shows CA basin of attraction fields as computer graphics diagrams, so that these objects may be as easily accessible as space-time patterns in experimental mathematics.

Construction of basin of attraction fields poses the problem of finding the complete set of alternative global states that could have preceded a given global state, referred to as its ***pre-images***. Solving this problem is recognised as being very difficult, other than by the explicit testing of the entire state space. Explicit testing becomes impractical in terms of computer time as the array size increases beyond modest values. Consequently, access to these objects has been limited.

This book introduces a *reverse algorithm* that directly computes the pre-images of a global state, resulting in an average computational performance that is many orders of magnitude faster than explicit testing. Two computer programs using the algorithm are described (and enclosed), to draw either basin of attraction fields or space-time patterns, for all 1-D, binary, *5-neighbour* CA rules, with *periodic boundary conditions*, and for the subsets of these rules, the *3-neighbour* rules, and the *5-neighbor totalistic* rules.

An atlas is presented (Appendix 2) showing the basin of attraction fields of all *3-neighbour* rules and all *5-neighbour* totalistic rules, produced using the program, for a range of array lengths. The atlas may be used as an aid to navigation in exploring the global dynamics of the 2^{32} rules in 5-neighbour rule space.

The book is divided into two parts. The first part (Chapters 1 through 4) gives the theoretical background and some implications of basin of attraction fields. The second part consists of appendices including the atlas and computer-program operating instructions.

Chapter 1 is an overview of the contents of the book.

Chapter 2 describes how CA global dynamics are represented by basin of attraction fields.

Chapter 3 looks in detail at CA architecture and rule systems, and the corresponding global dynamics. It is shown that *ordered architecture* and *periodic boundary conditions* impose restrictions on CA evolution in that *rotational symmetry* (and *bilateral symmetry* for *symmetrical rules*) are conserved. The rule numbering system and *equivalence classes* are reviewed. *Symmetry categories*, *rule clusters*, *limited pre-image rules*, and the *reverse algorithm* are introduced. The *Z parameter*, which reflects the *degree of preimaging*, or the convergence of dynamical flow in state space, is introduced.

Chapter 4 looks briefly at some implications of the above on current perceptions of the structure of *rule space*. The Z parameter is suggested as the mechanism underlying the λ *parameter*. A relationship between the Z parameter, basin field topology, and rule behaviour classes, based on the atlas, is proposed.

The idea of the rule table as *genotype* and the basin of attraction field as *phenotype* is examined. *Mutating* the rule table is found to result in *mutant* basin field topologies. Examples of sets of mutants are presented in Appendix 3.

We hope that the atlas of basin of attraction fields, and the program for exploring further into rule space, will provide new opportunities for CA research.

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We would like to especially thank Ronda Butler-Villa and Della Ulibarri for transforming a difficult manuscript into the present volume.

Table of Contents

ONE
Overview

CHAPTER ONE: Overview

Outlines what appears in this volume.

CHAPTER TWO: Cellular Automata and the Basin of Attraction Field

2.1 Cellular Automata

Briefly reviews CA architecture, space-time patterns, and CA dynamics.

2.2 Basin of Attraction Fields

Describes how CA global dynamics may be represented by the basin of attraction field. Shows how state transition graphs and basin of attraction fields are constructed and depicted, given a *reverse algorithm* for directly computing *pre-images.* A program that draws basin of attraction fields, providing immediate access to these objects, is introduced. The scope of the program and Atlas are described. The significance of basin of attraction fields is briefly discussed.

CHAPTER THREE: The Transition Function and Global Dynamics

Looks in detail at the relationship between the structure of the CA transition function and the corresponding CA global dynamics.

3.1 General CA Parameters

Reviews general rule parameters for 1-D CA, and narrows them down to a system with a *local rule* and *wiring diagram*, and periodic boundary conditions.

3.2 Rotation Symmetry

Shows that such rules have the property that *rotation symmetry* cannot decrease, and must remain constant inside the attractor cycle. This restriction of behaviour explains the sensitivity of the basin field topology to the number theoretic properties of the array size, apparent from the atlas.

3.3 Rule Clusters

Reviews the rule numbering system[33,34,42] and *equivalence classes*[28,29,42] and introduces the concept of rule *symmetry categories* and *rule clusters.*[42] It is shown that symmetric rules conserve *bilateral symmetry.*

3.4 Limited Pre-image Rules

Introduces the *limited pre-image rules*, which are a subset of rules with a special *rule table* structure, whereby the number of pre-images to any global state has a fixed upper limit irrespective of array size.[42] The rule table structure of the limited pre-image rules provides the basis for the reverse algorithm.

3.5 The Reverse Algorithm

Explains the detailed implementation of the reverse algorithm, starting with the limited pre-image rules, and extending to all 1-D local rules in general.

3.6 The Z Parameter

Introduces a parameter Z, that reflects the *degree of pre-imaging* typical of a given rule, according to the distribution of values in the rule table. The Z parameter measures the probability that the *next cell* in a *partial pre-image* that is being computed by the reverse algorithm, has a unique value.

CHAPTER FOUR: Implications of Basin of Attraction Fields

Looks briefly at some implications of basin of attraction fields.

4.1 Basin Field Topology and Rule Space

Discusses the implications of basin fields on the current perception of the structure of CA rule space. The degree of pre-imaging is suggested as a determinant of basin field topology, and thus of rule behaviour classes. The Z parameter is suggested as the mechanism underlying Langton's λ parameter.[16,17]

4.2 Mutation

Looks at the effect of mutating the rule table by a small Hamming distance. It is shown that this generally results in related basin structure. Mating rules by combining 1/2 of the rule table of two related rules to create an offspring rule is also investigated. Examples of sets of mutants are shown in Appendix 4.

4.3 Conclusion

APPENDICES

Appendix 1: The Atlas Program—Operating Instructions

Contains the operation instructions (and graphic conventions) for two programs, *Atlas1* and *Space1*, for drawing either basin of attraction fields, or space-time patterns.

Atlas1 draws either the entire basin of attraction field or just a single basin, for any rule in the set of 5-neighbour rules, for a range of array length, L. This includes the 5-neighbour *totalistic codes*[34] and the 3-neighbour rules, also called *elementary* rules.[33,39] The basin field version of the program draws a graphic image of all rotation inequivalent basins in state space, up to $L = 18$. A single basin may be drawn up to $L = 31$, from any seed state that forms part of the basin. Selected data may be optionally printed as basin fields are drawn. The colour plates presented elsewhere in this volume are examples of screen output.

Space1 draws space-time patterns in various graphic formats for an array length L, up to $L = 640$.

The program on diskette is enclosed inside the back cover.

Appendix 2: Atlas of Basin of Attraction Fields

The Atlas consists of two parts.

Part 1 presents the basin fields for all 88 *equivalence classes* of the 3-neighbour rules for $L = 1$ to 15.

Part 2 presents the basin fields for all 36 equivalence classes of the 5-neighbour totalistic codes for $L = 3$ to 16.

There is an index to rules and codes at the beginning of each section. *Complementary* rules and codes are shown on facing pages. Selected data is also presented concerning each basin field. The key to a typical Atlas page layout is shown at the start of Appendix 2.

Appendix 3: Mutants

A 5-neighbour rule table has 32 mutations separated from it by one bit. Sets of mutant basins of attraction, and entire fields, derived from the source rule and its 32 one-bit mutants are illustrated. Sets of mutants that diverge progressively from the source rule are also illustrated.

Appendix 4: The Rule-Space Matrix

Describes a *rule-space matrix* for the 3-neighbour (elementary) rules. Manipulations of the matrix simulate equivalence and cluster transformations described in Section 3.3.

TWO

Cellular Automata and the Basin of Attraction Field

2.1 Cellular Automata

A cellular automaton (CA) is a discrete dynamical system which evolves by the iteration of a simple deterministic rule; as in any dynamical system, the system's variables change as a function of their current values.

An alternative approach is to view a CA as a parallel processing computer[34] where the data is considered to be the initial CA configuration.

Yet a third approach is that a CA is a "logical universe...with its own local physics." [16] Such a CA universe, in spite of its mathematically simple construction, seems to be capable of supporting complex emergent behaviour.

2.1.1 CA Architecture

A CA is constructed as follows: Time is discrete and progresses in steps. A D-dimensional, potentially infinite space is partitioned into discrete "cells" (the CA array or lattice) according to a given geometry. Boundary conditions may be set to define a finite space. Each cell has one attribute (the cell's value) from a limited range of attributes, which may be labelled by an integer. The pattern of values across the whole array is the CA global state at a given time.

Any pattern may be set as an initial condition at time t_0. Each cell of the array simultaneously has its value updated to evolve a new global state at time t_1. The new value of any given cell (the target cell) at t_1 is a function of the values and locations of a set of cells (the neighbourhood) at t_0, typically situated locally in relation to the target cell (see Fig. 2.1). The neighbourhood may be defined by a neighbourhood template or wiring diagram. The CA evolves through a succession of global states (its *trajectory*) by the iteration of this global updating procedure (the *transition function*). Provided that the transition function is constant and the system is closed to noise (the updating is error free), then the evolution of the CA from its initial global state is uniquely determined.

Two types of CA may be distinguished, both deterministic: The more general case may be described as having varying degrees of *disordered* architecture (non-local[21]), where the wiring diagram and/or function at each cell may differ, for example Walker's networks of Boolean functions[27–32] and Kauffman's random Boolean networks.[14,15]

CA with *ordered* architecture are a special case, where the wiring diagram and function are the same over the entire array. In addition, the ordered wiring may be confined to a *local neighbourhood*, an uninterrupted zone of cells typically centred on the target cell. CA of this type will be referred to as having *local architecture*, for example, the architecture of Wolfram's "elementary rules." [33–40] This paper is relevant to deterministic CA in general, however it deals mainly with the simplest possible local CA architecture.

Von Neumann first proposed CA to model self-reproduction.[3,26] His relatively complicated CA architectures were local and two-dimensional with 29 cell values. The tendency since then has been to find simpler architecture that could nonetheless support complex emergent behaviour. For example Conways' "game of life" [2] is a 2-D local

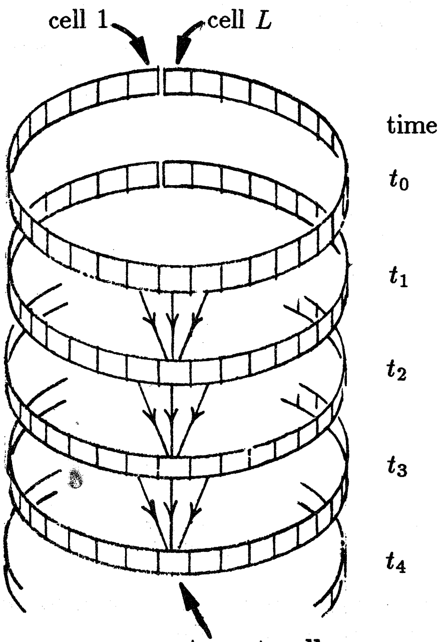

FIGURE 2.1 1-D, local binary CA with periodic boundary conditions, neighbourhood 3 (elementary rules), array length L.

CA with an orthogonal toroidal array, a 9-cell neighbourhood (the target cell and its eight nearest neighbours), and two cell values (binary value range).

2.1.2 Local 1-D Binary CA with Periodic Boundary Conditions

The simplest local CA architecture that we will investigate comprises a 1-D array of a small number of cells, a binary value range and a small local neighbourhood. The array is arranged in a circle; such a circular array is said to have *periodic boundary conditions*. Evolution of the CA may be represented as a sequence of global states on a cylinder, summed up by Fig. 2.1.

2.1.3 Space-Time Patterns

If the cylinder is split between cells 1 and L, and flattened out, it can be represented in 2-D as a space-time pattern, with space running across and time running down. (The space-time pattern of a 2-D CA could be represented in 3-D, for example[24]). Figure 2.2 shows examples of space-time patterns for various 1-D, local, binary, 5-neighbour rules. The rule numbers are indicated (see chapter 3, section 3.3.9).

Space-time patterns, which represent CA trajectories from given initial global states, have been the focus of statistical analysis and classification, and have been extensively illustrated in the literature, for instance.[17,18,33,34,39] Given the same CA architecture, different rules produce characteristic space-time patterns. For a given rule, patterns from different random initial global states are clearly recognisable as being similar by the human observer. Space-time patterns, in very broad terms, are said to display behaviour that is either static, periodic, complex (with interacting emergent structures), or chaotic.[17,34] CA rule classification schemes have been made on the basis of such space-time pattern phenomenology.

(a) (b) (c)

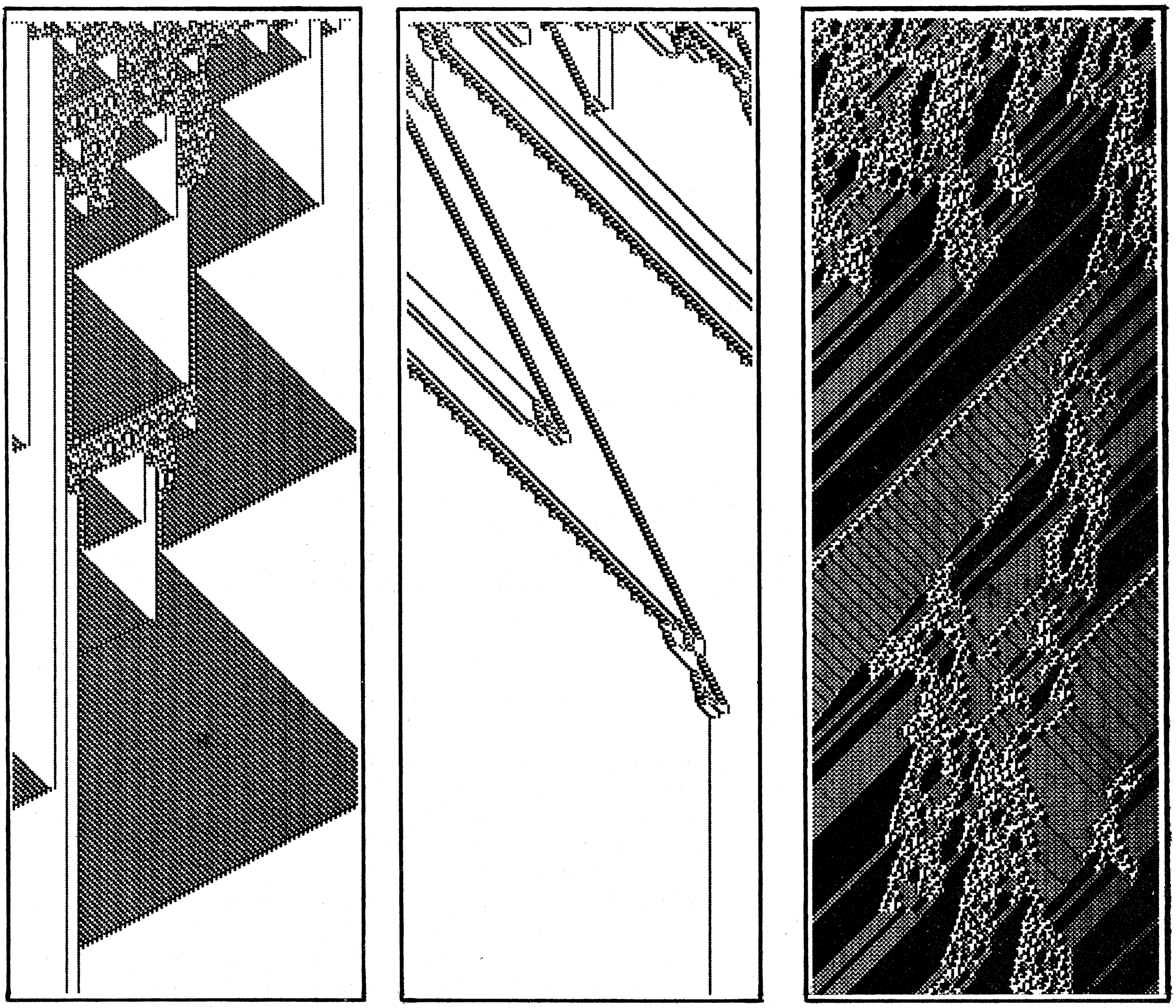

FIGURE 2.2 1-D, binary, local CA space-time patterns for 5-neighbour rules, with periodic boundary conditions. Array size 150, 420 time-steps from a random initial state. Rule numbers are (a) 3112581872; (b) 2334561936; and (c) 3583552890.

2.1.4 CA Dynamics

State space (also called *phase space*) is the set of all possible CA global states. In a finite CA, state space is finite; thus, any trajectory must eventually encounter a repeat of a global state that occurred at an earlier time. Because the system is deterministic, the trajectory will become trapped in this repeating sequence of states, a ***cyclic attractor***, with a specific period of 1 or more.

States are either part of the attractor or belong to a ***transient***, a sequence of states leading to the attractor. If transients exist, there must be states at their extremities (*garden-of-Eden states*), unreachable by evolution from any other state. The set of all possible transients leading to an attractor, plus the attractor itself, is the ***basin of***

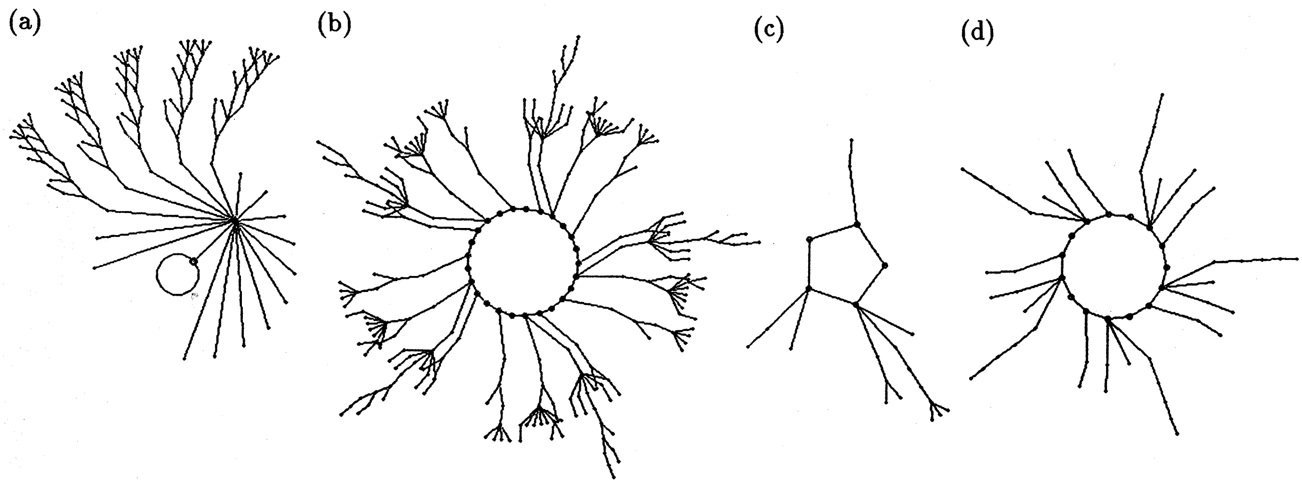

FIGURE 2.3 A basin of attraction field, local 3-neighbour rule 193, L = 10. The number of basins of each type is (a) 1, (b) 2, (c) 10, and (d) 2.

attraction of that attractor. State space is populated by one or more basins of attraction. These basins of attraction constitute the dynamical flow imposed on state space by the CA transition function.

A portrait of this global behaviour is the *basin of attraction field*, a discrete analogue of the familiar basin of attraction field found in the phase space of a continuous dynamical system, known as the system's *phase portrait.*

2.2 The Basin of Attraction Field

The *basin (of attraction) field* of a finite CA is the set of *basins of attraction* into which all possible states and trajectories will be organised by the cellular automaton transition function. The topology, or structure, of a single basin of attraction may be described by a diagram, the *state transition graph*. The set of graphs making up the field specifies the global behaviour of the system. Various other names have been used: state transition fragment,[39] contraction map,[7] topology of behaviour space,[29] and network of attraction.[42]

The notion of basin fields was proposed by Walker,[27] and examples[1] have been given by Martin et al.,[22] Pitsianis et al.,[25] Wolfram,[39,40] Feldberg and Rasmussen,[5] and by the authors[42] in an earlier edition of this atlas.

An example of a basin of attraction field is shown in Fig. 2.3 for the local, binary, 3-neighbour rule 193 (see chapter 3, section 3.3). The array length, L, equals 10, so that state space consists of $2^{10} = 1024$ global states. The CA transition function connects these states into a set of basins, the basin (of attraction) field. In this case there are four different types of basins in the field, some of which occur more than once. The number of each type is indicated.

2.2.1 The State Transition Graph

A state transition graph links up all the states belonging to a single basin of attraction according to their specific evolutionary location; this will typically have a topology of *trees rooted on attractor cycles.*[22] Global states are represented by *nodes* which are linked by *directed arcs.* Each node will have zero or more incoming arcs from nodes at the previous time step (*pre-images*), but because the system is deterministic, exactly one outgoing arc

[1]Martin et al.[22] have shown fields for rules 90 and 18. Pitsianis et al.[25] have shown fields for rule 90, with "null" boundary conditions. Wolfram has presented a table of sample basins for the 3-neighbour ($n = 3$) rules,[39] and the field for rule 30.[40] Feldberg and Rasmussen[5] have shown the field for the $n = 3$ rules for $L = 12$. In an earlier paper,[40] the authors produced an atlas of the fields for the $n = 3$ rules, and for totalistic codes 20 and 52.

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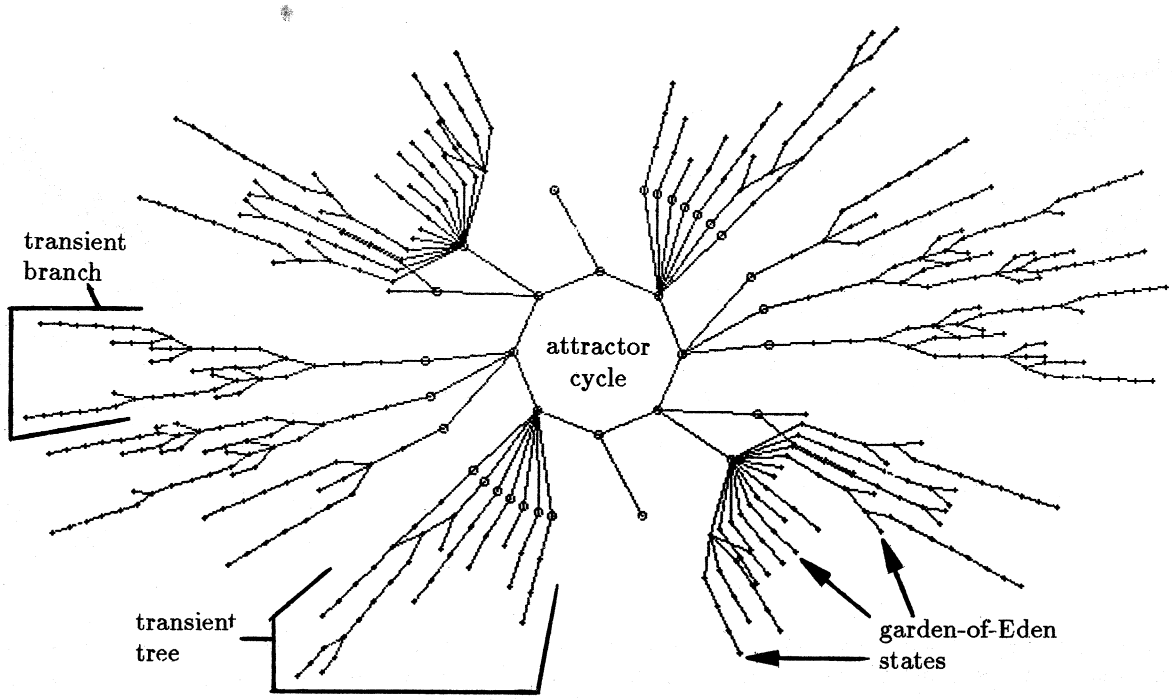


FIGURE 2.4 A state transition graph—basin of attraction (5-neighbour totalistic code 10, $L = 16$). Evolution proceeds inwards from garden-of-Eden states to the attractor, then clockwise. The graphic conventions are set out in Appendix 1.

("out degree") to a single node (the *successor state*), at the next time step. Nodes with no incoming arcs represent garden-of-Eden states. The number of incoming arcs is referred to as the *degree of pre-imaging* ("in degree").

For a given set of CA parameters, state space will, in a sense, crystallise into a set of one or more basins of attraction. The basin of attraction field is the set of state transition graphs representing all the basins in state space.

The make-up of a typical basin of attraction is illustrated by the state transition graph shown in Fig. 2.4 (it is part of the basin field shown in Fig. 2.6). In our graphic convention (see Appendix 2), the length of transition arcs decreases with distance away from the attractor, and the diameter of the graphic representation of the attractor asymptotically approaches an upper limit with increasing period, so that attractor cycles are drawn with approximately the same diameter irrespective of the number of nodes in the attractor. The forward direction of transitions is inward from garden-of-Eden states to the attractor, which is the only closed loop in the basin, and then clockwise around the attractor cycle.

Typically, the vast majority of nodes in a basin field, or a single basin of attraction, lie on *transient trees* outside the attractor cycle. A transient tree is the set of all paths from garden-of-Eden nodes leading to one node on the attractor cycle (an *attractor node*). A branch of the transient tree is termed a *transient branch*, and is the set of all paths from garden-of-Eden nodes leading to a state within a transient tree. A *transient* is one particular path from an arbitrary node in the transient tree leading to the attractor node. In all cases the attractor node itself is excluded from the definition.

2.2.2 Constructing the State Transition Graph

In our method of determining the topology of a single basin of attraction containing a particular global state, the attractor cycle is first isolated, then the topology of each transient tree is specified. This information is used to draw the state transition graph following a graphic convention described in detail in Appendix 1.

To isolate the attractor cycle, the CA is allowed to evolve forward in time from the selected initial state (the *seed*) until a repeat state is encountered. The number of steps to achieve this is the *disclosure length* (transient +

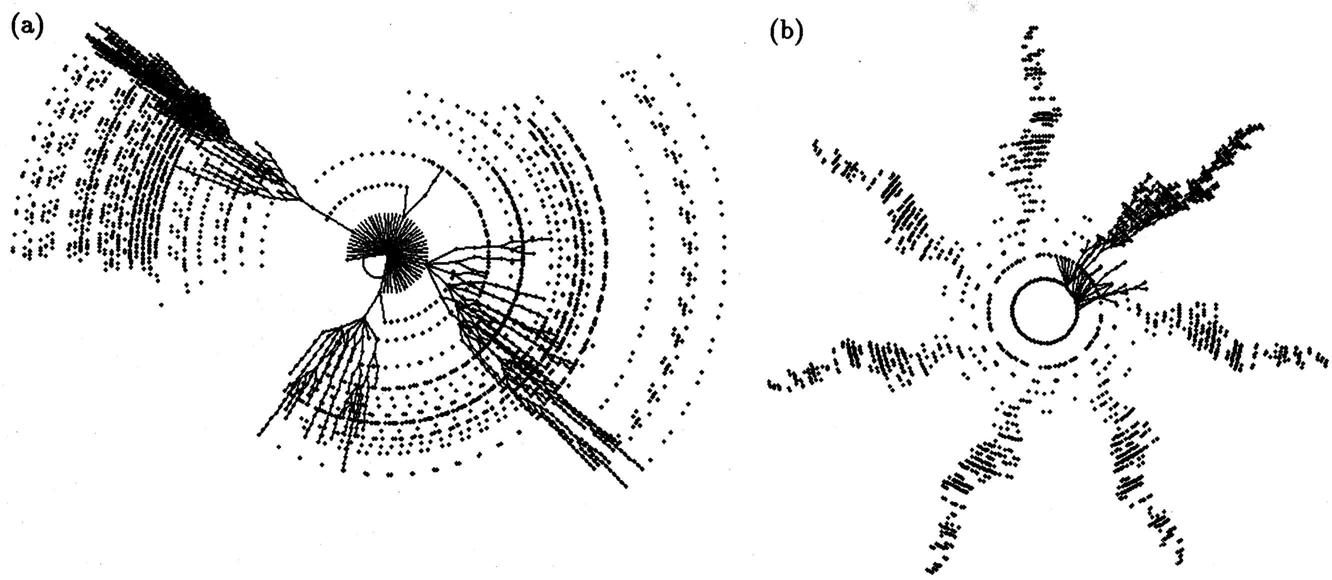


FIGURE 2.5 Examples of basins with (a) a *point attractor* (period 1, that cycles to itself), with suppressed equivalent transient branches [5-neighbour totalistic code 20, L=14, seed all 0s], and (b) a cyclic attractor (period 91), with suppressed equivalent transient trees [3-neighbour rule 193, L=14, singleton seed].

cycle period).[29] The sequence of states from the state that was repeated specifies the central attractor cycle of the basin.

The period of the attractor is generally between 1 and some simple multiple of L, the array length, but may diverge exponentially with L for *limited pre-image rules* (see chapter 3, section 3.4). The attractor period cannot, however, exceed $2^L - M$, where M is the number of states in state space made up of repeating segments on the circular array (see chapter 3, section 3.2).

Once the attractor cycle is known, it is drawn as a circle of nodes, evolving clockwise. To construct transient trees it is necessary to have a method of computing the complete set of pre-images of any node, in other words, to evolve the CA backwards in time. Given that such a *reverse algorithm* is available, the pre-images of an attractor state are computed, but the pre-image lying on the attractor cycle itself is deliberately excluded to prevent endlessly tracing pre-images "backwards" around the attractor cycle.

The reverse algorithm is reapplied repeatedly, to the pre-images of pre-images, until all the *garden-of-Eden states*,[12,14,22,23] those without pre-images, are reached. In this way the full description and topology of a *transient tree* is specified. The transient tree, if any, feeding into each state on the attractor cycle is derived in turn.

The construction of transient trees and branches is simplified by taking advantage of *shift invariance*.[40] There are global states that differ only by a rotation of the circular array. Such *rotation equivalent* states must have equivalent pre-images, rotated by the same amount, and must occupy equivalent positions in the same or an equivalent basin.

If the pre-images to a given state have been computed, the pre-images of its rotation equivalents are known, and by extension so is the entire transient tree (or transient branch), which does not need to be re-computed. If rotation equivalent states belong to separate basins, the basins will be equivalent, so only one example needs to be constructed.

Figure 2.5(a) shows a basin with equivalent transient branches from a point attractor (where all transients belong to a single transient tree). Figure 2.5(b) shows a basin with equivalent transient trees from an attractor cycle. In both cases only one example of a set of equivalent transient trees or branches has been drawn, and the remainder have been suppressed (apart from the point-attractor pre-images in Fig. 2.5(a)). Garden-of-Eden nodes, however, have been retained to indicate the footprints of the suppressed transients.

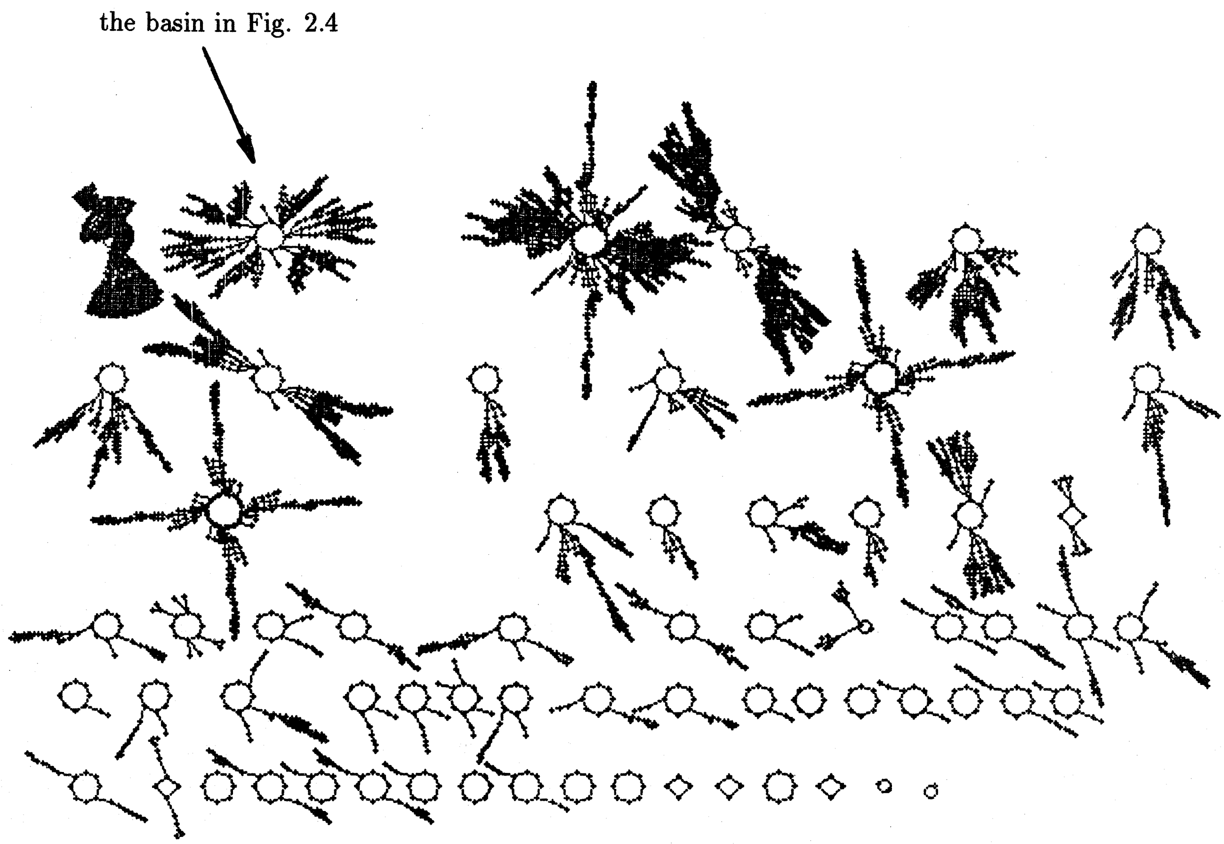


FIGURE 2.6 The basin of attraction field of the 5-neighbour totalistic code 10, $L = 16$.

2.2.3 Constructing the Basin of Attraction Field

In our method of determining the set of basins that make up the basin of attraction field, states that have not been assigned to basins are used to seed new basins until no unassigned states remain.

Figure 2.6 illustrates the basin field of the 5-neighbour rule with totalistic code 10 (see chapter 3, section 3.3), for array length 16. There are 2^{16} global states that are organised by the rule into a basin field consisting of 769 separate basins, in 64 distinct rotation equivalent sets of basins. Only one example of each rotation equivalent set of basins is shown, as they have an identical topology. The basin indicated (one of a set of eight) is illustrated at a larger scale in Fig. 2.4.

In our graphic convention, global CA states in the state transition graph are normally represented as nodes of decreasing size with distance from the attractor. However, the actual binary state of the node, or its decimal equivalent, may be displayed at the node position. (The resolution is subject to the scale of the diagram; see Appendix 1). This allows the explicit global behaviour of CA to be fully described. Figure 2.7 illustrates a basin field with numbered nodes.

2.2.4 Pre-images

The discrete and finite nature of CA allow their basin of attraction fields to be depicted by a diagram that may, in principle, be totally explicit. However, the difficulty in computing the required pre-images for constructing basin of attraction fields has prevented easy access to these objects.

As described above, the basin field can be constructed if the pre-images to each state in state space are known. This information can be derived in general by exhaustive testing; to find the pre-images of a global state S_1, the

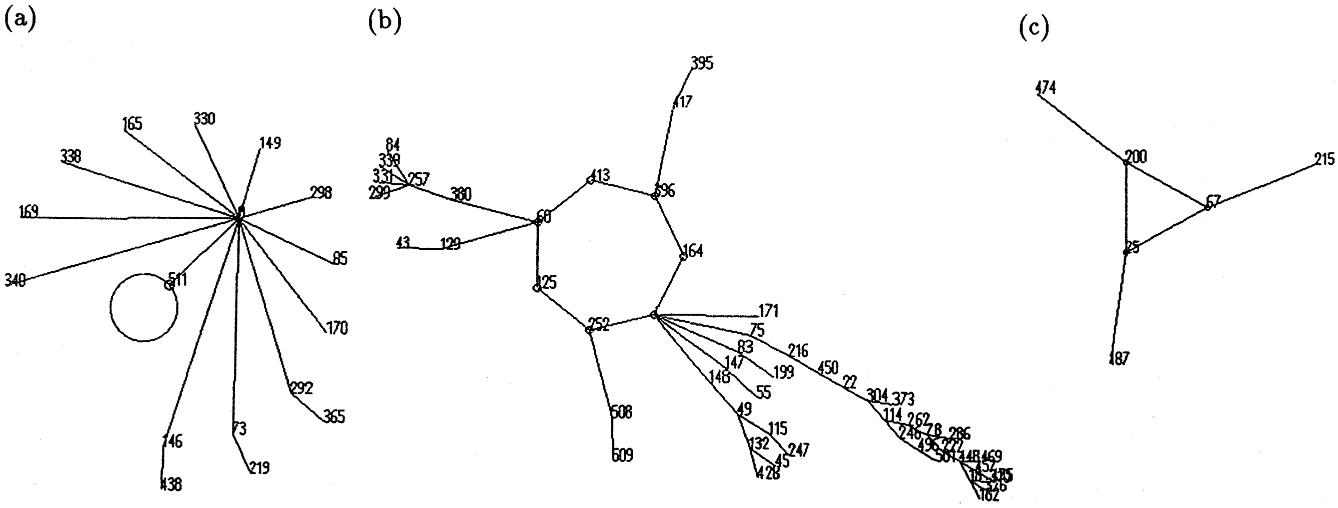


FIGURE 2.7 A basin field with numbered nodes representing the decimal equivalents of the CA binary global states. 3-neighbour rule 193, $L = 9$. The number of basins of each type is (a) 1, (b) 9, and (c) 3. Note that each of the set of basins of a particular type will consist of *rotation equivalent* states, and will have differently numbered nodes accordingly.

CA is evolved one step forward in time from all possible unallocated states. A one-step evolution that results in S_1 provides a pre-image of S_1. However, the number of states to be tested in a binary CA is equal to 2^L, where L is the array length. The corresponding exponential increase in the time required to compute pre-images for successively larger arrays rapidly makes their computation impractical.

Exhaustive testing has in general been the only method available for computing pre-images, thus effectively preventing a systematic view of basin fields. In Chapter 3 a computational shortcut, the *reverse algorithm*, is presented, which allows the pre-images of a CA state to be computed directly, with an average computational performance that is many orders of magnitude faster than explicit testing. This enables real-time computer generation of basin fields, thus providing a new opportunity for CA research.

2.2.5 The Program

A computer program that draws basin of attraction fields, or just a single basin, using the reverse algorithm, providing immediate access to these objects, is included with this book, together with a program that draws space-time patterns.

The programs work for any rule in the set of 1-D, binary, 5-neighbour rules. This includes the *totalistic codes*[34] and the 3-neighbour rules, also known as *elementary* rules.[33,39] The program operating instructions and the graphic conventions are described in detail in Appendix 1.

Space-time patterns are generated in various graphic formats for an array length L, up to $L = 640$. Space-time patterns can also be run "backwards," generating binary strings representing the pre-images of successive global states. Provided that the number of pre-images to a given state is within the limits of the computer system, this may be done for an array length up to $L = 80$.

The basin field version of the program draws a graphic image in real time of all rotation inequivalent basins in state space, for a selected range of L up to 18, together with relevant data. A single basin may be drawn up to $L = 31$, from any seed state that forms part of the basin. Alternatively, a fragment of a graph may be drawn representing only the transient branch leading to a given state. Selected data on each basin and on the basin field may be printed.

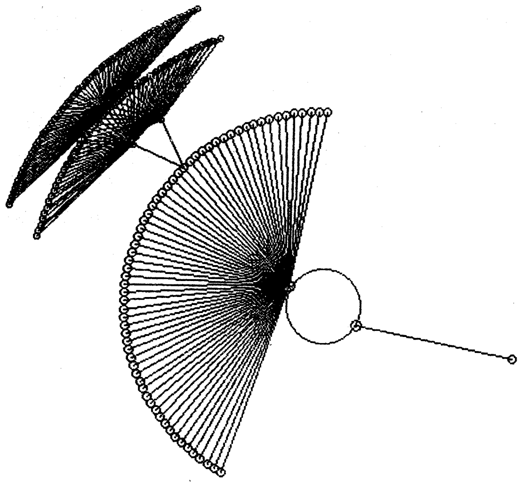


FIGURE 2.8 Example of a basin of the 3-neighbour rule 126, $L = 31$. The seed state is a single 0 followed by six copies of the string 01111.

2.2.6 The Atlas

The program has been used to produce an *atlas* of basin of attraction fields over a range of array lengths, presented in Appendix 2. The Atlas consists of two parts. Part 1 presents all 3-neighbour (elementary) rules. Part 2 presents all 5-neighbour totalistic codes. Selected data is also presented on each basin and basin field. There is an index to rules and codes at the beginning of each part.

A CA rule belongs to a set of up to four *equivalent* rules, the *equivalence class*, that differ only in that they have *negative* and/or *mirror image* space-time patterns, but which have identical basin field topology. Thus, only one rule representing each equivalence class is presented in the Atlas. Pairs of equivalence classes relate in that their rules have *complementary* rule tables, forming a *rule cluster*. The representative rules in the Atlas are presented according to their rule cluster, with complementary equivalence classes shown on facing pages (the relationships between rules are explained in detail in chapter 3, section 3.5)

The 3-neighbour rule clusters belong to one of three *symmetry categories—symmetric*, *semi-asymmetric*, and *fully asymmetric*—and are accordingly presented in three sections. The basin fields for the 88 *equivalence classes* of the 3-neighbour rules are shown for $L = 1$ to 15.

The 36 equivalence classes of the 5-neighbour totalistic codes (which belong only to symmetric clusters) are shown for $L = 3$ to 16.

The atlas may be used as an aid to navigation in exploring the basin field structure of any of the 2^{32} rules in 5-neighbour rule space, by mutating the rule table away from the subsets of rules with known basin fields, and moving into unknown territory.

2.2.7 Significance of Basin of Attraction Fields

The ability to represent basin of attraction fields may be significant in a number of areas such as CA theory, complex systems, dynamical systems, computational theory, artificial life, neural networks, and aspects of genetics.

Basin field topology represents a second order of complexity of CA behaviour, where the first order may be said to be the space-time patterns of particular trajectories. Easy access to a systematic presentation of basin fields, for a synoptic, qualitative, as well as explicit view of global behaviour, may provide insights into the dynamical theory of CA, and the structure of CA rule space. It may provide a useful analogue to the global behaviour of CA with more complex architecture, and dynamical systems in general.

The separate basins in a basin of attraction field classify state space. Attractors have been regarded as "memories," [13,14,15] with implications for the mechanism underlying neural networks.

Basin fields may be of interest in genetics because mutations of the CA *rule table* by a small Hamming distance typically result in altered but related basin structures. Analogies have been made between a CA rule table and a DNA sequence.[20] Kauffman and others study CA with non-local architecture as models in biology and genetics.[14,15]

Attractors have been interpreted as "cell types" in ontogeny. Evolution is said to occur in an optimal "fitness landscape" by mutation and selection of the CA "genotype" resulting in adapted dynamics or "phenotype."

A possible approach to basin of attraction fields is to see them as artificial morphologys, analogous to the "biomorphs" proposed by Dawkins.[4] Their explicit morphological form (including the global CA state at each node) is determined by the genetic code, the rule table; a mutation of the rule table results in a mutant morphology. In addition, the genetic code is implicit in the morphology because the rule table can be reconstructed from space-time patterns; this suggests possibilities for reproduction. Such a genotype-phenotype approach to CA may suggests applications in artificial life.

THREE
The Transition Function and Global Dynamics

3.1 General CA Parameters

In general, a cellular automaton is an array evolving by the iteration of a constant nonlinear function that independently determines the new value of each cell in the array according to the values of a predetermined subset of preceding cells. The array is restricted to one dimension and the definition is narrowed in stages. Fig. 3.1 illustrates a general space-time view of a CA adapted from the system proposed by Ashby.[1]

The cells of a finite array of size L, are restricted to k values (the *value range*), labelled for convenience $0, 1, 2, \ldots, k-1$. Each *target cell* at t_1 is wired to a subset of cells at t_0 and assigned a rule which acts on the set of cells specified by the *wiring diagram* (the order of input wires *is* significant), to determine the target cell's new value. The system may have *disordered architecture* in that both the wiring diagrams and the rules at each cell location may differ.

The updating of cell values is synchronous, and together specify the CA *transition function*, which is fixed over time. Note that *historical time reference* ("higher order in time"[34]), where the wiring diagram may extend to cells at t_{-1} and before, is excluded from this general scheme as it would result in qualitatively different behaviour from that described below.

Although these CA parameters are very broad, the following can be said about behaviour (and extended to two and higher dimensions), if the system is closed to external noise:

1. The global state of the array at t_0 has one and only one successor state at t_1.
2. The global state at t_0 may have 0, 1 or more pre-images at t_{-1}.
3. The total number of global states, the size of the state space, equals k^L.
4. Evolution will end at an attractor cycle, or at a point attractor which cycles on itself.
5. The transition function will organise all space-time trajectories into a basin of attraction field, which can, in principle, be represented by a set of state transition graphs, describing global behaviour.
6. The transition function will have equivalents: two equivalents by reflection (mirror-image space-time patterns, more equivalents for higher dimensions), and $k!$ equivalents by interchanging cell values.

The general scheme is complicated. This architecture may be drastically simplified without sacrificing complexity of behaviour. Walker's "networks of Boolean functions"[27–32] limit the value range to binary and the number of wiring inputs to 3. The target cell at t_1 is wired to itself at t_0, and the remaining two wires set at random, but with the same rule at each cell position. Walker has studied basin field structure by means of random sampling. He has sought to show the variation of basin topology for different rules and array sizes. In particular, he has investigated disclosure

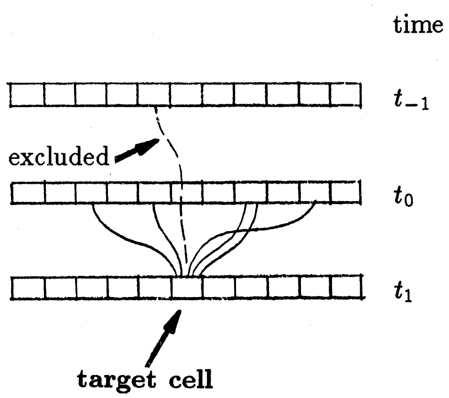


FIGURE 3.1 Disordered 1-D CA with arbitrary wiring from cells at t_0 to a target cell at t_1. The wiring and rule may differ at each target cell, but are constant over time. Wiring between t_1 and t_{-1} is excluded.

length (transient + cycle length) and the *dominance* of state space by the attractor,[32] which is the proportion of states in state space "drained" by an attractor.

Kauffman has investigated "random Boolean networks" as models of genetic systems and evolution.[14,15] In his CA architecture, both rule and wiring diagram (with typicaly only two wires) are set at random at each cell position. He has found that in spite of their random construction, "such systems can spontaneously crystallise enormously ordered dynamical behaviour." [14]

In such *disordered architecture* as described above, the question of defining the boundary conditions does not arise because each cell, including edge cells, has its wiring diagram individually specified.

3.1.1 Local Architecture

The situation is drastically simplified if the wiring diagram and rule are identical at each cell position. In the case of such *ordered architecture*, however, the wiring of cells at the edges of the array, the boundary conditions, needs to be defined. An additional simplification is *local architecture*, with nearest-neighbour wiring; the wiring is confined to an uninterrupted neighbourhood of cells, typically situated locally in relation to the target cell.

Wolfram has investigated local, binary CA, where every cell is wired uniformly to the three neighbouring cells centred on itself, (elementary rules).[33] Boundary conditions are specified as either infinite, fixed, or periodic (though fixing boundary conditions makes the system disordered, because the wiring is atypical at edge cell locations). Considerable descriptive and analytic work has focused on specific rules of this type.[8,9,20,22,33,39–42] Wolfram broadened his investigation by increasing the neighbourhood to 5 cells, and also increasing the value range to 3 for 3-neighbour rules.[34] Langton has investigated a wide variety of local CA architectures with various value ranges, and has suggested the existence of a *phase transition* in rule space.[16]

3.2 Rotation Symmetry

In this investigation, CA parameters are narrowed initially as follows. An ordered architecture, with periodic boundary conditions (edge cells wired to simulate a circular array). Each cell has the *same* (possibly randomly connected) wiring diagram. This architecture is summed up in Fig. 3.2.

Ordered architecture and periodic boundary conditions impose additional restrictions on dynamical behaviour. The *rotation symmetry*[42] (also known as *translational invariance*) is the maximum number of identical sequences of cell values, termed *repeating segments*, into which the circular array can be divided. The size of a repeating segment is the minimum number of cells through which the circular array may be rotated and still appear identical. The rotation symmetry of successive global states in CA evolution cannot decrease, and may only increase in a transient.

The potential for the array to acquire rotation symmetry depends on the *number theoretic properties* of the array length, L, which has general effects on the topology of basin of attraction fields and the length of attractor cycles. Martin et al.[22] drew attention to this feature in relation to the *additive rules*, and noted that it accounts

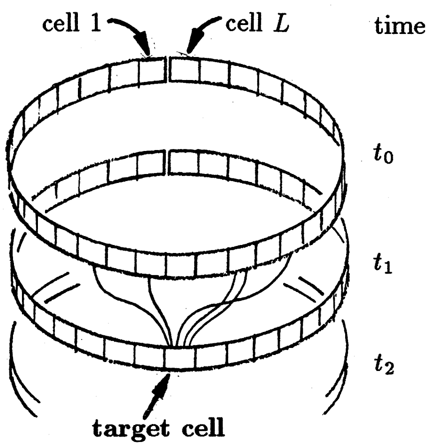


FIGURE 3.2 Ordered CA architecture with periodic boundary conditions. Each cell has the same rule/wiring.

for the irregular variation of attractor periods with L, and the "self-organising" behaviour of CA has been widely noted, for instance[22,34]

```
<--------------L-------------->
<--g-->
0010111,0010111,0010111,0010111
```

An example of a state with rotation symmetry, $s = 4$; value range, $k = 2$.

Consider an array size, L, value range, k, with a total number of states, k^L. If the rotation symmetry is s, and the length of each repeating segment is g, then $s = L/g$. If $s = 1$, then the whole line is one indivisible segment, a *disordered state*, and $g = L$. If all cells have equal value, for instance 11111..., then $s = L$ and $g = 1$. This is the highest possible degree of rotation symmetry, a ***uniform state***. The number of uniform states is equal to the value range, k. If $s > 1$, then the state is a ***segmented state***, where $g \leq L/2$.

Rotation equivalents are global states that differ only by a rotation of the circular array. Disordered states will have the maximum L rotation equivalents, whereas segmented states will have fewer g rotation equivalents.

There is no privileged segment or group of segments that can be treated differently from the rest in a circular array evolving under an ordered rule. Repeating segments are equivalent; therefore, evolution of all segments must be identical, and the following can be said in general about CA evolution:

1. Evolution cannot decrease s, as this would imply inequivalent evolution of segments, which is impossible if all segments must be treated identically.
2. On the attractor cycle, s must remain constant, because any increase would need to be balanced by a decrease as evolution returned around the cycle, and a decrease cannot occur as a consequence of 1.
3. In a transient, s will either remain constant or increase as a consequence of 1. As a consequence of 2, a trajectory where s increases must be a transient. As a consequence of 1, the location of states with greater s in the basin will be downstream, closer to the attractor. Transient evolution (see Fig. 3.3) can proceed in this order, but never in the opposite direction:

disordered states $\longrightarrow$ segmented states, s increasing $\longrightarrow$ uniform states

4. s can only increase by equivalent subdivisions of g into equal parts; therefore, if L or g is prime, s can only increase by a jump to the uniform state.

These general restrictions on behaviour result in the following limits to the maximum attractor cycle period, $P_{\max}$, and maximum disclosure length, $D_{\max}$, in the basin of attraction field of any rule.

1. The k uniform states occur closer to the end of evolution than any other state, and cannot form part of a cycle with a non-uniform state; only a uniform state may be *downstream* of another uniform state; therefore, for any state where $g = 1$ ($s = L$), $P_{\max} \leq k$ and $D_{\max} \leq k$.

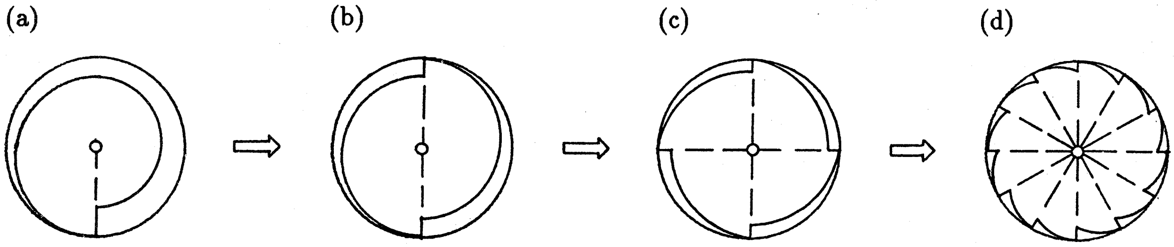


FIGURE 3.3 Possible transient evolution of a disordered state to states with an increasing degree of rotation symmetry. (a) $s = 1$, $g = L$. (b) $s = 2$, $g = L/2$. (c) $s = 4$, $g = L/4$. (d) $s = 12$, $g = L/12$.

2. For any state where $g > 1$, $P_{\max} \leq k^g - k$ and $D_{\max} \leq k^g$.
3. For any array of length L, $P_{\max} \leq k^L - k$ and $D_{\max} \leq k^L$.
4. For any state where $g > 1$, if m is the total of all states of length g which are made up of repeating segments, $P_{\max} \leq k^g - m$ and $D_{\max} \leq k^g$.
5. For any array of length L, if M is the total of all segmented states, $P_{\max} \leq k^L - M$ and $D_{\max} \leq k^L$.

The division of state space into segmented states M, and disordered states $k^L - M$, depends on the number of theoretic properties of the array length, L. The fact remains, however, that many basins are comprised only of disordered states; for prime L all are disordered apart from the uniform states. This leaves open the question of whether evolution among disordered states, and also the more general systems of Walker and Kauffman, follow parallel general principles

A possible approach is to consider *symmetric rules* (which include all *totalistic rules*) which conserve *bilateral symmetry* of the array. The degree of bilateral symmetry may increase in a transient, but must remain constant in the attractor. States with bilateral symmetry may be segmented, or disordered. For instance, the singleton state (a single 1 among 0s) is a disorderd state by our definition, but has bilateral symmetry (see section 3.3.6).

3.3 Rule Clusters

CA parameters may, finally, be restricted as follows: a binary value range ($k = 2$) and local architecture, thus nearest-neighbour wiring, and periodic boundary conditions (a circular array). For a neighbourhood size n, the number of different neighbourhoods equals 2^n, and the number of different rules equals 2^{2^n}.

3.3.1 Rule Numbering System, n=3 (Elementary Rules)

The $k = 2$, $n = 3$ rules have the form,

$$P_i^{t+1} = f(P_{i-1}^t, P_i^t, P_{i+1}^t)$$

where $P_i = 0$ or 1, i is the spatial position between 1 and L, and t is the time. For a circular array, length L, $P_1 = P_{L+1}$.

The system may be represented as follows:

t_0	– – A B C –	B C – – – A	C – – – A B	The neightbourhood **ABC** at t_0
t_1	– – – T – –	T – – – – –	– – – – – T	defines the cell **T** at t_1.

The $n = 3$ *rule table* with $2^3 = 8$ entries uniquly specifies each individual rule from a total of $2^{2^3} = 256$. Following Wolfram's convention[33] the rule table is ordered in descending values of the binary neighbourhood strings.

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

If the rule table is regarded as a binary number, the rule number, R, is its decimal equivalent; thus, the rule table will range from 00000000 to 11111111, and R from 0 to 255.

3.3.2 Complementary Transformation, n=3

Every rule, R, has a distinct *complementary* rule, R_c, where each entry in the rule table is inverted, so that for a given input line, the next line generated by R_c will be the *negative* of the next line generated by R. R and R_c may have equivalent behaviour (see collapsed clusters below).[1] In any case, their behaviour will be closely related. This will be reflected in the basin field structure. Deterministic structure and symmetry category (see below) will be common to R and R_c. In general, pre-imaging, cycle periods, and transient lengths will be related.

The $n = 3$ rules can be listed in 128 complementary pairs.

if $R =$	0,	1,	2,	$\cdots$,	127
	|	|	|		|
$R_c =$	255,	254,	253,	$\cdots$,	128

There are two types of symmetry that relate pairs of neighbourhoods:

Complementary neighbourhood pairs	$\cdots$	111, 000	T_7, T_0
(0s changed to 1s and vice versa)		110, 001	T_6, T_1
		101, 010	T_5, T_2
		100, 011	T_4, T_3
Reflected neighbourhood pairs	$\cdots$	110, 011	T_6, T_3
(mirror image)		100, 001	T_4, T_1

3.3.3 Negative Transformation, n=3

For any rule R and input line I, the CA will generate a space-time pattern P. There is a rule, R_n, that, given the negative input line, $\overline{I}$ will generate the negative space-time pattern, $\overline{P}$ (all cell values inverted).

Consider the rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

To find R_n suppose a *negative world* in which the rule table, including neighbourhoods, could be transformed by inverting all values into

	000	001	010	011	100	101	110	111	neighbourhoods
Rule table..	$\overline{T_7}$	$\overline{T_6}$	$\overline{T_5}$	$\overline{T_4}$	$\overline{T_3}$	$\overline{T_2}$	$\overline{T_1}$	$\overline{T_0}$	outputs

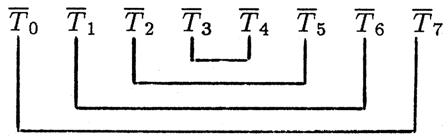
The conventional order of neighbourhoods has been altered and must be restored, giving

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	$\overline{T_0}$	$\overline{T_1}$	$\overline{T_2}$	$\overline{T_3}$	$\overline{T_4}$	$\overline{T_5}$	$\overline{T_6}$	$\overline{T_7}$	outputs

[1]Although negative and complement may have the same meaning, changing 0s for 1s (permutation of values), we use *negative* (as in photographic negative) for transformed CA space-time patterns, and *complement* for transformed rule tables or neighbourhoods.

The transformed rule table is R_n. Thus the procedure to transform R to R_n is as follows in either order.

1. Take the complement of the rule table, R_c.
2. Swap the output of complementary neighbourhoods.



For example, rule 193-11000001 is transformed to rule 124-01111100.

3.3.4 Reflection Transformation, n=3

For any rule, R, input line I and space-time pattern P, there is a rule R_r that, given the reflected input line, I_r will generate the reflected (mirror-image) space-time pattern P_r.

Consider the rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

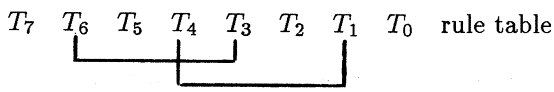
To find R_r suppose a *mirror image world* in which the rule table, including neighbourhoods would be transformed by reflection into

	000	100	010	110	001	101	011	111	neighbourhoods
Ruletable..	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	outputs

The conventional order of neighbourhoods has been altered and must be restored, giving

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_3	T_5	T_1	T_6	T_2	T_4	T_0	outputs

The transformed rule table is R_r. Note that only the outputs of asymmetric neighbourhoods are altered. Thus to transform R to R_r, swap the outputs of the two pairs of asymmetric reflected neighbourhoods.



For example, rule 193-11000001 is transformed to rule 137-10001001.

3.3.5 Symmetry Categories

The reflection transformation allows rules to be placed into one of three symmetry categories:

1. *symmetric rules* ($R = R_r$), if $T_6 = T_3$ and $T_4 = T_1$.
2. *semi-asymmetric rules*, if either $T_6 \neq T_3$ or $T_4 \neq T_1$, but not both.
3. *fully asymmetric rules*, if $T_6 \neq T_3$ and $T_4 \neq T_1$.

Symmetric rules have space-time patterns whose structures appear to have no bias to move left or right; for semi-asymmetric rules, there is a clear bias towards either the left or right; and for fully asymmetric rules, there is an intersecting bias towards both left and right.

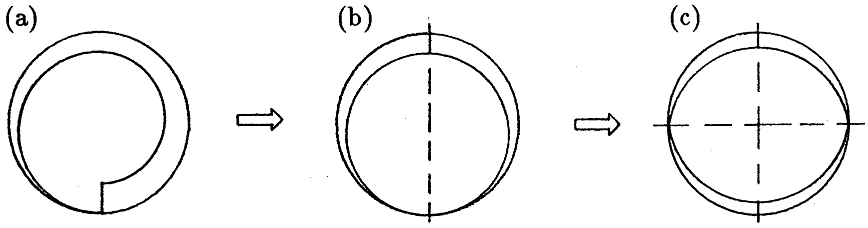


FIGURE 3.4 Possible transient evolution of a disordered state to a state with bilateral symmetry, *bs* (rotation symmetry, *s*). (a) $bs = 0$, $s = 0$. (b) $bs = 2$, $s = 0$. (c) $bs = 4$, $s = 2$.

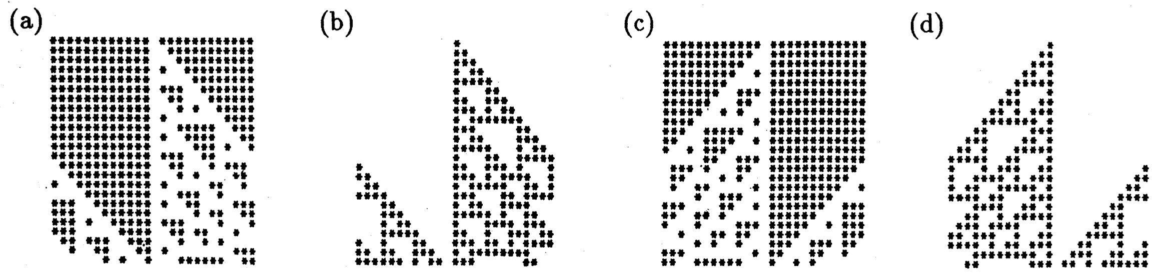


FIGURE 3.5 Equivalent space-time patterns, from equivalent rules. (a) rule R (193). (b) rule R_n (124). (c) rule R_r (137). (d) rule R_{nr} (110).

3.3.6 Bilateral Symmetry of Symmetric Rules

The space-time patterns of symmetric rules, given an input line with *bilateral symmetry* (*bs*), must conserve bilateral symmetry, because the rule acts equivalently on both sides of the axis of symmetry.

As with rotation symmetry (see section 3.2), the degree of bilateral symmetry cannot decrease; in an attractor, bilateral symmetry must remain constant, because any increase would have to be balanced by a decrease as evolution returned around the attractor cycle, and a decrease cannot occur. Bilateral symmetry may increase only in a transient (see Fig. 3.4).

The bilateral axis may divide a circular array, length L, as follows: if L is even, the axis may divide the array into two reflected halves, for instance 0001 – 1000. If the maximum disclosure length is $d_{\max}$, then $d_{\max} \leq 2^{L/2}$. If L is odd, the axis may divide the array into two reflected halves bisecting one cell, for instance 000$\underline{1}$000, then $d_{\max} \leq 2^{((L-1)/2)+1}$. Alternatively, if L is even, there may be more than one axis of bilateral symmetry, for instance 0$\underline{0}$0$\underline{1}$0$\underline{0}$0$\underline{1}$. Such an array has rotation symmetry as well as bilateral symmetry. The interaction between rotational and bilateral symmetry and consequences on behaviour is still unclear. All *totalistic rules* are symmetric (see 3.3.10).

3.3.7 Equivalence Classes

We have seen that a given rule, R, will have two equivalent rules, R_n and R_r. R will also have a third equivalent, R_{nr}, derived by performing the negative and reflection transformations successively, in either order. There will thus be a maximum of four rules grouped in an *equivalence class*; an example is shown in Fig. 3.5, based on rule 193.

The equivalent rules differ only in that they have negative and mirror-image space-time patterns; otherwise, CA behaviour is totally equivalent. There are 88 *equivalence classes* among the 256 $n = 3$ rules.[28,42] Basin field topology for the rules in each class will be identical, though the actual states will differ according to the transformations described.

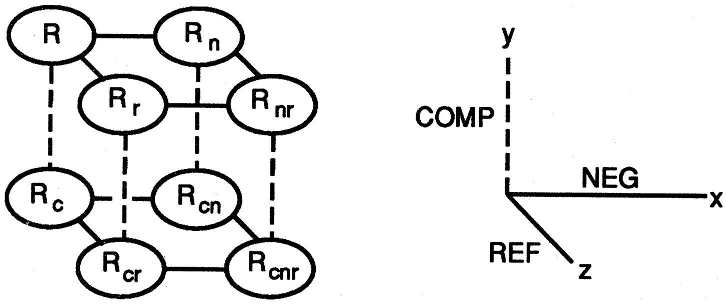


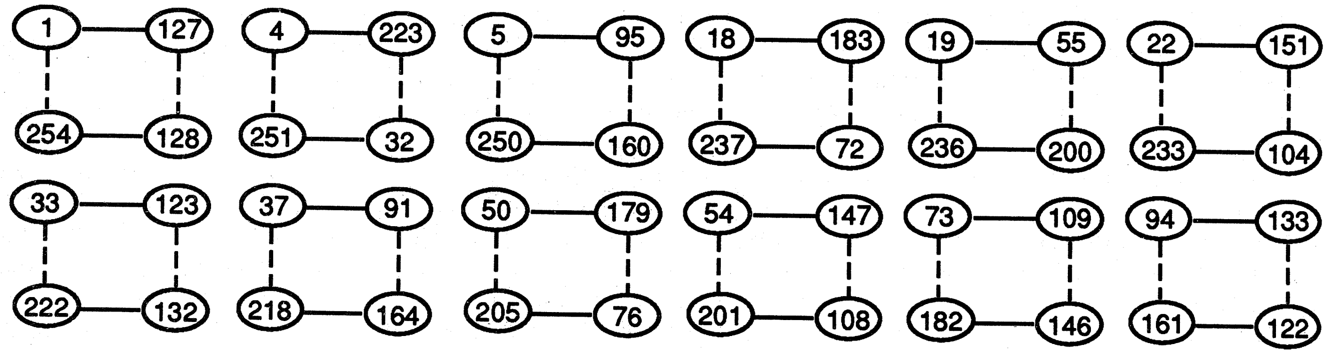
FIGURE 3.6 A rule cluster, two complementary sets of four equivalent rules.

3.3.8 Rule Clusters

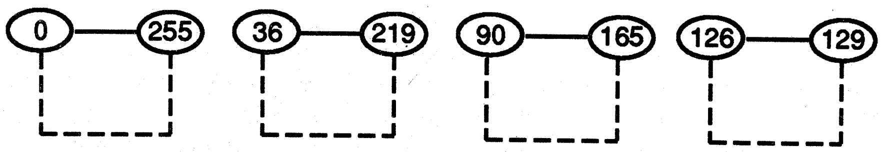
Every rule R has a distinct complement R_c. The 4-rule equivalence class relates to a complementary 4-rule equivalence class, resulting in an 8-rule cluster. We depict the cluster as a box with the rules at each corner (Fig. 3.6), with complementary links along the y axis (dashed line), negative links along the x axis, and reflection links along the z axis.

The rules in a rule cluster are described by two basin fields, one for the top and one for the bottom layer. The outcome of transformations may produce the same rule, resulting in more than one occurrence of that rule in the rule cluster. In this case the cluster is shown collapsed, so that each rule only occurs once. For the purpose of reference, the lowest rule number identifies the cluster, and is positioned in the top left-hand corner. The rule clusters for all $n = 3$ (elementary rules) are set out below.

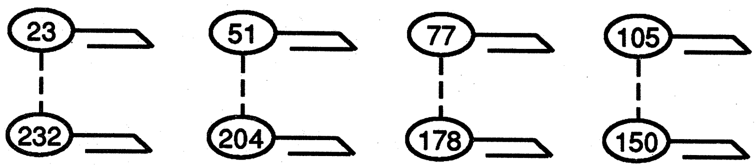
SYMMETRIC RULE CLUSTERS ($T_6 = T_3$ and $T_4 = T_1$). By definition, $R = R_r$, so the reflection links (z axis) will collapse.



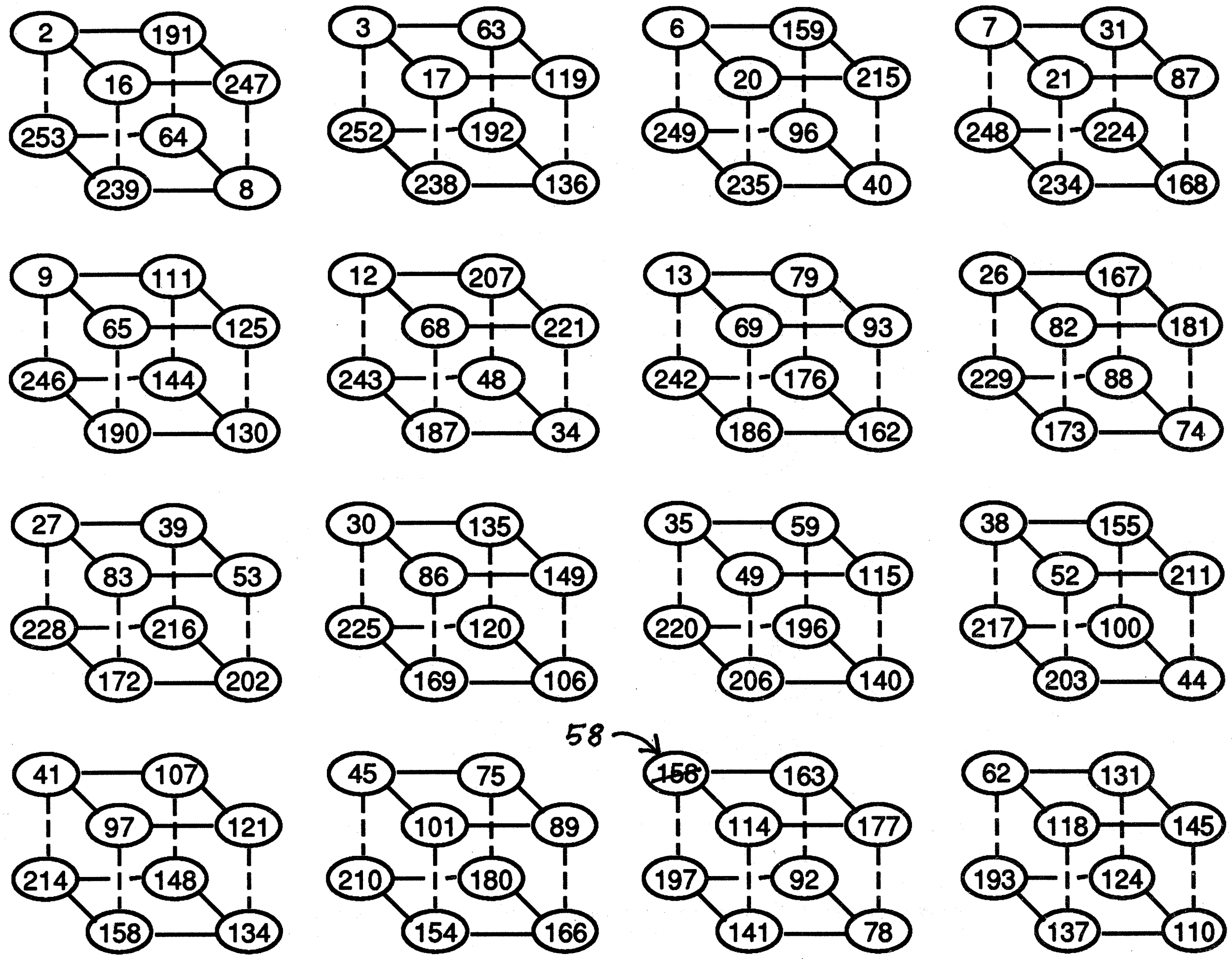
The cluster will collapse further, if for a given rule R, $R_c = R_n$,



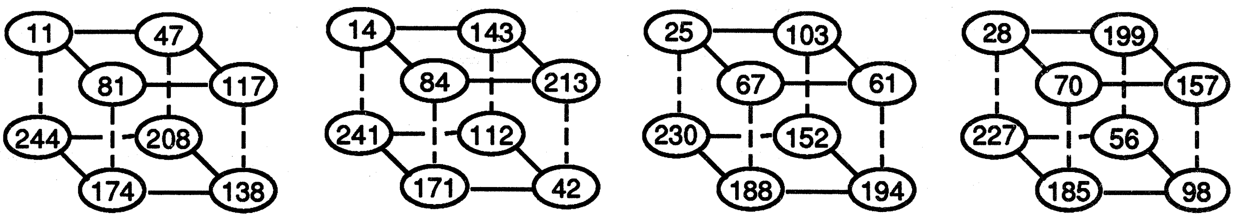
and also if $R = R_n$,



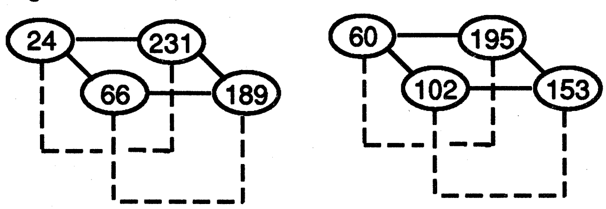
SEMI-ASYMMETRIC RULE CLUSTERS (either $T_6 \neq T_3$ or $T_4 \neq T_1$). There are no collapsed clusters among the semi-asymmetric rules.



FULLY ASYMMETRIC RULE CLUSTERS ($T_6 \neq T_3$ and $T_4 \neq T_1$).



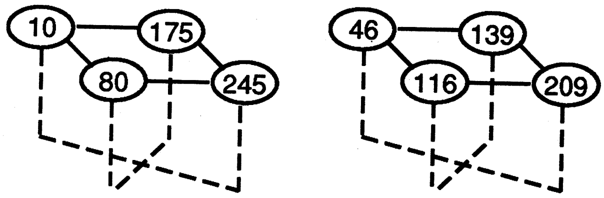
The cluster will collapse if, for a given rule R, $R_c = R_n$,



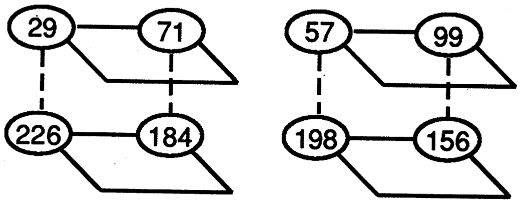
(continued)

Fully asymmetric rule clusters (continued):

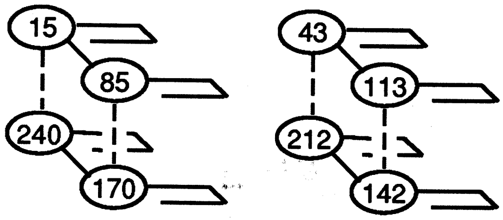
And if, for a given rule R, $R_c = R_{nr}$,



And if, for a given rule R, $R_n = R_r$,



And also if $R = R_n$,



To summarise, the 256 rules collapse into 88 equivalence classes[20,28,42] and 48 rule clusters in Table 3.1.

TABLE 3.1

	rules	equiv. classes	clusters
Symmetric rules	64	36	20
Semi-asymmetric rules	128	32	16
Fully asymmetric rules	64	20	12
Total	256	88	48

The 256 rules have been tabulated on a 16 × 16 "rule-space matrix" (Appendix 4). Manipulations of the matrix simulate the clustering transformations.

3.3.9 Rule Numbering System, n=5

The $k = 2$, $n = 5$ rules have the form,

$$P_i^{t+1} = f\left(P_{i-2}^t, P_{i-1}^t, P_i^t, P_{i+1}^t, P_{i+2}^t\right)$$

where $P_i = 0$ or 1, i is the spatial position between 1 and L, and t is the time. For a circular array, length L, $P_1 = P_{L+1}, P_2 = P_{L+2}, P_3 = P_{L+3}$.

The relationship between the target cell T and the neighbourhood, including periodic boundary conditions, is shown below.

t_0	–	–	$\mathbf{a_1}$	$\mathbf{a_2}$	$\mathbf{a_3}$	$\mathbf{a_4}$	$\mathbf{a_5}$	–	–
t_1	–	–	–	–	**T**	–	–	–	–

the neighbourhood $\mathbf{a_1} \ldots \mathbf{a_5}$ at t_0 defines the cell **T** at t_1

t_0	$\mathbf{a_2}$	$\mathbf{a_3}$	$\mathbf{a_4}$	$\mathbf{a_5}$	–	–	–	$\mathbf{a_1}$
t_1	–	**T**	–	–	–	–	–	–

and

$\mathbf{a_5}$	–	–	–	$\mathbf{a_1}$	$\mathbf{a_2}$	$\mathbf{a_3}$	$\mathbf{a_4}$
–	–	–	–	–	–	**T**	–

t_0	$\mathbf{a_3}$	$\mathbf{a_4}$	$\mathbf{a_5}$	–	–	–	$\mathbf{a_1}$	$\mathbf{a_2}$
t_1	**T**	–	–	–	–	–	–	–

and

$\mathbf{a_4}$	$\mathbf{a_5}$	–	–	–	$\mathbf{a_1}$	$\mathbf{a_2}$	$\mathbf{a_3}$
–	–	–	–	–	–	–	**T**

The $n = 5$ *rule table* with $2^5 = 32$ entries specifies a unique rule out of a total of $2^{2^5} = 2^{32} = 4,294,967,296$ rules. The rule table is ordered in descending values of the binary neighbourhood strings.

$\mathbf{a_1}$ –	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$\mathbf{a_2}$ –	1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0	1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
$\mathbf{a_3}$ –	1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0	1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0
$\mathbf{a_4}$ –	1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0	1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0
$\mathbf{a_5}$ –	1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0	1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
	|	|
T..	31...	...3 2 1 0

The rule number, R, is the decimal equivalent of the binary string $T_{31} \ldots T_0$. R will range from 0 to $2^{32} - 1$.

Note that $n = 3$ rules may be expressed as $n = 5$ rules by assigning each $n = 3$ rule table entry to four separate $n = 5$ rule table positions as specified below. For example, the $n = 3$ rule table entry T_7 is assigned to positions 31, 30, 15, 14 in the $n = 5$ rule table.

$n = 3, T \ldots$	T_7	T_6	T_5	$\cdots$	T_0
	|	|	|		|
$n = 5, T \ldots$	$31 = 30 = 15 = 14,$	$29 = 28 = 13 = 12,$	$27 = 26 = 11 = 10,$	$\ldots$	$17 = 16 = 1 = 0$

$n = 5$ rules with this rule table structure belong to the subset of the 256 $n = 3$ rules. For example, the $n = 3$ rule 193, 1100-0001, may be expressed as the $n = 5$ rule 4026789891, 1111000000000011-1111000000000011.

COMPLEMENTARY TRANSFORMATION, n=5. The $n = 5$ rules can be arranged in complementary pairs, shown linked by a vertical line,

if $R =$	0,	1,	2,	$\ldots$	$(2^{32}/2) - 1$
	|	|	|		|
$R_c =$	$2^{32} - 1,$	$2^{32} - 2,$	$2^{32} - 3,$	$\ldots$	$2^{32}/2$

Two symmetries relate pairs of neighbourhoods, complementary neighbourhood pairs and reflected neighbourhood pairs, following the same reasoning as in $n = 3$ rules.

NEGATIVE TRANSFORMATION, n=5. To obtain R_n for a given rule R, in either order,

1. Take the complement of the rule, R_c.
2. Swap the output of complementary neighbourhoods pairs, shown linked by a vertical line.

$\overline{T}$..	0	1	2	3	...	15
	|	|	|	|		|
$\overline{T}$..	31	30	29	28	...	16

REFLECTION TRANSFORMATION, n=5. To obtain R_r for a given rule R, swap the outputs of the pairs of asymmetric reflected neighbourhoods, shown linked by a vertical line (note that 3/4 of neighbourhoods are asymmetric):

T..	30	29	28	26	25	24	22	20	18	16	12	8
	|	|	|	|	|	|	|	|	|	|	|	|
T..	15	23	7	11	19	3	13	5	9	1	6	2

The reflection transformation allows rules to be placed into one of three symmetry catagories, as with $n = 3$ rules:

1. *Symmetric rules*: all pairs are equal; evolution will conserve bilateral symmetry from a symmetrical input line.
2. *Semi-asymmetric rules*: some, but not all, pairs are unequal.
3. *fully Asymmetric rules*: all pairs are unequal.

The rules will form rule clusters, equivalence classes and collapsed clusters, as with the three input rules.

All 1-D binary local CA rules cluster in this way. Two- and higher-dimensional CA will have more equivalents by reflection, and CA with a greater range of k will have k! equivalents by interchanging cell values, analogous to negative space-time patterns in binary CA.

3.3.10 Rule Numbering System, n=5 Totalistic Rules (Totalistic Code)

Totalistic rules are a small subset of the $n = 5$ rules where the value of the target cell depends only on the *total* of 1s in the neighbourhood. In $n = 5$ totalistic rules, the total of 1s can range from 0 to 5. These rules were investigated by Wolfram.[34] Following his *totalistic code* convention, the code table, with six entries, is made up of neighbourhoods that are arranged in descending order of the arithmetical sum of 1s. The neighbourhoods are shown together with their decimal equivalents.

			28 – 11100	3 – 00011		
			26 – 11010	5 – 00101		
			25 – 11001	6 – 00110		
			22 – 10110	9 – 01001		
			21 – 10101	10 – 01010		
		30 – 11110	19 – 10011	12 – 01100	1 – 00001	
		29 – 11101	14 – 01110	17 – 10001	2 – 00010	
		27 – 11011	13 – 01101	18 – 10010	4 – 00100	
		23 – 10111	11 – 01011	20 – 10100	8 – 01000	
neighbourhoods	31 – 11111	15 – 01111	7 – 00111	24 – 11000	16 – 10000	0 – 00000
total of 1s	5	4	3	2	1	0
code table	T_5	T_4	T_3	T_2	T_1	T_0

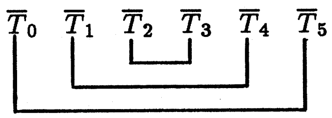
There are $2^6 = 64$ totalistic rules. The binary string $T_5 \ldots T_0$, with value 000000 to 111111, and decimal equivalents 0 to 63, is the totalistic code C. To transform a totalistic code table to an $n = 5$ rule table, the code table outputs are assigned to neighbourhoods as listed above.

Given an $n = 5$ rule table, if $T \ldots$ $(15 = 23 = 27 = 29 = 30)$ and $(7 = 11 = 13 = 14 = 19 = 21 = 22 = 25 = 26 = 28)$ and $(3 = 5 = 6 = 9 = 10 = 12 = 17 = 18 = 20 = 24)$ and $(1 = 2 = 4 = 8 = 16)$, then the rule is totalistic.

All totalistic rules are *symmetric rules* ($C = C_r$), because the total of 1s in pairs of reflected neighbourhoods must be equal. Totalistic rules thus conserve bilateral symmetry in their space-time patterns. Complementary and negative transformations are contained within the set of totalistic rules.

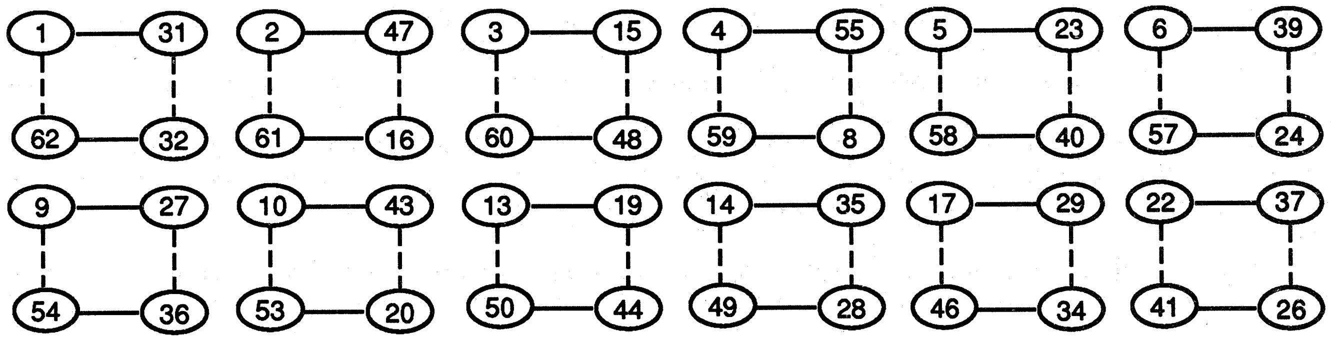
NEGATIVE TRANSFORMATION, n=5 TOTALISTIC CODE. To obtain C_n for a given code C, in either order,

1. Take the complement of the code, C_c.
2. Swap the output of complementary *totals* (totals of 0s and totals of 1s).

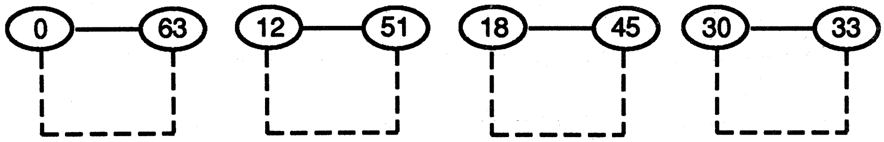


For example, code 53-110101 is transformed to code 20-010100.

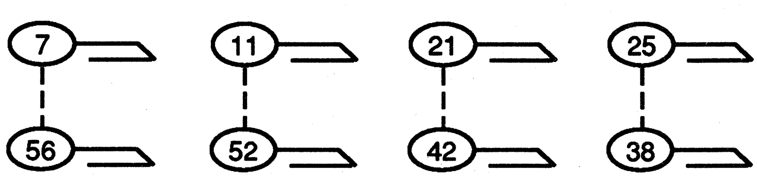
TOTALISTIC CODE CLUSTERS. The 64 totalistic codes can be organised into 36 *equivalence classes* and 20 *symmetric code clusters* as shown below. Codes on the top or bottom layer of a cluster are equivalent, and will produce negative space-time patterns from a negative input line; the topology of their basin fields will be identical. Codes 0 and 63 are the same rules as the $n = 3$ rules 0 and 255.



The cluster collapses further where, for a given rule C, $C_c = C_n$



and also if $C = C_n$



3.4 Limited Pre-image Rules

The CA rule determines the one successor state to a given state; thus, the CA's subsequent evolution is totally determined, though supposed in general to be computationally irreducible.[35-37,40] Constructing the basin of attraction field poses the problem of finding the set of all possible global states that could have preceded a given global state, its *pre-images*[12] (or predecessors[22,42]).

This problem is recognised as being very difficult,[35,36,40] requiring, in general, the exhaustive testing of the entire state space. Such a procedure becomes impractical in terms of computer time as the array length increases beyond modest values. It has been suggested that for particular rules with "simple, algebraic, structure, an efficient inversion procedure. . .may exist." [38] Algebraic analysis of this possibility has been pursued by Martin et al.[22] and Jen.[9,12] Jen has shown that rule specific formulae can be obtained for the exact number of pre-images for any sequence on an infinite array.[12]

The *reverse algorithm*,[42] described in this paper, directly computes all of the pre-images of a global state or directly computes that the state is a garden-of-Eden state. The average computational performance is many orders of magnitude faster than exhaustive testing.

The simplest form of the procedure applies to a subset of rules with simple algebraic structure, the *limited pre-image rules* (*limited branching rules*).[42] They include the *additive rules*[22,33] and correspond to the subset of rules that exhibit *deterministic structure* identified among the 3-neighbour rules by Jen,[8] using a similar approach.

3.4.1 Limited Pre-image Rules in General

A rule table is a list, in a conventional order, of all possible neighbourhoods and their outputs to the target cell (in the following argument, the order is irrelevant). Given a neighbourhood size of n, the rule table will have 2^n entries (examples in section 3.6). In 1-D local binary CA, the rule table may be organised into pairs of neighbourhoods that are identical except for their *rightmost* value; this identical segment is the *left start string* of the neighbourhood (conversely, the *right start string*). Such a pair of neighbourhoods may have the same or different outputs to the target cell.

If all the pairs of such neighbouhoods in the rule table are of the type that have different outputs, then the rule is defined as a *limited pre-image rule*. Note that in a limited pre-image rule the number of 0s and 1s in the rule table will, by definition, be equal, but a rule table with equal numbers of 0s and 1s is not necessarily a limited pre-image rule.

Consider a neighbourhood of size n, $\mathbf{a_1, a_2, \ldots, a_n}$, and target cell $\mathbf{T}$ (where $\longrightarrow$ signifies the output to the target cell, and $\overline{\mathbf{T}}$ signifies **not** $\mathbf{T}$). If a pair of neighbourhoods with the same *left start string* have *different* outputs...

$$\begin{aligned} &\mathbf{a_1, a_2, \ldots, a_{n-1}, 1} \longrightarrow \mathbf{T} \\ \text{and}\quad &\mathbf{a_1, a_2, \ldots, a_{n-1}, 0} \longrightarrow \overline{\mathbf{T}}, \end{aligned}$$

then each of the pair of rule table entries has a *left deterministic permutation*, because

$$\mathbf{a_n} \text{ is determined, given } \mathbf{a_1, a_2, \ldots, a_{n-1}} \text{ and } \mathbf{T}.$$

Conversely, if a pair of neighbourhoods with the same *right start string* have different outputs

$$\begin{aligned} &\mathbf{1, a_2, a_3, \ldots, a_n} \longrightarrow \mathbf{T} \\ \text{and}\quad &\mathbf{0, a_2, a_3, \ldots, a_n} \longrightarrow \overline{\mathbf{T}}, \end{aligned}$$

then each of the pair of rule table entries has a *right deterministic permutation*, because

$$\mathbf{a_1} \text{ is determined, given } \mathbf{a_2, a_3, \ldots, a_n} \text{ and } \mathbf{T}.$$

Following Jen's terminology[8] a rule table where *all* entries have a deterministic permutation (either left, right, or both), has *deterministic structure* of the corresponding direction. A rule with, say, left deterministic structure allows a simple inverse procedure for deriving pre-images.

If the *left start string* of the candidate pre-image line, $\mathbf{a_1, a_2, \ldots, a_{n-1}}$, is assumed, its continued derivation from *left to right* is completely determined, and its validity decided only by whether or not it complies with periodic boundary conditions. (The converse procedure is from *right to left*).

The length of the start string equals $n-1$. The number of possible start strings that need to be assumed is therefore 2^{n-1}, so this number of iterations of the procedure is required, resulting in, at most, one pre-image per iteration, irrespective of the length L of the array.

For limited pre-image rules, the maximum number of pre-images to any state in the basin field (*maximum pre-imaging*) is therefore 2^{n-1}, irrespective of L. However, it is usually less. To exhibit the maximum number of pre-images, a rule must conform to both the left *and* right deterministic structure. Rules with such *two-way* deterministic structure overlap with the much investigated[14,22,33] $n = 3$ *additive* rules,[2] that have proved amenable to algebraic analysis.[22] Such rules have highly regular and predictable unfolding of their basin fields with increasing array length L, and basin field topology is highly sensitive to the number theoretic properties of L.

The vast majority of $n > 3$ limited pre-image rules have either left or right deterministic structure, but not both. Such rules with *one-way* deterministic structure, say *right*, must necessarily have a rule table where some permutations of the *left start string* and $\mathbf{T}$ do not appear. Such *excluded permutations* will occur in conjunction with two *ambiguous permutations*, where a pair of neighbourhoods with the same *left start string* have the *same* output; a different output combined with the start string must be excluded. For example,

$$\begin{array}{ll} \text{if} & \mathbf{a_1, a_2, \ldots, a_{n-1}, 0} \longrightarrow \mathbf{T} \\ \text{and} & \mathbf{a_1, a_2, \ldots, a_{n-1}, 1} \longrightarrow \mathbf{T} \quad \text{(the same output)}, \end{array}$$

then each of the pair of rule table entries is described as having a *left ambiguous permutation*, because,

$$\text{given } \mathbf{a_1, a_2, \ldots, a_{n-1}} \text{ and } \mathbf{T},\ \mathbf{a_n} \text{ could be 0 or 1 with equal validity;}$$

consequently, the permutation

$$\mathbf{a_1, a_2, \ldots, a_{n-1}} \longrightarrow \overline{\mathbf{T}}$$

does not appear in the rule table, and is referred to as a *left excluded permutation.*

Conversely, if a pair of neighbourhoods with the same *right start string* have the same output, that is

$$\begin{array}{ll} \text{if} & \mathbf{0, a_2, a_3, \ldots, a_n} \longrightarrow \mathbf{T} \\ \text{and} & \mathbf{1, a_2, a_3, \ldots, a_n} \longrightarrow \mathbf{T} \quad \text{(the same output)}, \end{array}$$

then each of the pair of rule table entries is described as having a *right ambiguous permutation*, because,

$$\text{given } \mathbf{a_2, a_3, \ldots, a_n} \text{ and } \mathbf{T},\ \mathbf{a_1} \text{ could be 0 or 1 with equal validity;}$$

consequently, the permutation

$$\mathbf{a_2, a_3, \ldots, a_n} \longrightarrow \overline{\mathbf{T}}$$

does not appear in the rule table, and is referred to as a *right excluded permutation.*

In a *one-way* (say *left*) limited pre-image rule (with *right* ambiguous permutations), maximum pre-imaging must be *less* than 2^{n-1}, irrespective of array size. A limited pre-image rule, by definition, has an equal number of 1s and 0s in its rule table; therefore, for every pair of ambiguous permutations with an output of 1, there must be another pair with an ouput of 0. For each pair of ambiguous permutations, there is a corresponding excluded permutation. Therefore, a one-way limited pre-image rule must have at least two excluded permutations with different outputs, 0 and 1, in its rule table.

To derive pre-images of a global CA state (*left to right*), the 2^{n-1} start strings $\mathbf{a_1, a_2, \ldots, a_{n-1}}$ are assumed in turn. However, as has been shown, there will be at least two *right excluded permutations* of the form

$$\begin{array}{rl} & \mathbf{a_2, a_3, \ldots, a_n} \longrightarrow \mathbf{T} \\ \text{and} & \mathbf{a_2, a_3, \ldots, a_n} \longrightarrow \overline{\mathbf{T}}. \end{array}$$

Whatever the value of $\mathbf{T}$, at least one start string is therefore invalid, and maximum pre-imaging for a *one-way* limited pre-image rule must be less than 2^{n-1}.

[2]An example of an $n = 3$ additive rule is rule 90, $\mathbf{T} = \mathbf{a_1} + \mathbf{a_3} \bmod \mathbf{2}$. Other rules considered additive are 150 and 204. Rule 60 is also additive because $\mathbf{T} = \mathbf{a_1} + \mathbf{a_2} \bmod \mathbf{2}$.

In general, CA rules that are not limited pre-image rules will contain a mixture of deterministic and ambiguous (thus also excluded) permutations (both left and right), and maximum pre-imaging will increase, often exponentially, with increasing array size. Ambiguous permutations will amplify the rate of increase; deterministic and excluded permutations will inhibit the rate of increase.

The general reverse algorithm [42] (see section 3.5) applicable to local binary CA architecture uses a combination of these mechanisms, together with the idea of *partial pre-images* (unfinished, valid so far, start segments), also called partial predecessors,[42] to compute all pre-images of a given CA global state for any size of array, provided that the number of partial pre-images is within the limits of the computer system's memory.

3.4.2 Limited Pre-image Rules, n=3

In a 3-neighbour rule table, neighbourhoods that are identical except for their rightmost value, with the same *left start string*, are paired as follows:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

Part of the space-time pattern for one iteration of the CA is represented as

t_0	–	–	**A**	**B**	**C**	–	The neighbourhood **ABC** at t_0
t_1	–	–	–	**T**	–	–	defines the cell **T** at t_1.

If, say, $T_7 \neq T_6$, and the *left start string* $\mathbf{A\,B} = 1\,1$, **A B** and **T** determine **C**. For instance,

$$\text{if } T_7 = 0, T_6 = 1, \text{ and } \mathbf{T} = 0, \text{ then } \mathbf{C} = 1;$$
$$\text{if } T_7 = 0, T_6 = 1, \text{ and } \mathbf{T} = 1, \text{ then } \mathbf{C} = 0;$$
$$\text{if } T_7 = 1, T_6 = 0, \text{ and } \mathbf{T} = 0, \text{ then } \mathbf{C} = 0;$$
$$\text{if } T_7 = 1, T_6 = 0, \text{ and } \mathbf{T} = 1, \text{ then } \mathbf{C} = 1;$$

This covers all possibilities, so each of the pair of rule table entries, where the left start string equals 1 1, is a *left deterministic permutation.*

If all pairs are of the type that have different outputs, i.e.,

$$T_7 \neq T_6 \text{ and } T_5 \neq T_4 \text{ and } T_3 \neq T_2 \text{ and } T_1 \neq T_0,$$

then all rule table entries are *left deterministic permutations*. In these circumstances, any combination of values of **A B** and **T** determine **C**; the rule has *left deterministic structure* and is a limited pre-image rule. This permits pre-images of any given state to be easily derived.

To derive a pre-image of a known global state, a two-cell start segment of the pre-image line (the *partial pre-image*) is assumed, say 0 0. The rule table is consulted to find the known value of **T** in combination with $\mathbf{A\,B} = \mathbf{0\,0}$, giving a unique value of **C**, say 1, the value of the next cell in the partial pre-image. The partial pre-image has been extended to 0 0 1. The procedure is repeated for $\mathbf{A\,B} = \mathbf{0\,1}$ to determine **C**, the value of the next cell, and so on until the whole line plus two extra cells are derived.

If the two extra cells equal the assumed start segment, 0 0, then the pre-image is valid, because periodic boundary conditions are satisfied. Otherwise, the pre-image is not valid. As there are four possible two-cell start segments (00, 01, 10, 11), there are a maximum of four pre-images (2^{3-1}) in an $n = 3$ limited pre-image rule, irrespective of the array size.

Conversely, pre-images may be derived in the opposite direction, from right to left. The rule table, with pairs of neighbourhoods that are identical except for their *leftmost* value, with the same *right start string*, are paired as follows:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

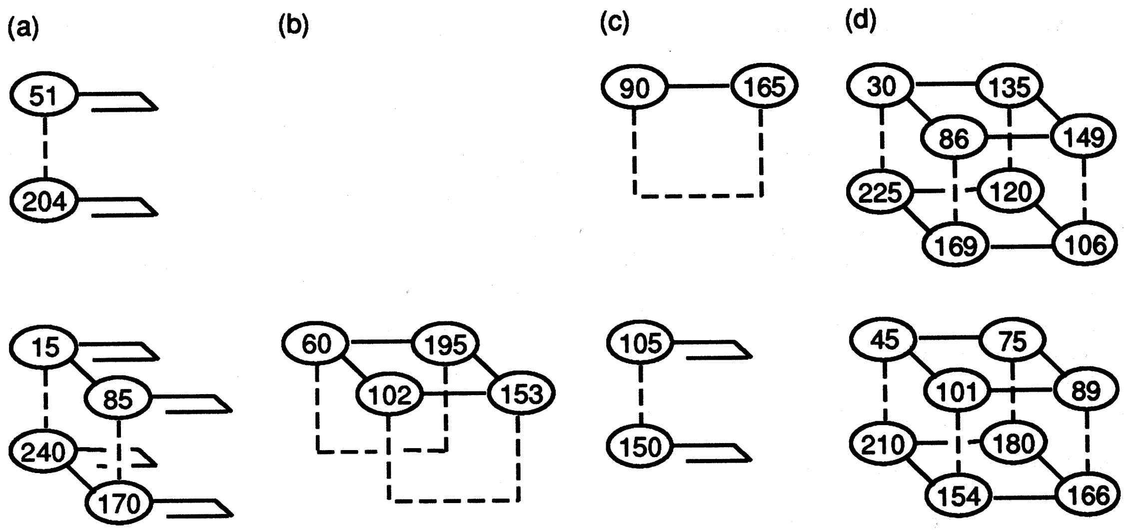


FIGURE 3.7 The set of limited pre-image rule clusters, $n \leq 3$. (a) $n = 1$, two-way, $mp = 1$; (b) $n = 2$, two-way, $mp = 2$; (c) $n = 3$, two-way, $mp = 4$; (d) $n = 3$, one-way, $mp = 3$.

An equivalent argument can be made for *right* (to *left*) deterministic permutations, deterministic structures, and limited pre-image rules.

An example of a rule with *two-way* deterministic structure, both left and right, is rule 150. Its rule table is shown below. Pairs of neighbourhoods with the same *left start string* are indicated with a dotted line (*right start string*, a solid line).

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	1 ••••	0	0 ••••	1	0 ••••	1	1 ••••	0	outputs

Rule 150 has exactly four pre-images for all states (other than garden-of-Eden states), for all array lengths exactly divisible by three (the neighbourhood size). Basin fields for all other array lengths consist only of attractor cycles without transients.[3]

An example of a rule with *one-way*, in this case *right*, deterministic structure is rule 30.

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	0 ::::	0	0 ••••	1	1 ::::	1	1 ••••	0	outputs

There are two pairs of *left* ambiguous permutations (indicated with a double dotted line), which must have different outputs, as the total of 0s and 1s in the rule table is equal (by definition). Thus, there are two *left excluded*

[3]Can a state cycle without transients justify the name *attractor*? After all, it does not *attract* anything outside of itself. We have chosen not to address this problem in terminology.

permutations with different outputs, (if "$*$" is a wildcard, equal to 0 or 1 with equal validity),

$$\text{the ambiguous permutations are..} \quad 11* \longrightarrow 0 \quad \text{and} \quad 01* \longrightarrow 1$$
$$\text{and the excluded permutations are...} \quad 11* \longrightarrow 1 \quad \text{and} \quad 01* \longrightarrow 0$$

To derive pre-images of a global CA state (*right* to *left*), the four possible start segments, 00, 01, 10, and 11, are assumed in turn. Whatever the global state, however, one of these assumptions must be invalid, so maximum pre-imaging is 3. In rule 30, maximum pre-imaging is generally 2; however, for the global states 11111... (all 1s), pre-imaging is 3 for array sizes on the series 3, 6, 12, etc. For further examples of one-way limited pre-image rules, see section 3.5.3.

Figure 3.7 shows all limited pre-image rules among the 3-neighbour rules in their respective *rule clusters*[42] (see section 3.3) and specifies *maximum pre-imaging, mp.* Note that $n = 1$ and $n = 2$ rules are a subsets of, and expressed as, $n = 3$ rules. $n = z$ are always a subset of $n > z$ rules.

3.4.3 Limited Pre-image Rules, n=5

An example of an $n = 5$ rule with *two-way* deterministic structure is the totalistic rule, code 21. The rule table is shown below, in two parts, one below the other.

T.. 31 – 16	0 1	1 0	1 0	0 1	1 0	0 1	0 1	1 0
T.. 15 – 0	1 0	0 1	0 1	1 0	0 1	1 0	1 0	0 1

If horizontally adjacent pairs (as grouped above) are unequal, these neighbourhoods have a *left* deterministic permutation; if vertically adjacent pairs are unequal, the neighbourhoods have a *right* deterministic permutation.

Code 21 has exactly 16 pre-images (the maximum) for all states (other than garden-of-Eden states), for all array lengths exactly divisible by five (the neighbourhood size). Basin fields for all other array lengths consist only of attractor cycles without transients.

An example of an $n = 5$ rule with *one-way* (left) deterministic structure is rule 1771465110. Its rule table is shown below in the same format.

T.. 31 – 16	0 1	1 0	1 0	0 1	1 0	0 1	0 1	1 0
T.. 15 – 0	0 1	1 0	1 0	0 1	1 0	0 1	0 1	1 0

The rule table has 32 left deterministic permutations and 32 right ambiguous permutations, and is based on code 21 but with complementry rule table entries for $T_{15} - T_0$.

3.4.4 Deterministic Permutations, K>2

Deterministic permutations may be generalised for any value range k. The permutation is deterministic (left or right) if, for a neighbourhood size n,

left given $\mathbf{a_1, a_2, \ldots, a_{n-1}}$ and $\mathbf{T}$, $\mathbf{a_n}$ is determined;
right given $\mathbf{a_2, a_3, \ldots, a_n}$ and $\mathbf{T}$, $\mathbf{a_1}$ is determined.

To determine if rule table entries have *left* deterministic permutation, the outputs of the set of k neighbourhoods with the same left start string, $\mathbf{a_1, a_2, \ldots, a_{n-1}}$, and k alternative values of $\mathbf{a_n}$ (*the k-set*) are examined. If the k-set has outputs that all differ, i.e., with one example of each permitted value in the k-set, then it is termed a *left deterministic k-set.* If all k-sets making up the rule table are left deterministic, then the rule is a *left* limited pre-image rule, with maximum pre-imaging less than k^{n-1} irrespective of array size (the converse argument applies to a *right deterministic k-set* and *right* limited pre-image rules). If the rule has two-way deterministic structure (left *and* right), then pre-imaging may equal the maximum of k^{n-1}.

For instance, if $n = 3$ and $k = 3$ (with values 0, 1 and 2), then a segment of the rule table, consisting of a left deterministic k-set, may appear as follows

002	001	000	neighbourhoods
0	1	2	outputs

where each permutation is deterministic, whereas the non-deterministic k-set

002	001	000	neighbourhoods
1	1	2	outputs

consists of two ambiguous permutations $002 \rightarrow 1$ and $001 \rightarrow 1$ and one deterministic permutation $000 \rightarrow 2$.

The example above demonstrates that if $k > 2$, deterministic permutations may exist within a non-deterministic k-set (This is not the case with a $k = 2$, binary CA). The number of deterministic permutations in a k-set depends on the distribution of the k values to the k outputs. If some values are missing, resulting in the same values assigned to several outputs, then there must be some ambiguous (and some excluded) permutations. The *degree of ambiguity* of ambiguous permutations depends on how many neighbourhoods in the k-set share the same output. If only part of the k values are used for the outputs to a k-set, then the number of deterministic permutations in the k-set will equal the number of outputs that occur once only.

As with $k = 2$ rules, ambiguous permutations will amplify the rate of increase of pre-imaging with increasing array size (with greater amplification for a greater degree of ambiguity); deterministic and excluded permutations will inhibit the rate of increase.

3.5 The Reverse Algorithm

This section describes the logic for directly generating pre-images, starting with the *limited pre-image rules*. It was shown in section 3.4 that limited pre-image rules have a fixed maximum number of pre-images, mp, to any node irrespective of array length. For a neighbourhood size n, $mp \leq 2^{n-1}$. The rules may be expressed as simple algorithms relating the target cell to neighbourhood values; these rule algorithms are included below. The limited pre-image rules are examined in the order of increasing n.

3.5.1 Pre-images of n=1 Limited Pre-image Rules

Consider the neighbourhood...	**A B C**
and the target cell	**T**

The trivial $n = 1$ rule subset, expressed as $n = 3$ rules, has neighbourhoods as follows (where "$*$" signifies a wild card, a member of the $n = 3$ neighbourhood that is irrelevant to the target)

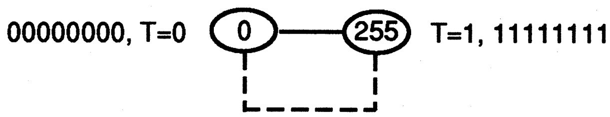
$$*\mathbf{B}*, \quad \mathbf{A}**, \quad **\mathbf{C}.$$

The $n = 1$ rule table consists of 2 entries.

	1	0	neighbourhoods
Rule table..	T_1	T_0	outputs

If $T_1 = T_0$ (an ambiguous permutation), then the rule belongs to the trivial $n = 0$ rule cluster (a subset of $n = 1$). The rule cluster 0 is shown below, where every state in state space is the pre-image of one of the two *uniform states*, all 0s or 1s.

SYMMETRICAL CLUSTER 0.



If $T_1 \neq T_0$, the rule has deterministic structure.

Consider the $n = 3$ rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

Matching wildcard neighbourhoods with the $n = 3$ rule table, we obtain the following $n = 3$ *deterministic structures* for $n = 1$ rules.

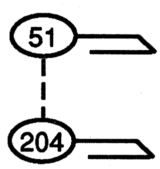
$* \mathbf{B} *$ rules....$(T_7 = T_6 = T_3 = T_2) \neq (T_5 = T_4 = T_1 = T_0)$;
$\mathbf{A} * *$ rules....$(T_7 = T_6 = T_5 = T_4) \neq (T_3 = T_2 = T_1 = T_0)$;
$* * \mathbf{C}$ rules....$(T_7 = T_5 = T_3 = T_1) \neq (T_6 = T_4 = T_2 = T_0)$.

Rules that satisfy these conditions are $n = 1$ limited pre-image rules. They are shown below expressed as $n = 3$ rules, in their respective rule clusters, with the rule table, rule number, and rule algorithm.

SYMMETRICAL CLUSTER 51.

(B), 00110011, T≠B

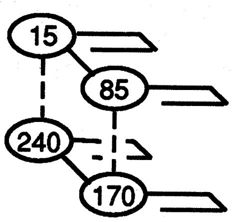
(B), 11001100, T=B



FULLY ASYMMETRIC CLUSTER 15.

(A), 00001111, T≠A

(A), 11110000, T=A



T≠C, 01010101, **(C)**

T=C, 10101010, **(C)**

The behaviour and basin field structure of $n = 1$ rules is predictable and $n = 1$ limited pre-image rules are reversible. Every state must have one pre-image; therefore, there are no garden-of-Eden states nor transients, and all states belong to attractor cycles.

3.5.2 Pre-images of n=2 Limited Pre-image Rules

Consider the neighbourhood... **A B C**
and the target cell **T**

Expressed as $n = 3$ rules, the $n = 2$ rules have neighbourhood as follows, where "$*$" signifies a wild card:

$$\mathbf{A}\,\mathbf{B}\,*, \quad *\,\mathbf{B}\,\mathbf{C}.$$

The $n = 2$ rule table consists of four entries.

	1 1	1 0	0 1	0 0	neighbourhoods
Rule table..	T_3	T_2	T_1	T_0	outputs

Given this rule table, a rule has deterministic structure

$$\text{if } (\textit{left})\ T_3 \neq T_2 \text{ and } T_1 \neq T_0,$$
$$\text{or } (\textit{right})\ T_3 \neq T_1 \text{ and } T_2 \neq T_0.$$

Consider the $n = 3$ rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

Matching wildcard neighbourhoods with the $n = 3$ rule table, we obtain the following $n = 3$ deterministic structures for $n = 2$ rules.

left **A B** $*$ rules.......$(T_7 = T_6) \neq (T_5 = T_4)$ and $(T_3 = T_2) \neq (T_1 = T_0)$;
right $*$ **B C** rules.......$(T_7 = T_3) \neq (T_5 = T_1)$ and $(T_6 = T_2) \neq (T_4 = T_0)$.

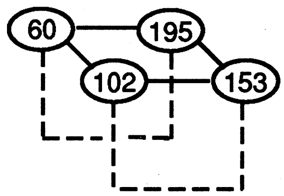
Looking back at the $n = 1$ rules, cluster 51 satisfies both conditions, and cluster 15 satisfies one or the other condition.

The $n = 2$ rules that satisfy these conditions are limited pre-image rules. They are shown below expressed as $n = 3$ rules, in their respective rule clusters, with the rule table, rule number, and rule algorithm.

DOUBLY ASYMMETRIC CLUSTER 60.

(AB), 00111100, if A=B, T=0 else 1 — if A=B, T=1 else 0, 11000011

(BC), 01100110, if B=C, T=0 else 1 — if B=C, T=1 else 0, 10011001



Note that the $n = 2$ rule table for these rules has *two-way* deterministic structure, but the $n = 3$ rule table has *one-way* deterministic structure. For **A B** rules, **A** and **T** will determine **B** (left to right); conversely, for **B C** rules, **B** and **T** will determine **C** (right to left).

Consider an unknown pre-image, $P_0, P_1, P_2, \ldots, P_L, P_{L+1}$ (with notional values P_0 and P_{L+1}) of the known successor line of length L, $I_1, I_2, \ldots, I_L$. As boundary conditions are periodic, $P_0 = P_L$ and $P_{L+1} = P_1$. To derive pre-images, the value of P_0 is assumed, and the rule table used to determine, from left to right, successive values of P_x according to P_{x-1} and I_x.

$P_0 \quad P_1 \quad P_2 \quad \ldots \quad P_{x-1} \quad P_x \quad \ldots \quad P_{L-1} \quad P_L$

$I_1 \quad I_2 \quad \ldots \quad I_x \quad \ldots \quad I_L$

When the final value P_L is derived, if $P_L = P_0$, then periodic boundary conditions are satisfied, and the pre-image is valid. As P_0 can have two assumed values, 0 and 1, each is assumed in turn to generate, at most, one pre-image, resulting in a maximum of two possible pre-images.

Conversely, pre-images may be derived by assuming P_{L+1}, and determining, from ***right*** to ***left***, successive values of P_x according to P_{x+1} and I_x.

$P_1 \quad P_2 \quad \ldots \quad P_x \quad P_{x+1} \quad \ldots \quad P_{L-1} \quad P_L \quad P_{L+1}$

$I_1 \quad \ldots \quad I_x \quad \ldots \quad I_{L-1} \quad I_L$

If $P_1 = P_{L+1}$, the pre-image is valid, and the same conclusions apply as with the left to right procedure.

3.5.3 Pre-images of n=3 Limited Pre-image Rules

Consider the neighbourhood... **A B C**
and the target cell **T**

Consider the $r = 3$ rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

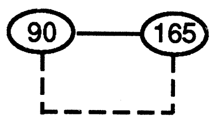
As shown in section 3.4, deterministic structures for $n = 3$ rules (left, right or two-way) are as follows:

left (**AB** rules)..........$T_7 \neq T_6$ and $T_5 \neq T_4$ and $T_3 \neq T_2$ and $T_1 \neq T_0$;
right (**BC** rules)..........$T_7 \neq T_3$ and $T_5 \neq T_1$ and $T_6 \neq T_2$ and $T_4 \neq T_0$;
two-way (**ABC** rules).......both of the above conditions are true.

Looking back at the $n < 3$ rules, the $n = 1$ cluster 15 and the $n = 2$ cluster 60 are either **AB** or **BC** rules. The $n = 3$ limited pre-image rules are set out below.

SYMMETRIC CLUSTER 90.

(ABC), 01011010, if A=C, T=0 else 1

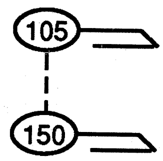


if A=C, T=1 else 0, 10100101

SYMMETRIC CLUSTER 105.

(ABC), 01101001, if A≠C, T=B else $\bar{B}$

(ABC), 10010110, if A=C, T=B else $\bar{B}$



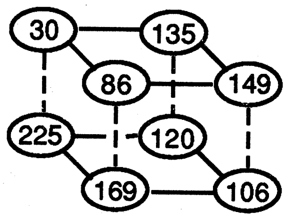
SEMI-ASYMMETRIC CLUSTER 30.

(BC), 00011110, if BC=00, T=A else $\bar{A}$

(AB), 01010110, if AB=00, T=C else $\bar{C}$

(BC), 11100001, if BC=00, T=$\bar{A}$ else A

(AB), 10101001, if AB=00, T=C else $\bar{C}$



if BC=11, T=A else $\bar{A}$, 10000111

if AB=11, T=C else $\bar{C}$, 10010101

if BC=11, T=$\bar{A}$ else A, 01111000

if AB=11, T=$\bar{C}$ else C, 01101010

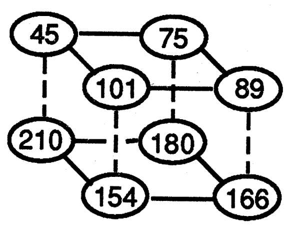
SEMI-ASYMMETRIC CLUSTER 45.

(BC), 00101101, if B<C, T=A else $\bar{A}$

(AB), 01100101, if A>B, T=$\bar{C}$ else C

(BC), 11010010, if B<C, T=$\bar{A}$ else A

(AB), 10011010, if A>B, T=C else $\bar{C}$



if B>C, T=A else $\bar{A}$, 01001011

if A<B, T=C else $\bar{C}$, 01011001

if B>C, T=$\bar{A}$ else A, 10110100

if A<B, T=$\bar{C}$ else C, 10100110

The rules have deterministic structure of the following type,

left **AB** rules, **AB** and **T** will determine **C**;
right **BC** rules, **BC** and **T** will determine **A**;
two-way **ABC** rules, both the above will apply.

To determine the set of pre-images of a given state for **A B** rules, consider an unknown pre-image, $P_0, P_1, P_2, \ldots, P_L, P_{L+1}$ (with notional values P_0 and P_{L+1}) of the known successor line of length L, $I_1, I_2, \ldots, I_L$. As boundary conditions are periodic, $P_0 = P_L$ and $P_{L+1} = P_1$. To derive pre-images, the value of the start string, P_0P_1, is assumed and the rule table used to determine, from left to right, successive values of P_{x+1} according to P_{x-1}, P_x, and I_x.

$$\begin{array}{cccccccccccc} P_0 & P_1 & P_2 & P_3 & \ldots & P_{x-1} & P_x & P_{x+1} & \ldots & P_{L-1} & P_L & P_{L+1} \\ & & & & & \searrow & \mid & \nearrow & \rightarrow & & & \\ & I_1 & I_2 & & \ldots & & I_x & & \ldots & & I_L & \end{array}$$

If $P_L = P_0$ and $P_{L+1} = P_1$, then periodic boundary conditions are satisfied, and the pre-image is valid. As the start string, P_0P_1, can have four values (00, 01, 10, 11), each is assumed in turn to generate at most one pre-image, resulting in a maximum of four possible pre-images.

Conversely, for **B C** rules, pre-images may be derived by assuming the start string, P_LP_{L+1}, and determining, from *right* to *left*, successive values of P_{x-1} according to P_{x+1}, P_x and I_x.

$$\begin{array}{cccccccccccc} P_0 & P_1 & P_2 & P_3 & \ldots & P_{x-1} & P_x & P_{x+1} & \ldots & P_{L-1} & P_L & P_{L+1} \\ & & & & \leftarrow & \nwarrow & \mid & \swarrow & & & & \\ & I_1 & I_2 & & \ldots & & I_x & & \ldots & & I_L & \end{array}$$

If $P_L = P_0$ and $P_{L+1} = P_1$, then the pre-image is valid, and the same conclusions apply as with the left-to-right procedure.

Clusters 30 and 45 have ***one-way*** deterministic structure, either left (**A B**), or right (**B C**). Such rules have ***excluded permutations*** that reduces maximum pre-imaging to 3 (see section 3.4).

Consider the neighbourhood... **A B C**
and the target cell **T**

and the rule tables for the **A B** rule 86 (cluster 30) and 101 (cluster 45), which have *left* deterministic structure.

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs
	0	1	0	1	0	1	1	0	rule 86
	0	1	1	0	0	1	0	1	rule 101

Every permutation of **A B** and **T** will give a unique value of **C**. However there are two permutations of **B C** and **T** which are forbidden, $\begin{array}{cc} 1 & 1 \\ 1 & \end{array}$ and $\begin{array}{cc} 1 & 0 \\ 0 & \end{array}$

Thus, for any value of **T**, there is one forbidden value of the string **B C**. To determine the set of pre-images, we assumed in turn the four alternative start strings, P_0P_1. However, one of these start strings must be invalid; therefore, maximum pre-imaging = 3. An equivalent argument can be made for all the other rules in the clusters 30 and 45.

3.5.4 The Reverse Algorithm for Pre-images of Any n=3 Rule

Consider the neighbourhood... **A B C**
and the target cell **T**

Consider the $n = 3$ rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

and the limited pre-image rules with one, or both, of the following deterministic structures:

left (**AB** rules).......$T_7 \neq T_6$ and $T_5 \neq T_4$ and $T_3 \neq T_2$ and $T_1 \neq T_0$;
right (**BC** rules).......$T_7 \neq T_3$ and $T_5 \neq T_1$ and $T_6 \neq T_2$ and $T_4 \neq T_0$.

A deterministic structure has a complete set of ***deterministic permutations***, with ***unequal pairs*** of outputs of the form $T_7 \neq T_6$; given a ***left start string*** **A B** = 11 and **T**, **C** is determined. If, however, there are ***ambiguous permutations*** with equal pairs of the form $T_7 = T_6$, and if **A B** and **T** is not an ***excluded permutation***, then **C** could be 0 or 1 with equal validity.

Most rules have a mixture of ambiguous permutations (which amplify pre-imaging), and deterministic and excluded permutations (which inhibit pre-imaging).

To determine the set of pre-images of a given state for ***any*** rule, consider an unknown pre-image, $P_0, P_1, P_2, \ldots, P_L, P_{L+1}$ (with notional values P_0 and P_{L+1}) of the known successor line of length L, $I_1, I_2, \ldots, I_L$. As boundary conditions are periodic, $P_0 = P_L$ and $P_{L+1} = P_1$. To derive the values of successive cells of a candidate pre-image, the value of the start string, P_0P_1, is assumed and the rule table used to determine, from left to right, successive values of P_{x+1} according to P_{x-1}, P_x, and I_x. As the start string accretes more cell values, we call it the ***partial pre-image.***

$$\begin{array}{cccccccccccc} P_0 & P_1 & P_2 & P_3 & \ldots & P_{x-1} & P_x & P_{x+1} & \ldots & P_{L-1} & P_L & P_{L+1} \\ & & & & & & & & \longrightarrow & & & \\ & I_1 & I_2 & & \ldots & & I_x & & \ldots & & I_L & \end{array}$$

At each step, the following procedure is enacted:

1. If P_{x-1}, P_x, and I_x make up an ***excluded*** permutation, abandon the start string or partial pre-image. Resume derivation of the next partial pre-image (step 5).

2. If P_{x-1}, P_x, and I_x make up a ***deterministic*** permutation, P_{x+1} has a unique value; proceed to the next cell (step 1).

3a. If P_{x-1}, P_x, and I_x make up an ***ambiguous*** permutation, P_{x+1} could be 0 or 1 with equal validity; assume 0.

3b. If P_x, P_{x+1}, and I_x make up an ***excluded*** permutation, change P_{x+1} to 1, and proceed to the next cell (step 1).

3c. If P_x, P_{x+1}, and I_x do ***not*** make up an ***excluded*** permutation, record the partial pre-image ending with $P_{x+1} = 1$, adding it to the ***partial pre-image queue.*** Reassume $P_{x+1} = 0$ and proceed to the next cell (step 1). At each new cell position, it may be necessary to add one partial pre-image to the queue.

4. Once the pre-image (ending with P_{L+1}) is completed, if periodic boundary conditions ($P_L = P_0$ and $P_{L+1} = P_1$) are ***not*** satisfied, the pre-image is abandoned; if satisfied, the pre-image is added to the list of valid pre-images.

5. Take the earliest partial pre-image, recorded in step 3, from the head of the partial pre-image queue and proceed to the next cell (step 1). More partial pre-images may be added to the end of the queue at step 3, and removed from the beginning of the queue at step 5, until none remain.

6. When the partial pre-image queue is exhausted, all possible pre-images, starting with the start string P_0P_1, have been derived without duplication. The procedure is repeated in turn for the three remaining values of P_0P_1.

The ***reverse algorithm*** will compute all possible pre-images, without duplication, to a given CA state for any rule and any array length, provided that the number of partial pre-images in the partial pre-image queue at any one time is within the limits of the computer system's memory. For an array size L, and a state with p pre-images, if the time required to output the full set of pre-images is t_p, it is estimated that, using this algorithm, t_p increases arithmetically with L and p (garden-of-Eden states are identified instantly), whereas, if the whole state space is exhaustively tested for pre-images, for any p, however small, t_p increases exponentially as 2^L.

3.5.5 Pre-images of n=5 Rules

Pre-images for $n = 5$ rules, or indeed $n > 5$ rules, may be computed with an extended reverse algorithm.

Consider the $n = 5$ neighbourhood... $\mathbf{a_1\ a_2\ a_3\ a_4\ a_5}$
and the target cell **T**

and consider the 32 neighbourhoods and their outputs $T_{31} \dots T_0$:

$\mathbf{a_1}$	–	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$\mathbf{a_2}$	–	1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0	1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
$\mathbf{a_3}$	–	1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0	1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0
$\mathbf{a_4}$	–	1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0	1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0
$\mathbf{a_5}$	–	1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0	1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
T..		31...	...3 2 1 0

Deterministic structures for $n = 5$ rules (left, right, or two-way) are as follows:

left $T_{31} \neq T_{30}$ and $T_{29} \neq T_{28}$ and ... $T_3 \neq T_2$ and $T_1 \neq T_0$,
generally, for odd $x = 1$ to 31, $T_x \neq T_{x-1}$;

right $T_{31} \neq T_{15}$ and $T_{30} \neq T_{14}$ and ... $T_{17} \neq T_1$ and $T_{16} \neq T_0$,
generally, for $x = 0$ to 15, $T_{x+16} \neq T_x$.

Given a *left* deterministic permutation, the start string $\mathbf{a_1a_2a_3a_4}$ and $\mathbf{T}$ will determine $\mathbf{a_5}$. The start string is four cells long, and thus has $2^4 = 16$ permutations, so maximum pre-imaging will be 16, irrespective of array size. Conversely, given a *right* deterministic permutation, the start string $\mathbf{a_2a_3a_4a_5}$ and $\mathbf{T}$ will determine $\mathbf{a_1}$, and the same conclusions apply.

The vast majority of $n = 5$ limited pre-image rules will have *one-way* deterministic structure, and will therefore have *excluded permutations*, resulting in less than maximum pre-imaging. Generally for a neighbourhood of size n, maximum pre-imaging is less than or equal to 2^{n-1}.

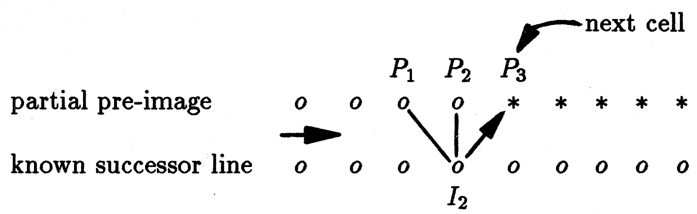
The procedure for deriving the pre-images of any $n = 5$ rule, takes into account deterministic, forbidden and ambiguous permutations, in an extension of the algorithm presented in section 3.5.4. In principle, the reverse algorithm may be applied to rules with any value of n, and may be extended for $k > 2$.

3.6 The Z Parameter

We will define *maximum pre-imaging* as the greatest number of incoming arcs to any node in a basin of attraction field. Maximum pre-imaging as a function of array length (see the data in the Atlas) would be expected to tie into other quantifiable measures of basin fields, for example, the density of garden-of-Eden nodes, and would reflect the degree of *convergence* of state space, discussed further in chapter 4. It would be useful to identify a quantifiable parameter implicit in the rule table that reflected such behaviour.

It is possible to quantify the *probability* that the *next unknown cell* in a partial pre-image is determined (i.e., has one solution, either 0 or 1); this will reflect the maximum pre-imaging exhibited and its variation with array length L, characteristic of a given rule. This probability is refered to as the *Z parameter*.

Consider the reverse algorithm for computing a pre-image (either from left to right, or conversely from right to left) as described in section 3.8. The left-to-right computation is represented by the diagram below:



where "o" represents a known cell value, and "*" an unknown value.

As a first approximation, the minimum value of Z is the proportion of *deterministic permutations* in a rule table, left or right, whichever is greater. It was demonstrated in section 3.4 that, if the rule table has a *deterministic* *structure*, i.e., 100% deterministic permutations, then the probability that the next cell is determined equals 1, and the rule is a *limited pre-image rule.* Maximum pre-imaging, *mp*, is constant with increasing *L*.

$$Z = 1, \qquad mp \leq 2^{n-1}.$$

If the rule table has no deterministic permutations, for instance, the trivial rule 0, then the probability that the next cell is determined equals 0, and *mp* diverges exponentially as a function of *L*

$$Z = 0, \qquad mp = 2^L.$$

The overwhelming majority of possible rules will have a rule table that has an intermediate proportion of deterministic permutations; Z and *mp* will have intermediate values.

For example, the $n = 3$ rule 193 has the rule table,

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	1	1	0	0	0	0	0 ••••	1	outputs

Left deterministic permutations are linked by a dotted line; the total, DP_L, equals 2. Right deterministic permutations are linked by a solid line; the total, DP_R, equals 6. The greater value as a proportion of the total of eight neighbourhoods equals 6/8, so $Z = .75$.

Maximum pre-imaging data for rule 193, taken from the Atlas, is tabulated below in relation to L, with the intermediate function $\sqrt{2^L}$ listed as a rough yardstick for assessing the divergence of *mp* with L.

L..	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
mp..	2	2	3	2	5	5	7	10	12	17	22	29	39	51	68
$\sqrt{2^L}$		2		4		8		16		32		64		128	

For an example of an $n = 5$ rule, consider the totalistic rule, code 50. The rule table is shown below in two parts, one below the other.

```
T.. 31 – 16   1     1   1 – 0   1 – 0   0     0   1 – 0   0     0   0     0   0 – 1
                    |   |       |             |   |             |         |   |   |
T.. 15 – 0    1 – 0   0     0   0     0   0 – 1   0     0   0 – 1   0 – 1   1 – 0
```

Left deterministic permutations are linked by a horizontal dash; the total, DP_L, equals 18. Right deterministic permutations are linked vertically; the total, DP_R, equals 18. Therefore, $Z = 18/32 = 0.5625$ (note that *symmetric* rules must have the same number of left and right deterministic permutations, and all totalistic rules are symmetrical). Maximum pre-imaging data for code 50, taken from the Atlas, is tabulated below, with the intermediate function $\sqrt{2^L}$ listed as a rough yardstick for assessing the divergence of *mp* with L.

L..	3	4	5	6	7	8	9	10	11	12	13	14	15	16
mp..	2	7	21	21	15	31	55	123	155	205	313	549	951	1423
$\sqrt{2^L}$		4		8		16		32		64		128		256

The divergence of *mp* with L relates inversely to Z (see chapter 4). For instance, the divergence of *mp* for rule 193 ($Z = .75$) is less than $\sqrt{2^L}$, whereas the divergence of *mp* for code 50 ($Z = .5625$) is greater than $\sqrt{2^L}$.

3.6.1 Corrected Z Parameter: n=3

Within the rule table of, say, an $n = 3$ rule, there may be ***hidden deterministic permutations*** present relating to the smaller neighbourhoods, $n = 2$ and $n = 1$. Within an $n = 5$ rule table there may be hidden deterministic permutations relating to the smaller neighbourhoods, $n=4$, $n=3$, $n=2$, and $n=1$. Generally, an $n=z$ rule table may include deterministic permutations relating to neighbourhoods smaller than z. The first approximation of the value of the Z parameter must be corrected upward in many cases to take this into account.

Consider the $n=3$ neighbourhood... **A B C** In a left deterministic
permutation, **AB** and **T** will
and the target cell **T** determine **C** (conversely, right).

The $n=3$ rule table for rule 51, with neighbourhoods rearranged vertically, is shown below,

A..	1 1	1 1	0 0	0 0	
B..	1 1	0 0	1 1	0 0	
C..	1 0	1 0	1 0	1 0	neighbourhoods **ABC**
T..	0 0	1 1	0 0	1 1	rule table, rule 51

Although the rule table has no deterministic permutations (left or right), relating to the neighbourhood **A B C**, it has 100% ***left deterministic permutations*** relating to the neighbourhood **A B**, where row **C** can be ignored; thus,

the neighbourhood... **A B C** Given **A** and **T**
B is determined;
and the target cell **T** **C** is irrelevant.

For an $n=3$ rule , there will be three types of deterministic permutations (left and right), labelled $n3$, $n2$, and $n1$, as tabulated below:

neighbourhood size	left	right	
$n3$	**ABC**	**ABC**	Neighbourhoods to which the
$n2$	**AB**	**BC**	deterministic permutations
$n1$	**A**	**C**	relate.

To count the number of the three types of deterministic permutations in a rule table, it is convenient to visualise a set of deterministic ***templates*** that are applied to the rule table to check if specific deterministic permutations exist. For $n=3$ rules there will be a left and a right set of three templates, $n3$, $n2$, and $n1$.

n=3. The $n3$ ***template*** (left or right) follows the definition described in section 3.4. The template size equals 2 and occupies 1/4 of the rule table, with a maximum of four positions. (In all the following examples, left and right templates and the corresponding deterministic permutation diagrams are illustrated.)

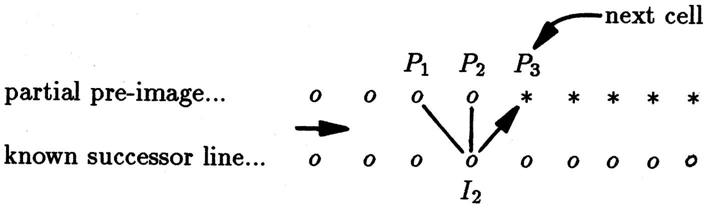
Left Right

The curved link indicates rule table entries are different:

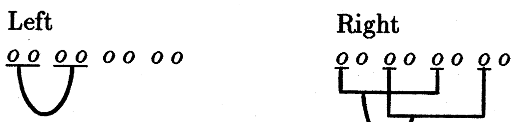
Left
A B C
T Given **AB** and **T**,
C is determined.

Right
A B C
T Given **BC** and **T**,
A is determined.

Let n_3 be the number of *left* $n3$ template positions. Then the probability that $P_1P_2P_3$ belongs to a neighbourhood on an $n3$ template equals $n_3/4$. If so, P_1, P_2, and I_2 will determine the *next cell* P_3, illustrated in the diagram below (conversely, *right*).



n=2. The $n2$ *template* (left or right), size 4, has two possible positions and occupies 1/2 the rule table.

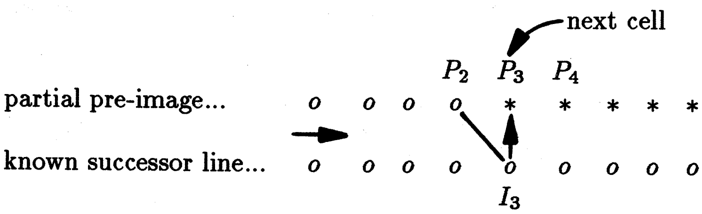


The underline (or linked underlines) indicated entries are equal; the curved link indicates the two sets of equal entries are different.

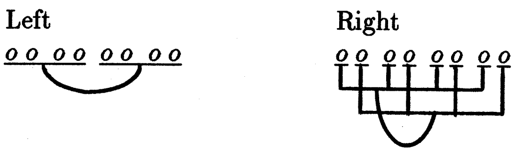
Left
A B C — C is irrelevant; given **A** and **T**, **B** is determined.
T

Right
A B C — **A** is irrelevant; given **C** and **T**, **B** is determined.
T

Let n_2 be the number of *left* $n2$ template positions. Then the probability that $P_2P_3P_4$ belongs to a neighbourhood on an $n2$ template equals $n_2/2$. If so, P_2 and I_3 will determine the *next cell* P_3, illustrated in the diagram below (conversely, *right*).



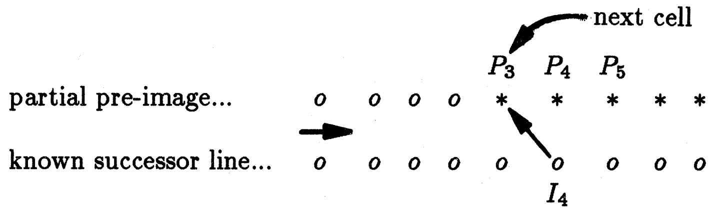
n=1. The trivial $n1$ *template* (left or right), size 8, has one possible position and occupies the entire rule table.



Left
A B C — **BC** is irrelevant; given **T**, **A** is determined.
T

Right
A B C — **AB** is irrelevant; given **T**, **C** is determined.
T

If a *left* $n1$ template fits, then $n_1 = 1$. The probability that $P_3P_4P_5$ belongs to a neighbourhood on the $n1$ template equals n_1. If so, I_4 will determine the *next cell* P_3, illustrated in the diagram below (conversely, ***right***).



To summarise, given an $n = 3$ rule, the number of positions where the three types of template fit the rule table are designated as follows,

n_3 (max 4)... $n3$ template (size 2)
n_2 (max 2)... $n2$ template (size 4)
n_1 (max 1)... $n1$ template (size 8)

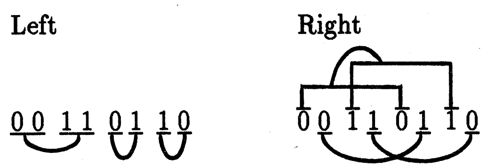
Templates relating to a given direction are mutually exclusive; a given rule table entry may not connect to more than one left template and one right template. The following method will give the corrected probability that the *next cell* is determined, thus the corrected value of Z. The method described below computes Z_L, from left to right. A converse, but otherwise identical method computes Z_R, from right to left.

- $n3$: let p_3 be the probability that the *next cell* is determined by $n3$, $p_3 = n_3/4$; the probability that it is ***not*** determined, $\overline{p}_3 = 1 - p_3$
- $n2$: if *not* determined by $n3$, let p_2 be the probability that the *next cell* is determined by $n2$, $p_2 = n_2/2 \times \overline{p}_3$
- $n1$: this is a special case, as the $n1$ template takes up the whole rule table. Let p_1 be the probability that the *next cell* is determined by $n1$, $p_1 = n_1$ (note: if $n_1 = 1$, Z_L and $Z_R = 1$)

$$Z_L = p_3 + p_2 + p_1.$$

The corrected parameter $Z = Z_L$ or Z_R, whichever is greater.

As an example, consider the $n = 3$ rule 54. The rule table is set out below; left template positions are $n_3 = 2$ and $n_2 = 1$ and right template positions are $n_3 = 2$ and $n_2 = 1$



Left procedure $p_3 = n_3/4 = 2/4 = 1/2$. $\overline{p}_3 = 1 - 1/2 = 1/2$
(same for right) $p_2 = n_2/2 \times \overline{p}_3 = 1/2 \times 1/2 = 1/4$
$Z_L = p_3 + p_2 = 1/2 + 1/4 = 3/4$
$Z_L = Z_R$, so $Z = .75$

3.6.2 Corrected Z Parameter: n=5

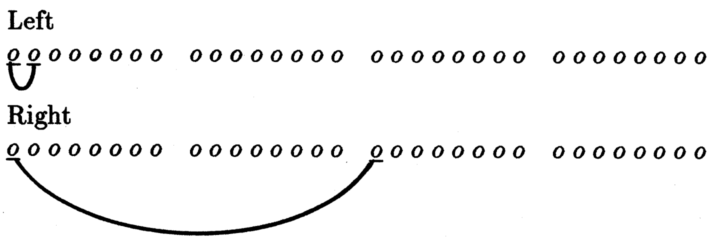
Consider the rule table for $n = 5$ rules, with 32 neighbourhoods and their outputs $T_{31} \ldots T_0$:

a1 –	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
a2 –	1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0	1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
a3 –	1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0	1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0
a4 –	1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0	1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0
a5 –	1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0	1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
T..	31...	...3 2 1 0

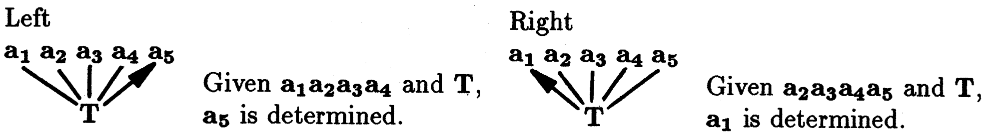
There will be five types of deterministic permutations (left and right), labelled $n5$, $n4$, $n3$, $n2$, and $n1$, as tabulated below:

neighbourhood size	left	right	
$n5$	**a1 a2 a3 a4 a5**	**a1 a2 a3 a4 a5**	
$n4$	**a1 a2 a3 a4**	**a2 a3 a4 a5**	
$n3$	**a1 a2 a3**	**a3 a4 a5**	Neighbourhoods to which the
$n2$	**a1 a2**	**a4 a5**	deterministic permutations
$n1$	**a1**	**a5**	relate.

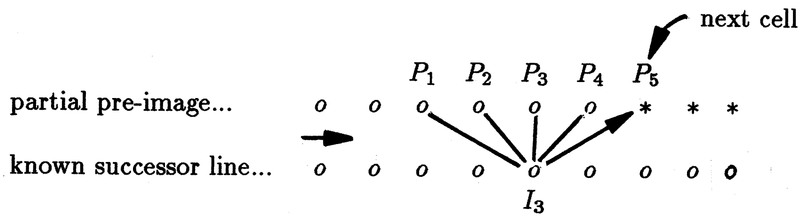
n=5. The $n5$ ***template*** (left or right) follows the definition described in section 3.4. The template size equals 2 and occupies 1/16 of the rule table, with a maximum of 16 positions. It is shown below.



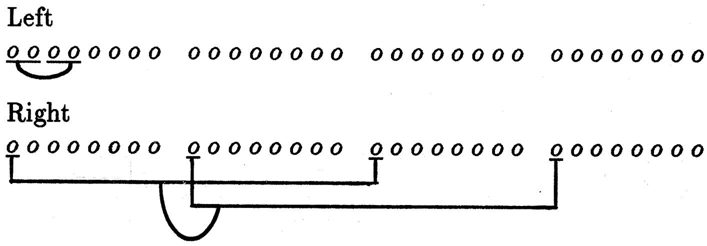
The curved link indicates rule table entries are different:



Let n_5 be the number of *left* $n5$ template positions. Then the probability that $P_1P_2P_3P_4P_5$ belongs to a neighbourhood on an $n5$ template equals $n_5/16$. If so, $P_1P_2P_3P_4$ and I_3 will determine the ***next cell*** P_5, as illustrated in the diagram below (conversely, ***right***).

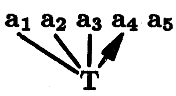


n=4. The $n4$ *template* (left or right), size 4, has eight possible positions and occupies 1/8 the rule table.



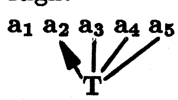
The underline (or linked underlines) indicates entries are equal; the curved link indicates the two sets of equal entries are different.

Left



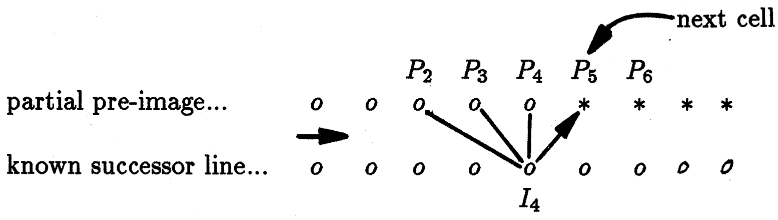
a_5 is irrelevant; given $a_1a_2a_3$ and T, a_4 is determined.

Right

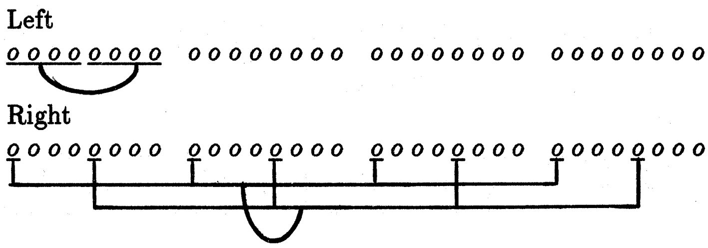


a_1 is irrelevant; given $a_3a_4a_5$ and T, a_2 is determined.

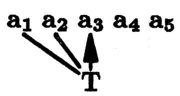
Let n_4 be the number of *left* $n4$ template positions. Then the probability that $P_2P_3P_4P_5P_6$ belongs to a neighbourhood on an $n4$ template equals $n_4/8$. If so, $P_2P_3P_4$ and I_4 will determine the *next cell* P_5, as illustrated in the diagram below (conversely, *right*).



n=3. The $n3$ *template* (left or right), size 8, has four possible positions and occupies 1/4 the rule table.

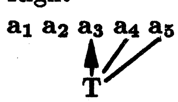


Left



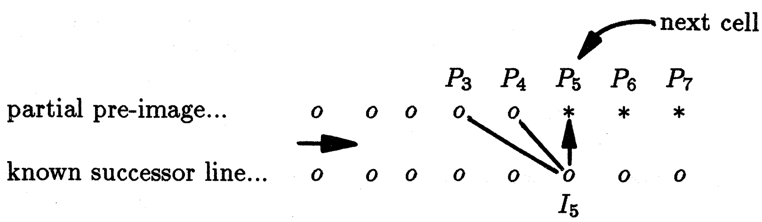
a_4a_5 are irrelevant; given a_1a_2 and T, a_3 is determined.

Right

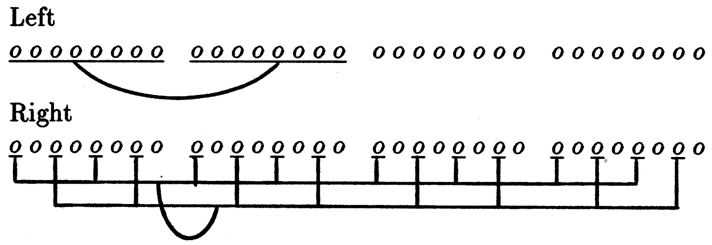


a_1a_2 are irrelevant; given a_4a_5 and T, a_3 is determined.

Let n_3 be the number of *left* $n3$ template positions. Then the probability that $P_3P_4P_5P_6P_7$ belongs to a neighbourhood on an $n2$ template equals $n_3/4$. If so, P_3P_4 and I_5 will determine the *next cell* P_5, as illustrated in the diagram below (conversely, *right*).



n=2. The $n2$ ***template*** (left or right), size 16, has two possible positions and occupies 1/2 the rule table.



Left

$\mathbf{a_1\ a_2\ a_3\ a_4\ a_5}$ — T

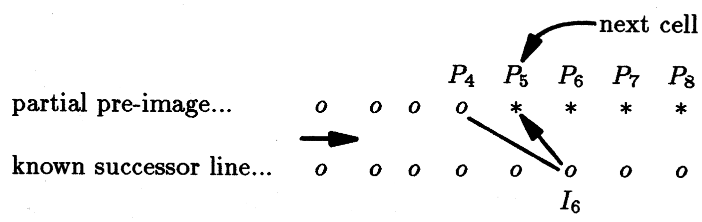
$\mathbf{a_3a_4a_5}$ are irrelevant; given $\mathbf{a_1}$ and $\mathbf{T}$, $\mathbf{a_2}$ is determined.

Right

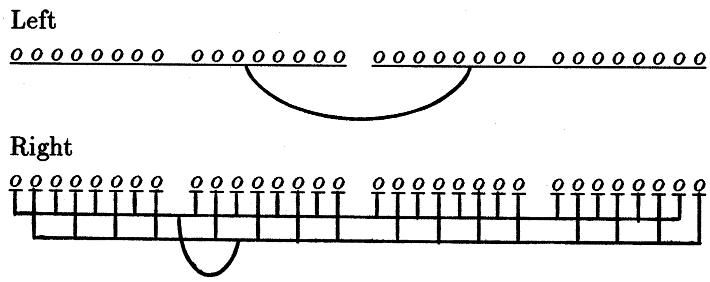
$\mathbf{a_1\ a_2\ a_3\ a_4\ a_5}$ — T

$\mathbf{a_1a_2a_3}$ are irrelevant; given $\mathbf{a_5}$ and $\mathbf{T}$, $\mathbf{a_4}$ is determined.

Let n_2 be the number of *left* $n2$ template positions. Then the probability that $P_4P_5P_6P_7P_8$ belongs to a neighbourhood on an $n2$ template equals $n_2/2$. If so, P_4 and I_6 will determine the ***next cell*** P_5, as illustrated in the diagram below (conversely, ***right***).



n=1. The trivial $n1$ ***template*** (left or right), size 32, has one possible position and occupies the entire the rule table.



Left

$\mathbf{a_1\ a_2\ a_3\ a_4\ a_5}$ — T

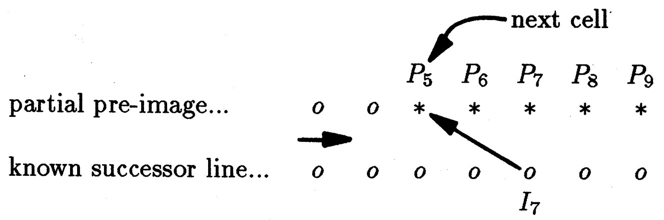
$\mathbf{a_2a_3a_4a_5}$ are irrelevant; given $\mathbf{T}$, $\mathbf{a_1}$ is determined.

Right

$\mathbf{a_1\ a_2\ a_3\ a_4\ a_5}$ — T

$\mathbf{a_1a_2a_3a_4}$ are irrelevant; given $\mathbf{T}$, $\mathbf{a_5}$ is determined.

If a *left* $n1$ template fits, then $n_1 = 1$. The probability that $P_5P_6P_7P_8P_9$ belongs to a neighbourhood on an $n1$ template equals n_1. If so, I_7 will determine the ***next cell*** P_5, as illustrated in the diagram below (conversely, ***right***).



To summarise, given an $n = 5$ rule, the number of positions where the five types of template fit the rule table are designated as follows:

n_5 (max 16)... $n5$ template (size 2)
n_4 (max 8)... $n4$ template (size 4)
n_3 (max 4)... $n3$ template (size 8)
n_2 (max 2)... $n2$ template (size 16)
n_1 (max 1)... $n1$ template (size 32)

Templates relating to a given direction are mutually exclusive; a given rule table entry may not connect to more than one left template and one right template. The following method will give the corrected probability that the *next cell* is determined, thus the corrected value of Z. The method described below computes Z_L, from left to right. A converse but otherwise identical method computes Z_R, from right to left.

- $n5$: let p_5 be the probability that the *next cell* is determined by $n5$, $p_5 = n_5/16$; the probability that it is *not* determined, $\overline{p}_5 = 1 - p_5$.
- $n4$: if *not* determined by $n5$, let p_4 be the probability that the *next cell* is determined by $n4$, $p_4 = n_4/8 \times \overline{p}_5$; the probability that it is *not* determined, $\overline{p}_4 = 1 - p_5 - p_4$.
- $n3$: if *not* determined by $n4$, let p_3 be the probability that the *next cell* is determined by $n3$, $p_3 = n_3/4 \times \overline{p}_4$; the probability that it is *not* determined, $\overline{p}_3 = 1 - p_5 - p_4 - p_3$.
- $n2$: if *not* determined by $n3$, let p_2 be the probability that the *next cell* is determined by $n2$, $p_2 = n_2/2 \times \overline{p}_3$.
- $n1$: this is a special case, as the $n1$ template takes up the whole rule table. Let p_1 be the probability that the *next cell* is determined by $n1$, $p_1 = n_1$. (Note: if $n_1 = 1$, Z_L and $Z_R = 1$)

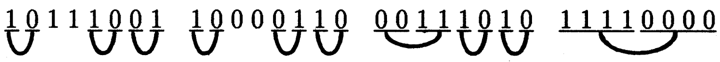
$$Z_L = p_5 + p_4 + p_3 + p_2 + p_1.$$

The corrected parameter $Z = Z_L$ or Z_R, whichever is greater.

As an example, consider the $n = 5$ rule ~~2334561936~~, which has a space-time pattern illustrated in Fig. 2.2. The rule table is set out below:

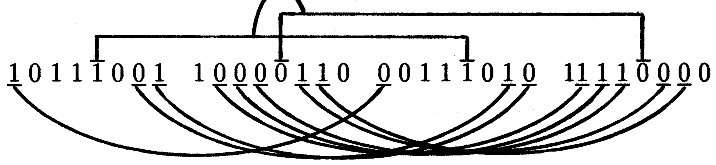
3112581872
(hex) b9863af0

Left



Left templates :
$n_5 = 8, \quad n_4 = 1, \quad n_3 = 1$

Right



Right templates :
$n_5 = 8, \quad n_4 = 1$

Left procedure

$p_5 = n_5/16 = 8/16 = .5$ $\qquad \overline{p}_5 = 1 - .5 = .5$

$p_4 = n_4/8 \times \overline{p}_5 = 1/8 \times .5 = .0625$ $\qquad \overline{p}_4 = 1 - .5 - .0625 = .4375$

$p_3 = n_3/4 \times \overline{p}_4 = 1/4 \times .4375 = .109375$

$$Z_L = p_5 + p_4 + p_3 = .5 + .0625 + .109375 = .671875$$

Right procedure

$$p_5 = n_5/16 = 8/16 = .5 \qquad \overline{p}_5 = 1 - .5 = .5$$
$$p_4 = n_4/8 \times \overline{p}_5 = 1/8 \times .5 = .0625$$

$$Z_R = p_5 + p_4 = .5 + .0625+ = .5625$$
$$Z_L > Z_R, \text{ so } Z = .671875$$

The method of computing Z gives the same result for an $n = z$ rule expressed as an $n > z$ rule. For instance, the $n = 3$ rule 54, 0011-0110, may be expressed as the $n = 5$ rule 255594300, 0000111100111100-0000111100111100. Z is the same for both rule tables.

Z is equal for rules belonging to the same rule cluster, and $Z_L = Z_R$ for *symmetric* and *fully asymmetric* rules. $Z_L \neq Z_R$ for *semi-asymmetric* rules.

The Z parameter could be extended for rules with greater neighbourhood size n, and generalised for greater value range k. The relationship between Z and Langton's λ parameter[16,17] is discussed in chapter 4.

A set of tables, 3.2 and 3.3, showing the λ parameter, λ ratio (see chapter 4), and Z parameter for $n = 3$ rules and $n = 5$ totalistic codes appears below. Only the lowest rule number or code in each cluster is listed, and represents the whole cluster.

TABLE 3.2 $n = 5$ **totalistic codes**

rule code	λ parameter	λ ratio	Z parameter
0	0/32	0	0
1	1/32	0.0625	0.0625
2	5/32	0.3135	0.3125
3	6/32	0.375	0.25
4	10/32	0.625	0.625
5	11/32	0.6875	0.6875
6	15/32	0.9375	0.4375
7	16/32	1	0.375
9	11/32	0.6875	0.6875
10	15/32	0.9375	0.9375
11	16/32	1	0.875
12	20/32	0.75	0.5
13	21/32	0.6875	0.5625
14	25/32	0.4375	0.3125
17	6/32	0.375	0.375
18	10/32	0.625	0.625
21	16/32	1	1
22	20/32	0.75	0.75
25	16/32	1	0.5
30	30/32	0.125	0.125

TABLE 3.3 $n = 3$ rules

	rule number	λ parameter	λ ratio	Z parameter
symmetric rules	0	0/8	0	0
	1	1/8	0.25	0.25
	4	1/8	0.25	0.25
	5	2/8	0.5	0.5
	18	2/8	0.5	0.5
	19	3/8	0.75	0.625
	22	3/8	0.75	0.75
	23	4/8	1	0.5
	33	2/8	0.5	0.5
	36	2/8	0.5	0.5
	37	3/8	0.75	0.75
	50	3/8	0.75	0.625
	51	4/8	1	1
	54	4/8	1	0.75
	73	3/8	0.75	0.75
	77	4/8	1	0.5
	90	4/8	1	1
	94	5/8	0.75	0.75
	105	4/8	1	1
	126	6/8	0.5	0.5
semi-asymmetric rules	2	1/8	0.25	0.25
	3	2/8	0.5	0.5
	6	2/8	0.5	0.5
	7	3/8	0.75	0.75
	9	2/8	0.5	0.5
	12	2/8	0.5	0.5
	13	3/8	0.75	0.75
	26	3/8	0.75	0.75
	27	4/8	1	0.75
	30	4/8	1	1
	35	3/8	0.75	0.625
	38	3/8	0.75	0.75
	41	3/8	0.75	0.75
	45	4/8	1	1
	58	4/8	1	0.75
	62	5/8	0.75	0.75
fully asymmetric rules	10	2/8	0.5	0.5
	11	3/8	0.75	0.75
	14	3/8	0.75	0.75
	15	4/8	1	1
	24	2/8	0.5	0.5
	25	3/8	0.75	0.75
	28	3/8	0.75	0.75
	29	4/8	1	0.5
	43	4/8	1	0.5
	46	4/8	1	0.5
	57	4/8	1	0.75
	60	4/8	1	1

FOUR
Implications of Basin of Attraction Fields

4.1 Basin Field Topology and Rule Space

In this section, some implications of the emerging basin field landscape on the current perception of the structure of rule space are examined. Wolfram has proposed that all CA rules belong to one of four universality classes.[34,36,39] These classes are essentially phenomenological,[8] based on the characteristic appearance of typical space-time patterns. Wolfram's classes, and their analogues in continuous dynamical systems, are said to exhibit the behaviour shown below.

CA behaviour	*dynamical systems analogue*
class 1 evolves to a fixed, homogeneous state	limit points
class 2 evolves to separated periodic regions	limit cycles
class 3 evolves to *chaotic*, aperiodic patterns	strange attractors
class 4 evolves to *complex*, localised structures	long transients, no analogue

It is accepted that many rules show "intermediate" behaviour,[36] and there is also scope for defining subclasses,[19,20] but the discussion below uses a loose definition of the four classes listed. Of these classes, it is conjectured that the supposedly rare[16] *complex rules* (class 4) may in some cases be capable of supporting *universal computation*.[17,34] The space-time patterns of the complex rules contain interacting, static, and propagating structures, sometimes called *information structures*, similar to those illustrated in Figs. 2.2 and 4.1.

Langton proposed that class 4 rules are located at a *phase transition* in rule space. He suggested that the long transients typical of these rules have potential for information processing, and implications for understanding the origin and evolution of life.[17]

4.1.1 The λ Parameter

Rule space has been characterised by the λ parameter,[16,17] the proportion of *non-zero* entries in the rule table. If the value range is k, with possible values $0, 1, 2, \ldots, k-1$, then a particular value, say 0, is selected as the *quiescent* value. The λ parameter is the proportion of rule table entries other than 0.

Langton and others[17,20,41] have shown that the rule classes can be roughly selected by adjusting λ between 0 and $1-(1/k)$. A rule table may be constructed by assigning one of the k values to each entry with equal probability, so that the density of all values, including the quiescent value, will be roughly equal, and $\lambda \simeq 1-(1/k)$. At this value of λ, space-time patterns are most likely to appear chaotic.

Rules may be selected according to other values of λ. This is typically done by assigning the quiescent value with a selected probability x, so that $\lambda = 1-x$. The remaining values are then assigned with equal probability $\lambda/(k-1)$.

For binary rules where $k = 2$, λ is simply the density of 1s in the rule table. Maximum chaos in the appearance of space-time patterns is likely to occur at $\lambda = 1/2$, a rule table with an equal number of 0s and 1s.

As the λ parameter is varied from 0 to 1/2, the various classes of behaviour are traversed. Complex, class 4 behaviour occurs at a ***phase transition*** between periodic, class 2 and chaotic, class 3 behaviour, reordering Wolfram's classes as follows[16,17]:

						complex rules |			
class :	1	$\longrightarrow$	2	$\longrightarrow$		4		$\longrightarrow$	3
in general, λ:	0	...	...	...		...		...	$1-(1/k)$
binary, $k=2$, λ:	0	...	...	...		...		...	$1/2$

As λ increases from 1/2 to 1 the sequence is reversed, so that the complex rules occur in two limited regions on either side of $\lambda = 1/2$.

4.1.2 Convergence of State Space

It is suggested that the λ parameter is modulating the same aspect of CA global behaviour as the Z parameter (introduced in chapter 3, section 3.6)—the ***maximum pre-imaging*** (***mp***) as a function of array length. In general this will reflect the ***degree of pre-imaging*** typical of a rule. The degree of pre-imaging is apparent in the topology of a rule's basin of attraction field, and reflects the ***convergence of state space***,[14] which may be equivalently measured as the density of garden-of-Eden nodes in state space (or in one basin).

Low pre-imaging implies low convergence and a low density of garden-of-Eden nodes; high pre-imaging implies high convergence and high density of garden-of-Eden nodes. The degree of pre-imaging and the density of garden-of-Eden nodes are available in the Atlas (or program).

4.1.3 The Z Parameter and the λ Parameter

The Z parameter is the probability that the ***next cell*** of a partial pre-image has a unique value (see chapter 3, section 3.6). For binary rules, when $Z = 1$, the quantity of 0s and 1s in the rule table is equal, $\lambda = 1/2$; when $Z = 0$, $\lambda = 0$. As Z varies between 0 and 1, λ varies between 0 and 1/2 or conversely between 1 and 1/2.

The Z parameter may possibly be the mechanism underlying the operation of the λ parameter, because the probability of a rule table having a given value of Z depends on the proportions of 0s and 1s assigned at random to the rule table according to the setting of λ. Thus λ seems to be a measure of the ***probability*** of a particular value of Z. The Z parameter is concerned not only with the numbers of 0s and 1s in the rule table, but also their position and thus might be expected to modulate behaviour more closely.

For a more direct comparison between the values of λ and Z for a given binary rule table, the λ parameter may be modified as follows. The quiescent state is taken as the majority value in the rule table; the minority value is taken as the active state. The modified λ parameter, referred to as the ***λ ratio***, is the ratio of the active values to 1/2 of the rule table (the potential maximum of active values). For example, in an $n = 5$ rule table with 32 entries, say 10 entries in the rule table are 0, then the λ ratio equals 10/16.

$$\text{If } \lambda \leq 1/2, \text{ then } \lambda \text{ ratio } = 2\lambda.$$
$$\text{If } \lambda > 1/2, \text{ then } \lambda \text{ ratio } = 2(1-\lambda).$$

The value of the λ ratio varies between 0 and 1 roughly in line with, but never smaller than, the Z parameter. $Z \leq \lambda$ ratio, and $Z = \lambda$ ratio $= 1$ only for limited pre-image rules.

As an example of the λ ratio and the Z parameter, consider the space-time patterns with complex structures that were illustrated in Figs. 2.2(a–c). The relevant rules have the following values for the λ ratio and Z:

a. rule 3112581872 λ ratio $= 1$, $Z = .671875$
b. rule 2334561936 λ ratio $= .8125$, $Z = .6875$
c. rule 3583552890 λ ratio $= .875$, $Z = .75$

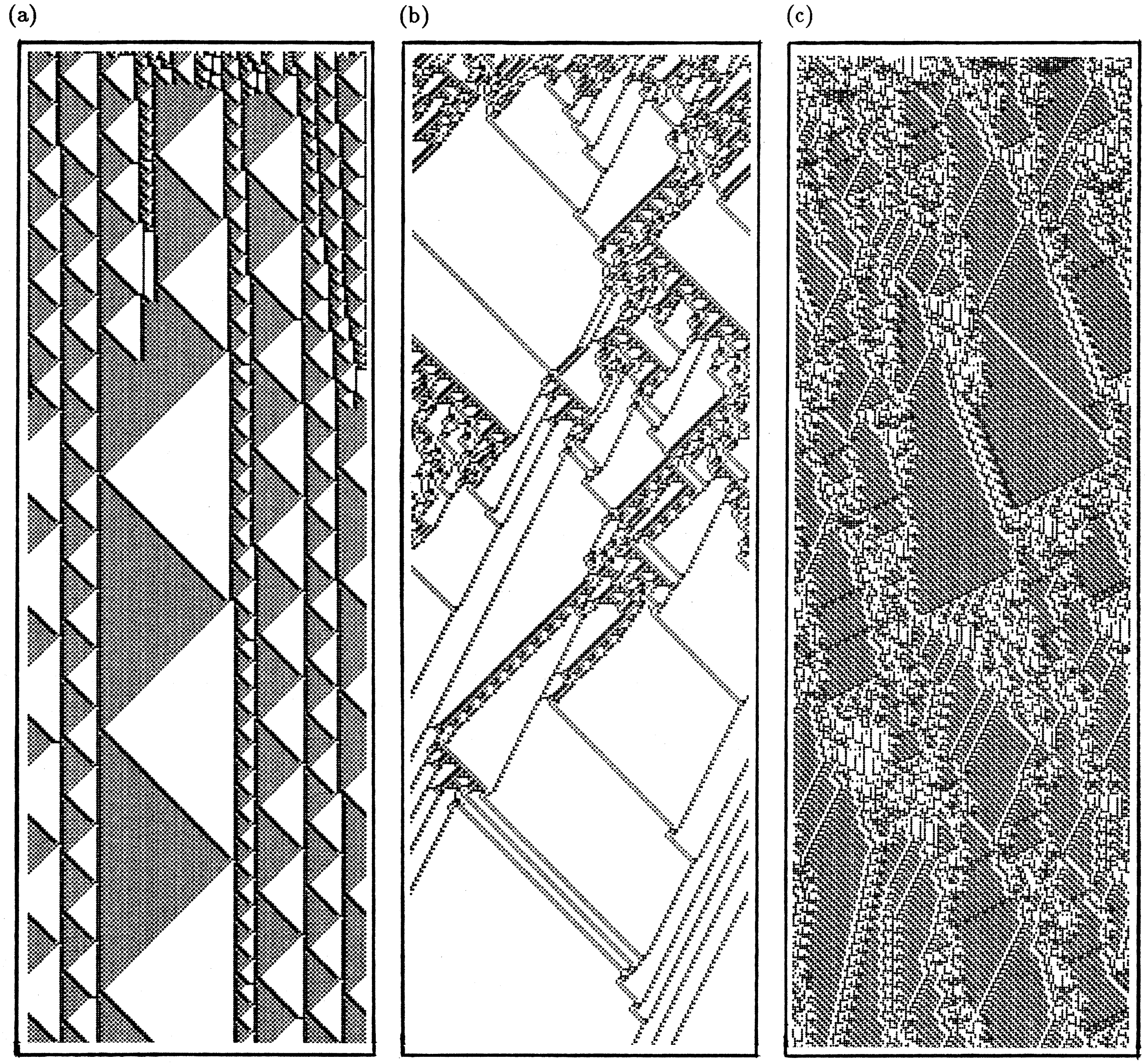


FIGURE 4.1 Space-time patterns for $n = 5$ rules. Array size is 150, 440 time steps from a random initial state. Rule numbers, λ ratio, and Z parameter are: (a) rule 1598319856, λ ratio = 1, $Z = .58984375$. (b) rule 1550470552, λ ratio = .9375, $Z = .7265625$. (c) rule 2912711218, λ ratio = 1, $Z = .75$.

There are many examples of rules where the λ ratio equals 1 or close to 1, suggesting a chaotic space-time pattern, when in fact the pattern appears complex. In such cases the value of Z will typically be between .6 and .8. In Figure 4.1 are some examples of space-time patterns for various $n = 5$ rules that appear to belong to class 4, with complex interacting large-scale structures. The values of λ ratio and Z are shown.

	HIGH convergence and garden of Eden density				*phase transition* mp and (mc, ml, nb) *in balance*		*LOW* convergence and garden of Eden density
class	1	→	2	→	4	→	3
Z :	0	...	...	...	...	...	1
λ ratio :	0	...	...	...	...	...	1
λ :	0	...	...	...	...	...	$1-(1/k)$
mp :	diverges exponentially with L	...	...	...	...	...	fixed upper limit
(mc, ml, nb) :	fixed lower limit	...	...	...	...	...	diverges exponentially with L

FIGURE 4.2 Relationship between the Z parameter (and λ) and the change in basin of attraction field topology as L increases, reflecting convergence of state space.

4.1.4 Basin Field Topology and the Z Parameter

As Z is varied between 0 and 1, it may be possible to predict how the following quantifiable features of basin field topology will vary with increasing array length L (see data in the Atlas), and correlate this with behaviour classes.

1. *mp*, maximum pre-imaging (will reflect the density of garden-of-Eden states in state space) .
2. *mc*, maximum period of attractor cycles.
3. *ml*, maximum length of transient trees.[1]
4. *nb*, the number of separate basins in the field.

mc, *ml*, and *nb* will vary together, because a preponderance of any one will tend to diminish the other two.

Taking the general case of a CA with neighbourhood n and value range k, the possible ways that the finite number of states, k^n, can be connected together into basins depends critically on the maximum pre-imaging, *mp*. Kauffman has observed that "high convergence in state space...is associated with short cycles." [14] If *mp* diverges exponentially with L, ($Z < 1/2$), most states will be locked up in pre-image fans, so that the scope for *mc*, *ml* and *nb* to diverge with L will be severely limited. This is characteristic of class 1 and 2 behaviour.

Conversely, if *mp* is held constant irrespective of L ($Z = 1$, limited pre-image rules), then some combination of *mc*, *ml*, and *nb* must diverge exponentially with L. Such exponential divergence is characteristic of chaotic, class 3 behaviour.

At an intermediate value of Z, *mp* and some combination of *mc*, *ml*, and *nb* will be finely balanced and will diverge by some intermediate function with L. An example of an intermediate function for a binary rule is $\sqrt{2^L}$, and in general $(k^L)^{1/k}$.

[1]In this paper, the length of transient trees is interpreted literally as the number of time steps from the furthest garden-of-Eden node to the attractor. However, beyond the "phase transition," the space-time patterns may quickly settle to what appears to be a "chaotic steady state." The number of time steps to reach this chaotic state (typically becoming *shorter* with increasing λ) is an alternative, descriptive interpretation of transient length, for instance.[17]

Following our literal interpretation, transient length typically continues to *increases exponentially* to an ever greater degree after the phase transition. Although the characteristic appearance of the space-time patterns, whether on a long transient or in the attractor, may be indistinguishable, the difference is clear in a state transition diagram.

It has been suggested that as the λ parameter is increased, this balancing point will emerge abruptly, and locates the class 4 complex rules at a ***phase transition*** in rule space.[17]

It is evident from the atlas that there is a certain trade-off between *mc*, *ml*, and *nb*, as Z approaches 1, and that often only one or two of these variables will diverge with L. For example, consider the complementary $n = 3$ limited pre-image rules, 45 and 210, where $Z = 1$, and *mp* = 3 irrespective of L. In rule 45, *mc* and *ml* diverge exponentially with L; *nb* diverges to a limited extent. By contrast, in rule 210, *mc* and *ml* diverge to a limited extent with L, whereas *nb* diverges exponentially. The mechanism of the trade-off between *mc*, *ml* and *nb* is unclear.

Figure 4.2 summarises the expected variation of the characteristics of the basin field with array size L, as Z is varied between 0 and 1, and the corresponding rule classes.

4.1.5 Examples of Typical Basin Topology in Relation to Rule Class

To illustrate the relationships in Fig. 4.2, an example of a typical basin of attraction for $n = 3$ rules characteristic of each rule class is given in Figs. 4.3–4.6, including the following data: *mp*, maximum pre-imaging; *g-density*, density of garden-of-Eden nodes; *period*, of attractor cycle; and *ml*, maximum transient length.

In these examples, rotation equivalent transients are suppressed for the sake of clarity. General examples of the complete basin field over a range of array length L are presented in the Atlas.

CLASS 1. Rules where $Z = .25$ will have a large proportion of ambiguous permutations in the rule table, so *mp* will diverge exponentially with L; *mc*, *ml*, and *nb* will either remain fixed or relate arithmetically to L. Most states are locked into pre-image fans flowing into point attractors, or two-state attractors, especially the ***uniform*** states (all 0s or 1s). The few states outside the influence of these attractors will only have enough scope to form basins with short cycles and transients. (See Fig. 4.3.)

CLASS 2. Rules where $Z = .5$ may have many basins separated from the uniform attractor. *mp* will diverge exponentially with L, but to a lesser extent than for class 1. *mc*, *ml*, and *nb* will relate arithmetically to L. Again many states are locked up in pre-image fans leaving enough scope for only short cycles or transients. (See Fig. 4.4.)

CLASS 4. The complex, class 4 rules occur typically at $Z = .75$. *mp* and some combination of *mc*, *ml*, and *nb* will be finely balanced and will diverge by some intermediate function with L. Controlled pre-imaging allows enough scope for moderately long cycles and transients. (See Figure 4.5.)

CLASS 3. The chaotic, class 3 rules are limited pre-image rules where $Z = 1$. *mp* is fixed irrespective of L. Some combination of *mc*, *ml*, and *nb* diverge exponentially with increasing L. For instance, clusters 30 and 45, where *mp* = 3, have the greatest values of *mc* and *ml* among the $n = 3$ rules. (See Fig. 4.6.)

In general, the space-time patterns of the $n = 3$ and $n = 5$ rules have been found to correlate with the Z parameter; however, as with the λ parameter, there are exceptions. It has been noted that the transition to chaotic behaviour may occur at different λ values, although there is a well-defined distribution around a mean value.[17,19] Chaotic space-time patterns may also occur at low values of Z. For example, the $n = 3$ rules 18 and 126 have a value of $Z = .5$. Although the typical basin topology of these rules correlates with Z, their space-time patterns appear chaotic, but are confined to only a subset of possible configurations of neighbourhoods. This is clearly seen when colors are assigned to cells in space-time patterns according to the neighbourhood which determined the cell's value, implemented in the program Space1 following a suggestion by Warrell.[43] The process whereby active neighbourhoods are eliminated requires further investigation.

The space-time patterns of the class 2, 4, and 3 rules used in the examples are shown in Figure 4.7.

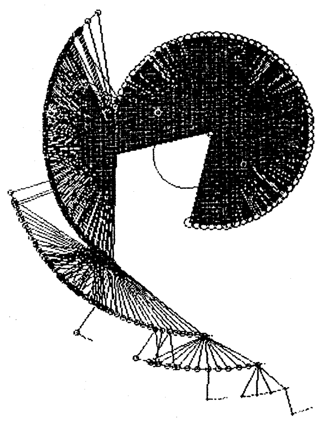


FIGURE 4.3 Class 1, rule 251. $L = 12$, seed 111111111111. $mp = 853$, *g-density* = .92, period = 1, $ml = 6$. mp diverges exponentially with L; $Z = .25$; (λ ratio = .25). Equivalent transients above level 1 are suppressed.

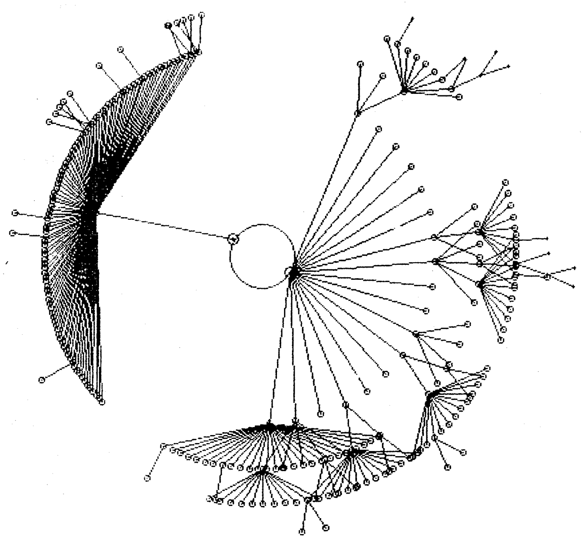


FIGURE 4.4 Class 2, rule 33. $L = 16$, seed 0111111111100000. $mp = 97$, *g-density* = .84, period = 2, $ml = 5$. mp diverges exponentially with L; $Z = .5$; (λ ratio = .5).

4.2 Mutation

Kauffman and others have made the analogy between CA behaviour and biological systems.[14,15,20] The rule table is regarded as the *genotype* and the dynamics shown by the rule, the *phenotype*. The rule, as expressed by its rule table, is compared to a DNA sequence. Experiments may be done to see how changing, or mutating, the elements in a rule table may change behaviour in terms of typical space-time patterns.

The same investigation may now be applied to basin of attraction fields, which in principle represent all possible space-time patterns or global behaviour. The effects of a mutation of the rule table on the topology, or form, of the basin field has added significance for biology because it is analogous to mutation of the genetic code altering the morphological form of an organism.

The *Hamming distance* between two rules specifies the extent of a mutation of a binary rule table, and is a measure of the number elements, or bits, in the rule table that have been changed. A Hamming distance of one bit is the smallest possible mutation of a given rule table.

4.2.1 Mutant Basins of Attraction

A binary rule table has 2^n entries. A given rule will have a set of 2^n mutants separated from it by one bit. In the

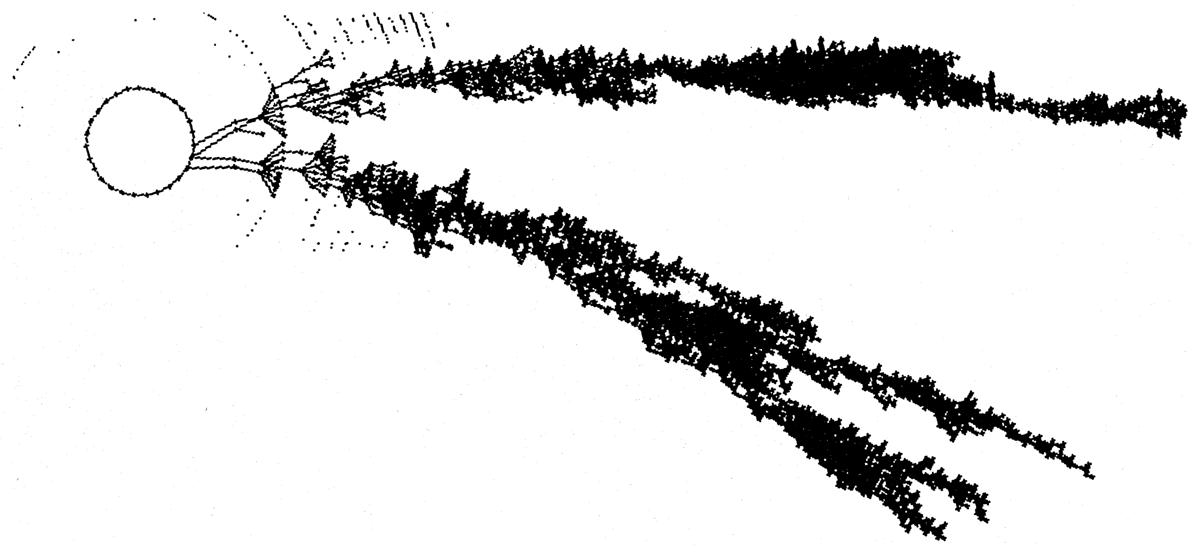


FIGURE 4.5 Class 4, rule 193. $L = 18$, seed 011010001110000010. $mp = 70$, $g\text{-}density = .61$, period $= 27$, $ml = 120$. mp, mc, and ml diverge with L; $Z = .75$; (λ ratio $= .75$). Equivalent transients are suppressed.

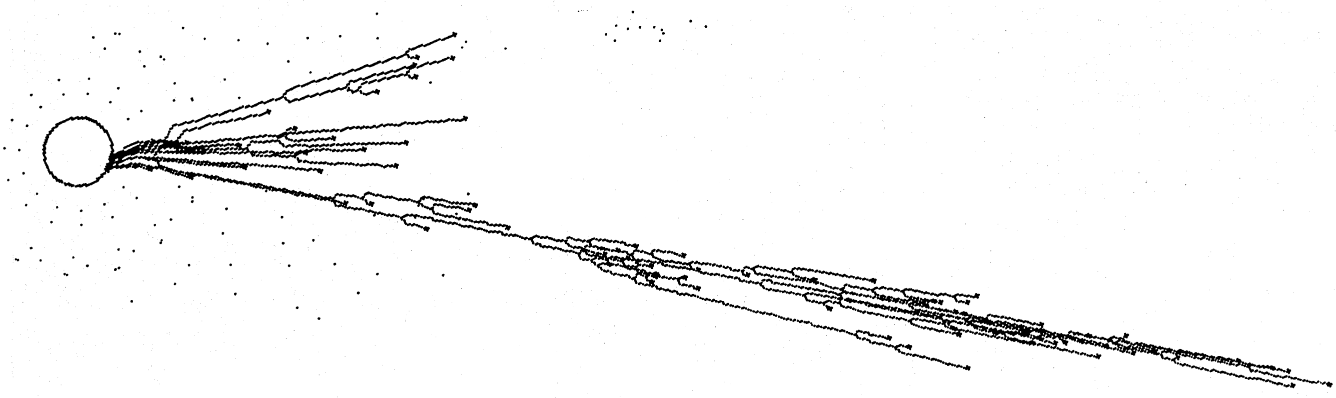


FIGURE 4.6 Class 3, rule 30. $L = 15$, seed 110110111000000. $mp = 2$, $g\text{-}density = .04$, period $= 1455$, $ml = 321$. mp and ml diverge exponentially with L; $Z = 1$; (λ ratio $= 1$). Equivalent transients and pre-image nodes are suppressed; angle between pre-image arcs is increased.

case of the $n = 3$ rule table, each rule will have 8 one-bit mutants. An $n = 5$ rule table will have 32 one-bit mutants.

In the $n = 3$ rule table, with eight entries, the smallest mutation of the rule table, one bit, is likely to result in a very significant change to the basin field topology, because a relatively large change, 12.5%, has been made to the rule table.

As the size of the rule table, 2^n, is increased, the rule space becomes "smoother." The significance of the smallest mutation decreases. Mutating the $n = 5$ rule table, with 32 entries, by one bit, results in a relatively smaller change to the rule table, 3.125%, and a correspondingly less significant change in basin field topology. The set of 32 one-bit mutant basin topologies are recognisably related. If further mutations are made to the rule table, the divergence increases.

It is possible to mutate an $n = 3$ rule by a change of less than 12.5%, if it is expressed as an $n = 5$ rule (see chapter 3, section 3.3.9). Mutating the $n = 5$ rule table by one bit means moving in rule space away from the $n = 3$ rule into the space of $n = 5$ rules that are as close as possible to the $n = 3$ source rule. Rules in the $n = 3$ rule

(a) (b) (c)

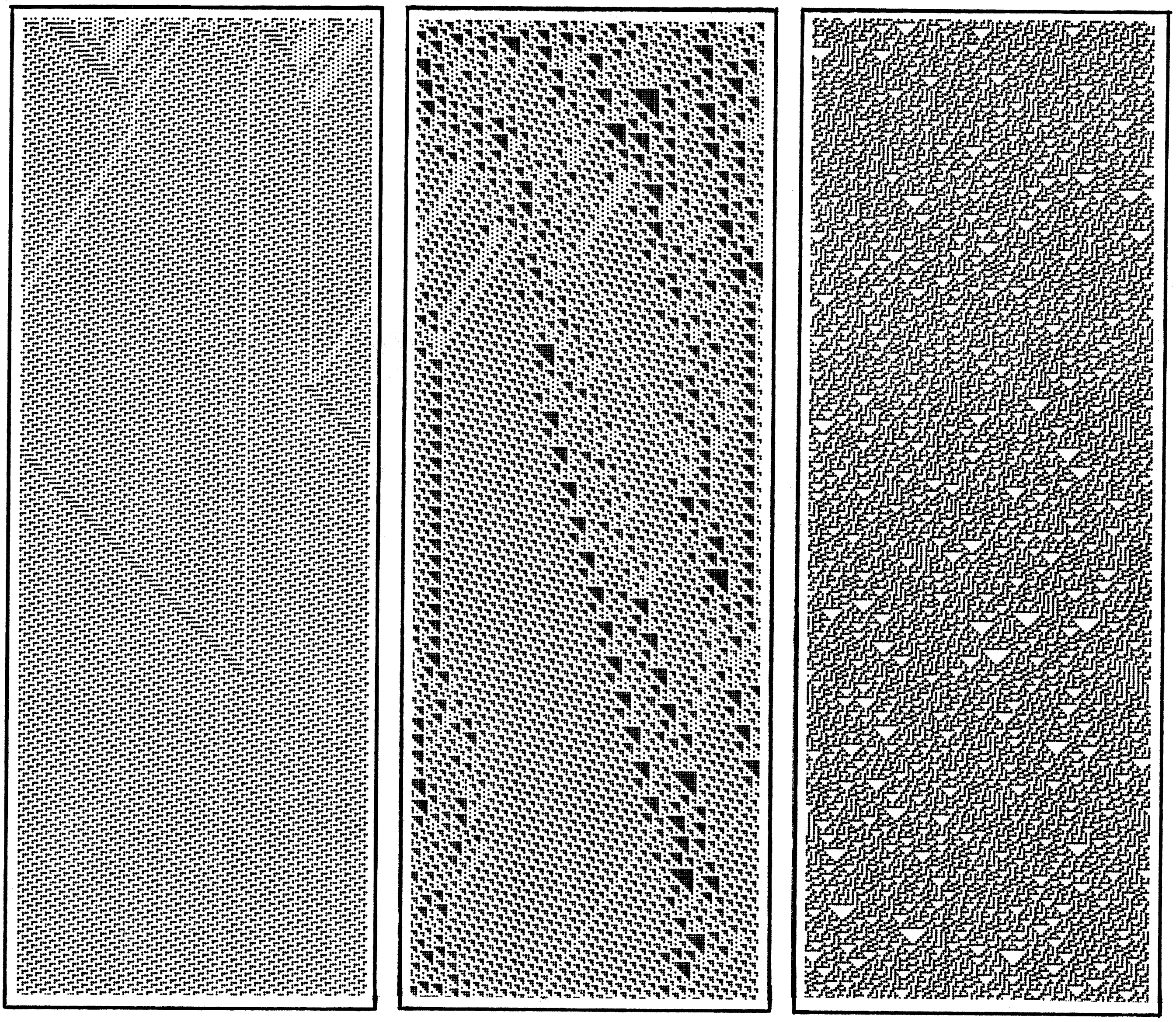


FIGURE 4.7 Space-time patterns for $n = 3$ rules. Array size is 150, 420 time steps from a random initial state. Rule numbers and Z parameter are: (a) Class 2: rule ~~33~~ 9, $Z = .5$; (b) Class 4: rule 193, $Z = .75$; (c) Class 3: rule 30, $Z = 1$.

subset, far from forming a cohesive group, are scattered within $n = 5$ rule-space. In the same way, $n = 5$ rules are scattered within $n = 7$ rule space, and so on.

Fig. 4.8 shows the basin field for the $n = 3$ rule 195, $L = 8$ (the source rule), located in the lower left corner of the diagram. In fact, the basin field consists of a single basin, with a point attractor whose global state is all 1s. The other 32 panels in the diagram show the single basin of attraction containing the seed all 1s, according to a rule mutated by one bit from the $n = 5$ expression of the source rule. The mutations are made to successive bits in the rule table from left to right, numbered 1 to 32. The corresponding one-bit mutant basins are located in ascending rows from left to right.

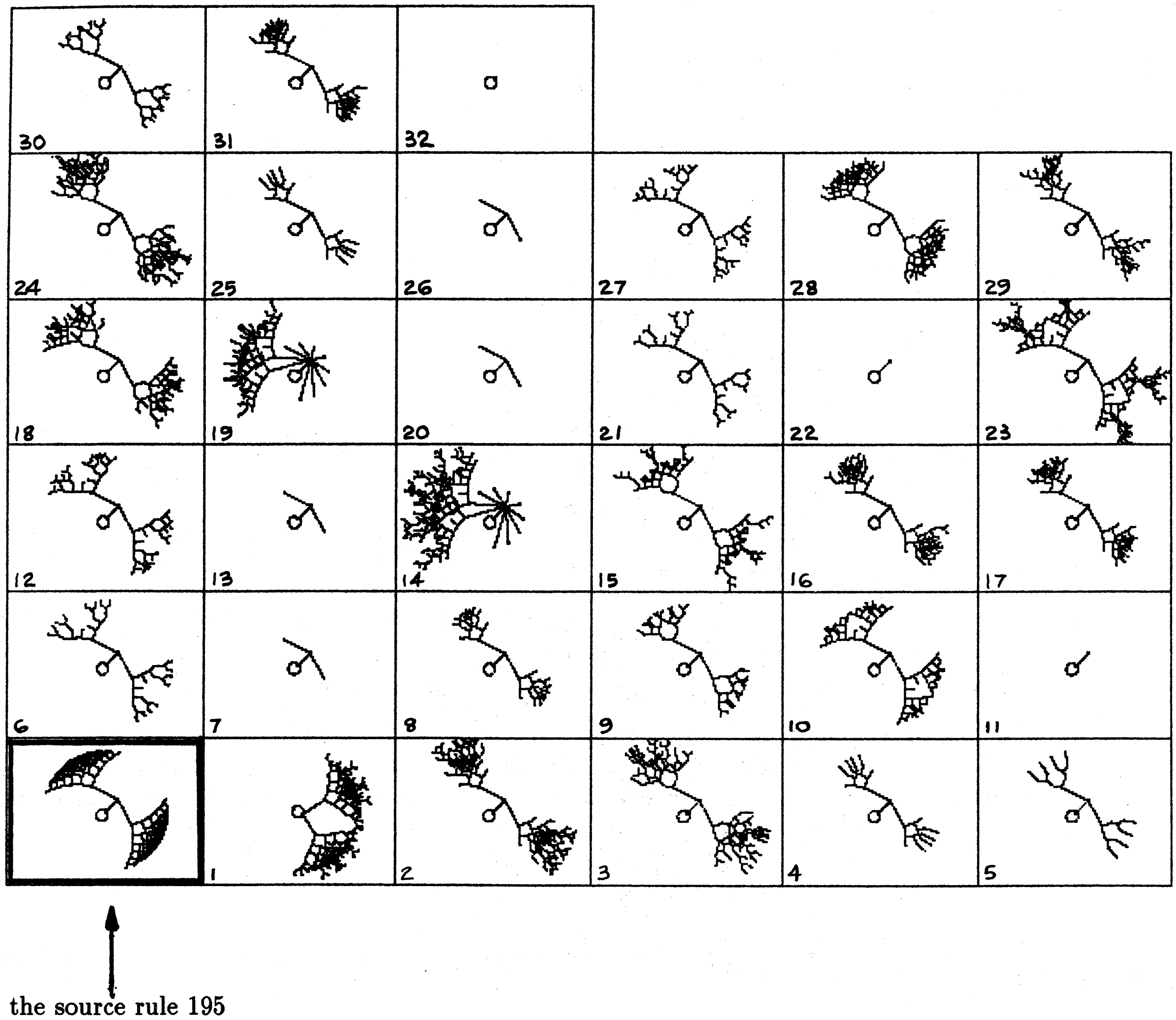


FIGURE 4.8 Rule 195 and 32 one-bit mutants. Seed state—all 1s; $L = 8$. The source rule is at the lower left corner. Successive mutants (1–32) are in ascending rows from left to right (angle between pre-image arcs increased).

In general, $n = 5$ rule space has been found to have a measure of continuity from a subjective appraisal of basin field topology, but sharp changes are also present. Rule space for $n > 5$, with a larger rule table, would be expected to be proportionately smoother.

Rules may be mutated cumulatively by 32 one-bit steps, until the rule table is turned into its complement. Appendix 4 shows further examples of sets of one-bit mutants and cumulative mutants.

The program allows automatic mutation of rules (see Appendix 1) to produce mutated basin of attraction fields (or single basins), or mutated space-time patterns. Sets of 32 mutants as illustrated may be assembled. Space-time patterns may be mutated by one bit (and mutated back) in mid run to allow the evolution and selection of a preferred pattern. Complex space-time patterns may be easily found in this way.

4.2.2 Mated Rules

Rules may be mated by combining the left half of one rule table with the right half of another. If the "parent" rule tables are very different, having different basin structure, then the *offspring* will have an intermediate basin

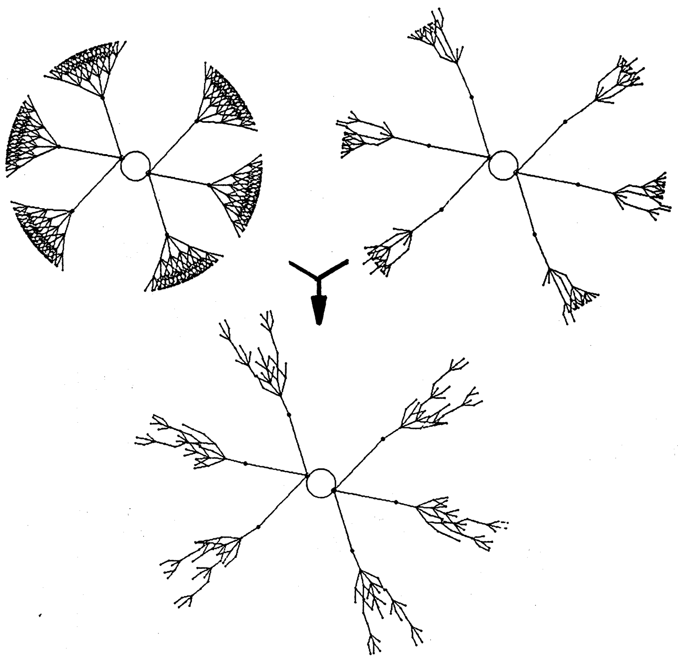


FIGURE 4.9 The $n = 3$ parent rules 105, 01101001, and 73, 01001001, and their offspring, the $n = 5$ rule 818101443, 0011110011000011-0011000011000011, L = 12, seed all 0s.

structure. If the parents are separated by only one bit, then mating rules will result in an offspring that is identical to either one parent rule or the other. Slightly greater distance between parent rules results in greater variation, but with the offspring basin structure bearing a closer family resemblance to each parent than the parents to each other. The parallels with sexual reproduction are clear. An example is given in Fig. 4.9.

4.3 Conclusion

This book demonstrates the complex unfolding of basin of attraction fields, for CA constructed with simple parameters. This complements, in terms of global behaviour, the well-documented space-time patterns of individual trajectories. Access to basin field structure may allow insights into the global behaviour of CA with more complex parameters, and indeed to any dynamical model or real system that evolves within a basin of attraction field. The basin field is self-organised according to a linear code, and responds to mutations of the code, a poignant analogue to genetic systems.

A synoptic view of basin structure has suggested a number of principles underlying CA behaviour and the structure of rule space. It has been shown that rotational symmetry, and bilateral symmetry for symmetrical rules, are conserved in CA evolution. The organisation of rule equivalence classes into rule clusters and symmetry categories has been proposed. A general reverse algorithm for generating all pre-images to a given CA state has been presented.

The subset of limited pre-image rules has been identified, and the mechanism within a rule table that modulates the degree of pre-imaging has been quantified by a proposed Z parameter. It has been shown that the degree of pre-imaging is likely to be a key variable controlling the topology of basin of attraction fields, and may explain the operation of the λ parameter used to classify rule space.[17]

The raw data in the atlas, and the program for generating basin fields, will, it is hoped, provide further avenues of research.

APPENDIX 1

The Atlas Program

The programs included with this volume are a research tool under development, and not presented as a consumer product. The programs run only on PC-compatible computers in DOS; Apple Macintosh computers can be made to emulate DOS, but the authors have not experimented with what kind of hardware would be required. The software is purchased "as is." Those interested in receiving future developments of the software (including versions for the Apple Macintosh and Sun Microsystems computers) should write to: Mr. A. Wuensche, ~~48 Esmond Road, London W4 1JQ, U.K~~. For the latest features update, run the "update" file included with the diskette. andy@ddlab.com

A1.1 Operating Instructions

A1.1.1 Hardware/Software Recommendations/Requirements

- *IBM PC or compatible, with DOS 2.0 or above
- 80286 chip or above, math co-processor recommended
- 640k memory available
- *VGA (640 × 480) 16-colour graphics card
- Epson MX-80 dot matrix printer, or compatible

Variation from the above may cause problems. Items marked "*" are essential.

A1.1.2 The Programs

There are three programs on a 3.5-inch diskette enclosed inside the back cover:

- ATLAS1.EXE: for drawing basin of attraction fields.
- SPACE1.EXE: for drawing space-time patterns.
- UPDATE.EXE: for the latest features not described in the operating instructions.

A1.1.3 The Setup and Graphics Screens and the Interrupt Key (F12)

The Atlas1 and Space 1 programs divide into two parts, the setup screen where parameters are specified and the graphics screen where the graphics are drawn. In both screens, except when printing, loading, or saving, **key F12 + RETURN** will interrupt operation and allow options as follows:

The interrupt key (F12)

`abort`	To the beginning of the setup screen
`dump`	The screen image to the printer
`continue`	From point of interrupt

for ATLAS1 only

`save`	The screen image to disk
`load`	A screen image file, or series of files, from disk

for SPACE1 only

`revise`	The revision menu

key (F11) will interrupt while printing

At the foot the screen, the following prompt and title appears:

F12-interrupt **Networks of Attraction/copyright © A.Wuensche & M.Lesser/May 1990.**

or

F12-interrupt **Space-Time Patterns/copyright © A.Wuensche & M.Lesser/May 1990.**

A1.2 ATLAS1: Basin of Attraction Fields

A1.2.1 Running the Program; the Set-up Screen

To run: insert the diskette into a drive, or save onto hard disk; if in drive A, at the DOS prompt, enter `A:ATLAS1`. The setup screen (grey background) will appear, with this message:

`free date bytes 460048` (or similar), at least 180000 required.
`to quit Atlas1 enter 99 + RETURN, else RETURN:`

The following is a brief commentary on each setup choice. Press RETURN for the next choice.

In general, 99 selects an option, and RETURN the default. A series of prompts are presented to specify the rule parameters, graphics screen layout, and the seed line. (Note that NAT, short for Network of ATtraction, is used to denote a basin of attraction/state transition diagram.)

A1.2.2 Select Either Single NAT (or transient branch only) or NAT Field

A single NAT is one basin in the basin field, whereas the NAT field is the entire set of nonequivalent basins making up the basin field.

```
for single NAT enter 99, else for NAT field RETURN:
transient branch only enter 99, else for complete NAT RETURN:
```

A transient branch is a fragment of the NAT consisting of all upstream states from a given seed state. Note that if the seed state is on the attractor, the sequence of upstream states will be endless.

A1.2.3 NAT Information to the Printer

Selects automatic printing of NAT data (see A1.2.14) as NATs are being drawn. Selecting this option when the printer is not ready will cause "`error - 57 - printer not ready`."

```
for NAT info to printer enter 99, else RETURN:
```

A1.2.4 Select the Line Length or Range of Line Lengths

A single NAT or the NAT field may be drawn for either one particular line length, or over a range of line lengths. For a range of line lengths, specify the first and last line lengths; for one line length only, specify the same line length for first and last. There are different defaults according to *single NAT* or *NAT field* selected in A1.2.2. For instance, for a NAT field,

```
select first line length, (default 1), 1-18:
select last line length, (default 10), 1-18:
```

The maximum line length for a *single NAT* is 31 and for a *NAT field*, 18. The minimum line length is 1. "`error - 14 - out of memory`" may occur, if the configuration of rule and seed, combined with an excessive line length, results in:

a. too great a *disclosure length*,

b. too many *partial pre-images*, or

c. too many *pre-images* at the same level on one transient tree.

Note that the time taken to draw a NAT field tends to increase as 2^{L-1}, where L is the line length, rather than 2^L. This is because equivalent basins, equivalent transient trees, and equivalent transient branches from uniform states do not need to be regenerated.

A1.2.5 Select the Type of Rule and Notation

The rule type may be selected as an $n = 5$ rule (in decimal or binary), an $n = 3$ rule (the default), or an $n = 5$ totalistic code (the specific rule will be chosen in A1.2.7). There is also the choice of a random $n = 5$ rule.

```
Select notation to specify 5-rule or subset
1)  5-rule Decimal
2)  5-rule Binary
3)  3-rule (default)
4)  5-rule Totalistic code
5)  5-rule Random
enter 1, 2, 3, 4, 5 or RETURN:
```

A1.2.6 Specify Automatic Mutation, Screen Save, or Printer Dump

When the graphic image is complete, RETURN will result in a new graphic image according to a mutated rule, retaining the current setup. The default is a one-bit mutation of the source rule, starting from the leftmost position in the rule table. After 32 mutations you are returned to the setup screen.

Mutations are made to the $n = 5$ expression of totalistic codes and $n = 3$ rules. A random $n = 5$ next rule may also be selected. Mutations may be automatically saved, or printed (see A1.2.17). Selecting "`next rule without pause`" and option 1 or 4 results in an endless succession of graphic output.

```
to specify mutated or automatic next rule
or automatic screen save or printer dump,
enter 99, else RETURN:
```

If 99 is selected, the various options for mutation and automatic saving or printing are presented:

```
1)  increment rule number by +1
2)  32 single mutations from left (default)
3)  32 cumulative mutations from left
4)  a new rule selected at random
enter 1, 2, 3, 4 or RETURN:
```

If option 2 or 3 is selected, the position of the first bit to be changed, from the left, may be specified. The default is 1, i.e., the leftmost bit.

```
enter start mutation 1-32 (default =1):
```

NATs for successive rules may be drawn without a pause:

```
next rule without pause enter 1, else RETURN
```

Each complete screen for successive rules may be automatically saved. This permits a set of 32 mutant NATs to be assembled (see A1.2.17).

```
for automatic screensave enter 1, else RETURN:

enter mutant filename max 6 characters
default is d:myfile (? to list):
```

Each complete screen for successive rules may be automatically dumped to the printer, with an optional pause before each dump:

```
for automatic printer dump enter 1, else RETURN:
to pause before dump enter 1, else RETURN:
```

A1.2.7 Select the Rule

According to the choice of rule type and notation in A1.2.5, the rule is selected in decimal or binary (the default is a random rule). For example,

```
select 3-rule notation 1) decimal (default), 2) binary
enter 1, 2 or RETURN:

enter 3-rule number 0 to 255 (default random):
```
or
```
enter 0s and 1s to make up an 8-bit binary number
```

A1.2.8 Reposition or Rotate

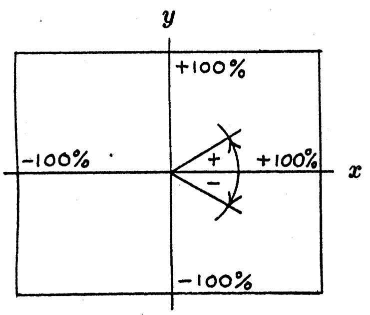
The centre of the first NAT may be located anywhere on (or off) the screen. The screen is centrally divided by notional x, y coordinates. The centre of the screen is at $x = 0$, $y = 0$, the left and right edges at $x = -100\%$ and $+100\%$, and the top and bottom edges at $y = +100\%$ and -100%.

Defaults for the initial position vary according to parameters previously selected. The position may be specified *off screen*, to allow partial views of NATs at greater resolution (the % sign is optional).

```
to reposition or rotate NATs enter 99, for default RETURN
```

If 99 is selected, the position and angle may be specified; for example,

```
enter x position, (default 0%) displace from centre - or +:
enter y position, (default 0%) displace from centre - or +:
```



The default orientation of NATs may be rotated by a selected angle, + for anti-clockwise and – for clockwise. For *cycle lengths* greater than 2, the first transient tree (if any) is attached to the node lying due east (0 degrees). For cycles of 1 and 2, the default orientation is as illustrated in A1.3.

```
alter angle of initial transient from cycle, for default RETURN:
enter angle (degrees) anti-clockwise from +x axis:
```

A1.2.9 Alter Presentation

Various options are offered for altering the default presentation of NATs, which may be used in any combination (see A1.2.13 for examples). Suppressing copies of equivalent transient trees will result in drawing only the first of each equivalent set of transient trees (or branches from the uniform states, all 0s or 1s). However, garden-of-Eden nodes may be retained, as the footprints of the suppressed transient trees.

The option to mark singleton states (a single 1 among 0s) will put a white circle around such nodes. Numbering the nodes according to the global state will place the decimal equivalent of the binary state (or the binary state itself) at the node. Beyond a certain scale, about 12 levels (see A1.2.10), these numbers become indistinct.

Garden-of-Eden nodes may be reduced to a dot, or suppressed. Pre-image nodes (i.e., nodes in transient trees that are not garden-of-Eden nodes) may be suppressed. Increasing the angle between transient arcs can be useful for a clearer presentation, if pre-imaging is restricted (for example, Fig. 4.6).

```
to alter presentation of NATs enter 99, else RETURN:
```

If 99 is selected, the following options are offered:

```
to suppress equivalent transients enter 99, else RETURN:
to mark singleton states enter 99, else RETURN:
to number nodes according to state enter 99, else RETURN:
to reduce size of Garden of Eden nodes enter 99,
               to suppress enter 88, else RETURN:
to suppress pre-image nodes enter 99, else RETURN:
to increase angle between pre-image arcs, enter 99, else RETURN:
```

A1.2.10 Set the Number of Levels (Scale)

This scales the NATs by setting the approximate number of levels (time steps to the attractor) that would fit the screen if a NAT was positioned centrally. The default varies according to previously selected parameters. For practical reasons of node density and transient length, the gap between levels decreases by a logarithmic function of the distance (in levels) from the attractor.

```
set number of levels, for default ( varies )-RETURN:
```

A1.2.11 Set the Separation between NATs

This may be required to amend the default separation between multiple NATs on the screen, for instance, for a basin of attraction field over a range of line lengths. An initial x- and y-axis separation factor is specified. A factor of 1 ensures that two NATs, each with transients trees about eight levels long, will not touch. As transients trees tend to become longer with increasing line length, the separation factor can be set to automatically increase by a given compound percentage.

```
enter x separation factor, for default (.5)-RETURN:
enter y separation factor, for default (.5)-RETURN:
```

If a range of line lengths was selected, specify the percentage increase (the % sign is optional)

```
enter compound increase x separation (default 10%):
enter compound increase y separation (default 10%):
```

A1.2.12 Select the Seed State for Single NAT(s)

Various options are offered for specifying the seed state for single NATs (NAT fields are seeded automatically). Any state that is part of the NAT will seed the entire NAT. The default is a singleton state. Note that specific binary or decimal seeds are selected on the graphics screen, in the top left dialogue area.

```
select seed line
1)  all 0s
2)  all 1s
3)  01.. repeating
4)  001... repeating
5)  0001.... repeating
6)  singleton - positive (default)
7)  singleton - negative
8)  random
9)  random-symmetric
10) decimal
11) binary
enter 1-11 or RETURN:
```

If "`transient branch only`" was selected in A1.2.2, an option to start the transient branch from a seed at a given number of steps forward in time from the selected seed state is offered.

```
transient branch from future time step,
how many steps forward (default 0, max (varies)):
```

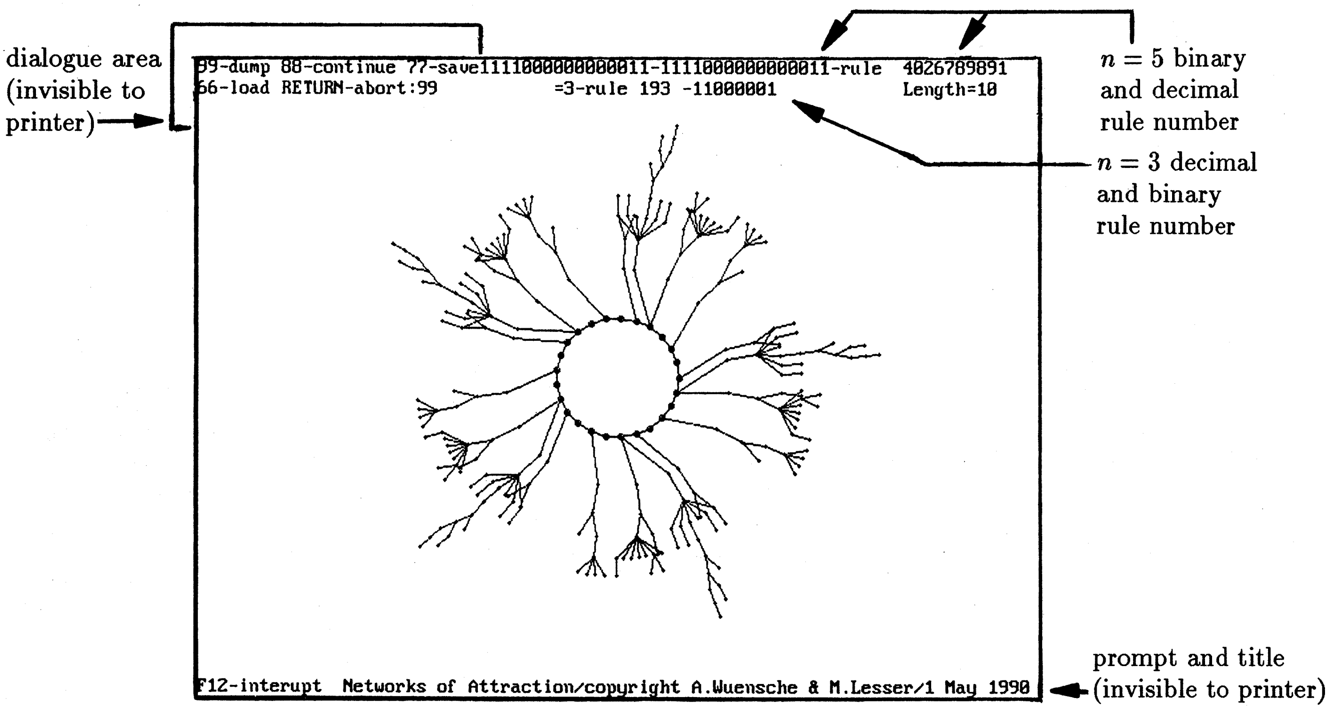
Running forward by at least one step before seeding the transient branch will ensure the existence of pre-images. A random state is very likely to have no pre-images (a garden-of-Eden state).

A1.2.13 The Graphics Screen and Examples

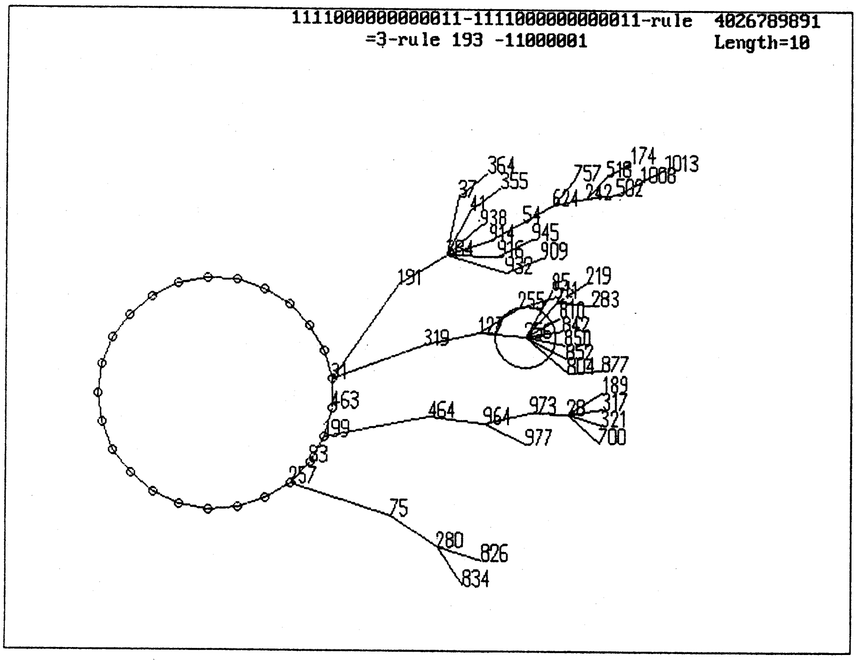
When the setup choices are complete, the following option to revise choices from A1.2.10, or RETURN for graphics, is presented:

```
99 to revise, RETURN for graphics:
```

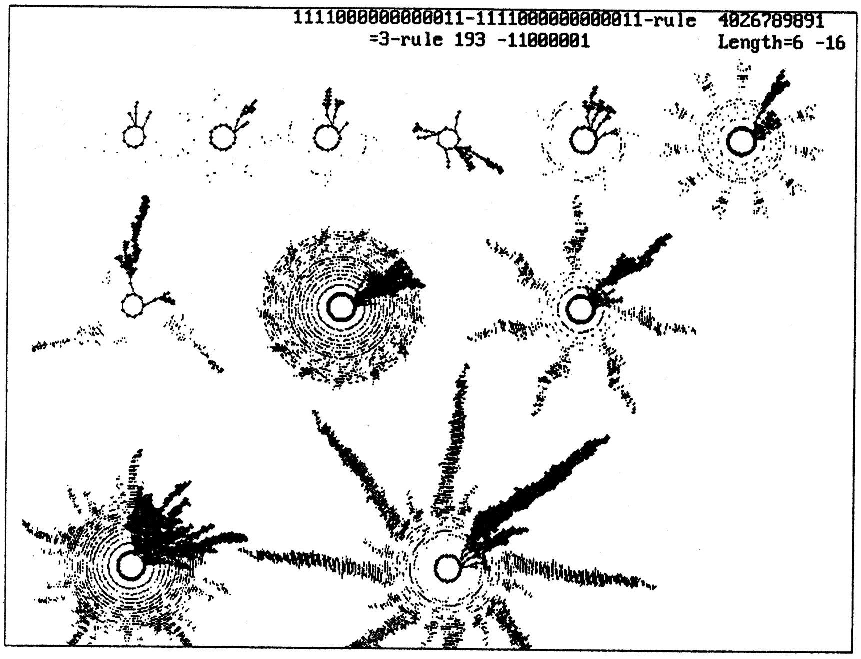
After RETURN, the graphics screen will appear (black background), laid out as shown below. The attractor cycle will be generated first, followed by transient trees in turn, growing outward from the attractor. Successive NATs will be generated in successive rows from the left.



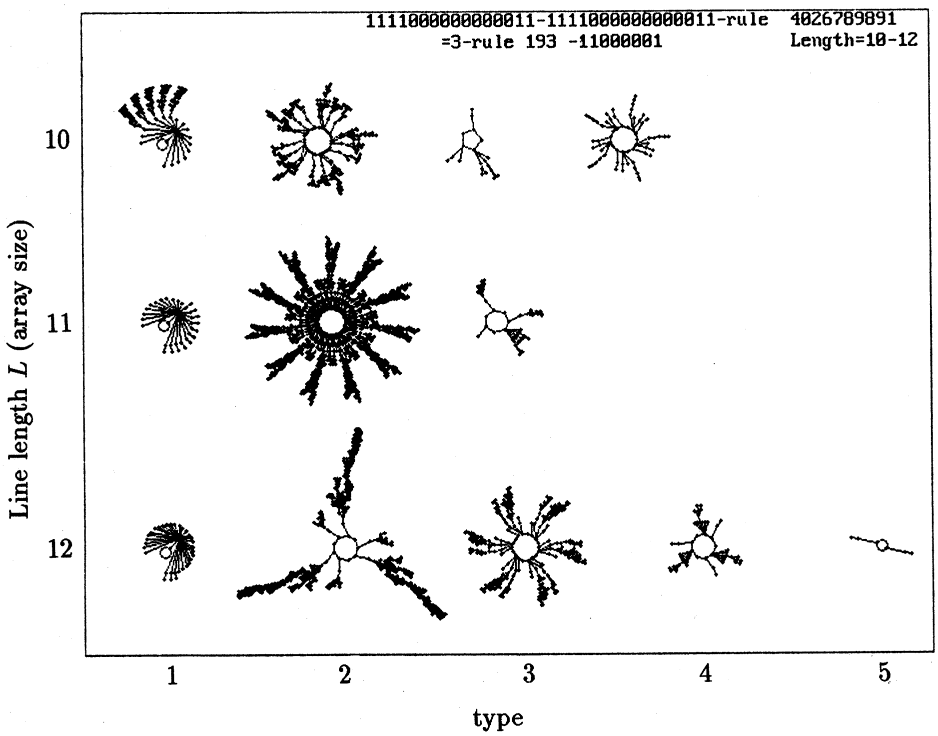
Example of a *single NAT*, line length, $L = 10$, singleton seed. Position: $x = 0$, $y = 0$. Levels: 15.



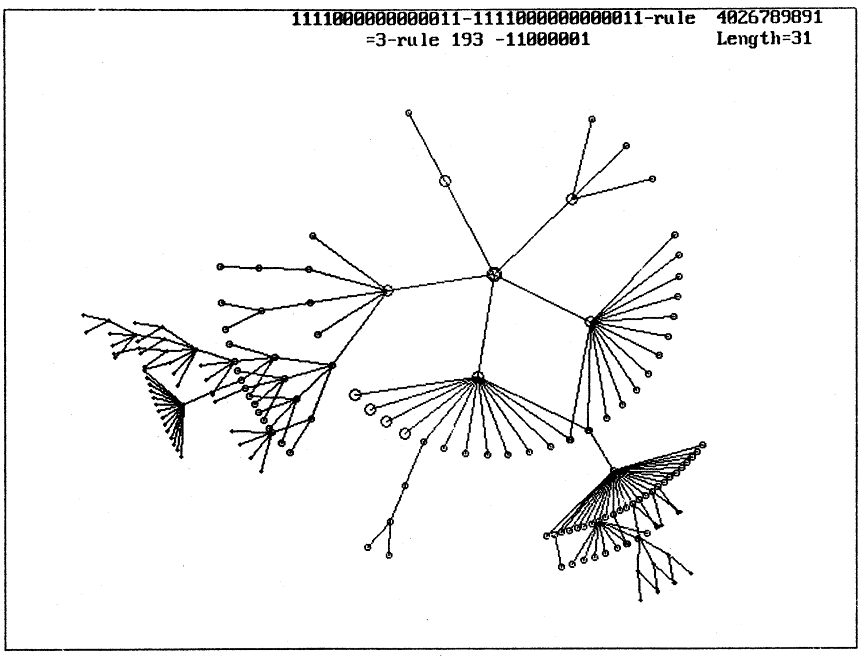
Example of *single NAT*, $L = 10$, transients suppressed, garden-of-Eden nodes suppressed, singleton marked, nodes numbered, singleton seed. Position: $z = -50$, $y = -20$. Angle: rotated -50 degrees. Levels: 4.



Example of single NATs, for range of $L = 6$ to 16, transients suppressed, garden-of-Eden nodes reduced, singleton seed. Position: $x = -70$, $y = 60$. Levels: 130. Separation: $x : 1$, $y : 1$. % increase: $x : 15$, $y : 15$.



Example of *NAT fields*, for range of $L = 10$ to 12. Note that only one example of each set of *rotation equivalent* NATs is drawn. Printed data on these NAT fields (see A1.2.3), and key to this data, is shown in A1.2.14. Position: $x = -80$, $y = 60$. Levels: 120. Separation: $x : 1.8$, $y : 1.8$. % increase: $x : 8$, $y : 25$.



Example of a ***transient branch only*** from the seed two steps forward in time from the state 1997870096. $L = 31$. Position: $x = 15$, $y = 17$. Levels: 6.

A1.2.14 Key to Printed Data

If the "`NAT info to printer`" option (A1.2.3) is selected, data will be printed as NATs are drawn. The key to the printed data is set out below.

Rule type, decimal and binary rule number

Line length

Total NATs in field

```
3-rule 193 =11000001
ty. at no(p)s g ml mp
L= 10
 1.1111111111 1(1)134 72 9 17
 2.0001111100 2(25)270 120 9 7
 3.0000001001 10(5)22 9 5 4
 4.1000100000 2(15)65 20 5 4
 total NATs = 15
L= 11
 1.11111111111 1(1)35 22 3 22
 2.11000100000 1(110)1518 682 20 10
 3.00000000111 11(7)45 21 5 6
 total NATs = 13
L= 12
 1.111111111111 1(1)34 29 3 29
 2.110001111100 4(9)831 402 35 13
 3.100011100000 2(18)312 138 10 6
 4.000001000001 2(9)51 27 4 7
 5.000100010001 2(2)6 2 2 2
 total NATs = 11
```

Reminder of data order:

- **ty** - type of basin
- **at** - a binary state belonging to the attractor cycle
- **no** - the number of equivalent basins of the type
- **(p)** - the attractor period
- **s** - the total states (nodes) in the basin
- **g** - total of garden-of-Eden states
- **ml** - maximum number of levels of the longest transient
- **mp** - maximum number of preimages to any state

A1.2.15 The Interrupt Message: To Print, Load, or Save the Screen Image

Except when printing, loading, or saving, "`key F12` + `RETURN`" will interrupt operation and the following interrupt message will appear in the top left-hand corner of the graphics screen, the dialogue area; in the setup screen the message appears in the text:

```
99-dump 88-continue 77-save
66-load RETURN-abort:99
```

`RETURN-abort`	▪ Returns to the beginning of the setup screen.
`99-dump`	▪ Prints the screen image to the printer. To continue from point of interrupt after printer dump, select 88 and RETURN twice. `key F11 interrupts printing` "error - 57 - printer not ready", may occur
`88-continue`	▪ Continues from the point of interrupt.
`77-save`	▪ Saves the screen image to disk.
`66-load`	▪ Loads a screen image file, or series of files, from disk. To continue from point of interrupt after loading, select 88 and RETURN twice.

The extension .NAT is automatically included in all graphics screen filenames. Omit the extension when entering a filename. Where indicated, *.NAT* files may be listed by responding with ? when prompted for the filename.

A1.2.16 Saving the Screen Image to Disk

at the interrupt message enter...	77
you will be prompted for...	`filename:`
answer with any legal name	
without extension, i.e....	d:rule193
while the file is being saved...	`d:rule193 - being saved`
will appear in the dialogue area.	

After saving, the interrupt message will reappear in the dialogue area. You may continue from the point of interrupt with 88.

A1.2.17 Automatic Screen Save

A sequence of graphics screens of a rule and its mutants may be saved automatically. This procedure is initiated in the setup screen (see A1.2.6).

Files will be saved automatically after each screen image is complete, with the mutant number added to the filename, i.e.,

myfile0	source rule
myfile1	1st mutant
myfile2	2nd mutant
⋮	⋮
myfile32	32nd mutant

If the start position was altered, for example, to 20, the file name of the first mutant will be "myfile20." A message in the dialogue area will indicate that a file is being saved, i.e., `d:myfile25-being saved`. When the last file has been saved, the interrupt message will reappear in the dialogue area.

A1.2.18 To Load a Screen Image File

at the interrupt message enter...	66
you will be prompted...	`for automutant-99 else RETURN:`
(for automutant see 2.19 below)	`filename (? to list, 99 to exit):`
(if the drive is omitted, the default is c:, or the last drive specified)	
enter the filename, i.e....	d:rule193
a chance for revision is offered...	`d:rule193.NAT -to revise enter 99:`
for overlay of images...	`keep screen-99 else RETURN:`

On RETURN, the image will be loaded. "`error - 53 - file not found`" may occur.

A1.2.19 To Load a Sequence of "Mutant" Files Automatically

at the interrupt message enter...	66
at the prompt...	`for automutant-99 else RETURN:`
enter 99...	`one screen-99 else RETURN:`
enter 99...	`filename (? to list, 99 to exit):`
enter the "mutant" filename,	
omitting the mutant number (see 2.17)...	d:myfile
a chance for revision is offered....	`d:myfile.NAT - to revise enter 99:`
you will be prompted...	`start mutant 1-32 (default=1):`
(select the start mutant number)	

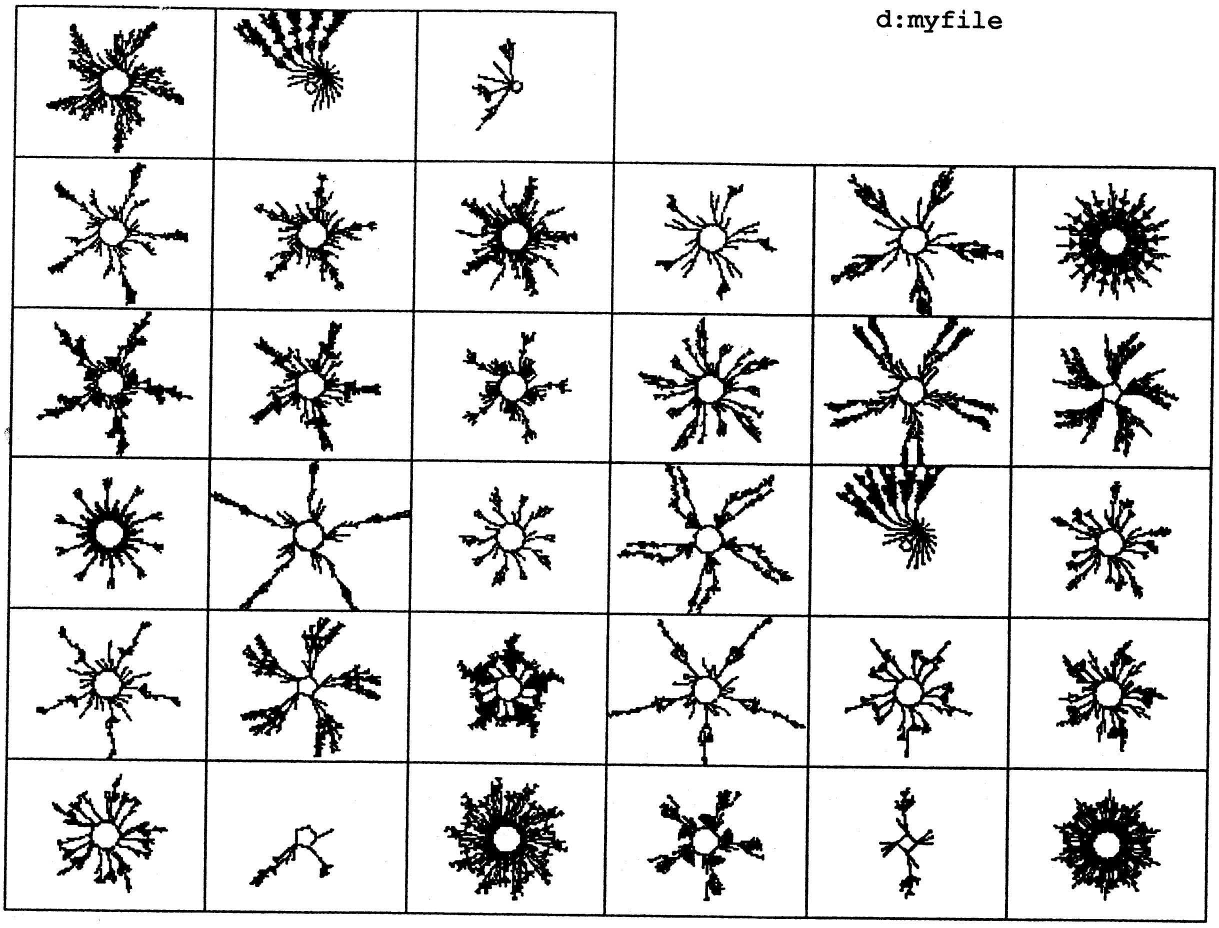


FIGURE A1.1 Composite screen: original rule, bottom left and the 32 mutants in ascending rows from left to right. Setup: rule 193, single NAT, $L = 10$, levels 15.

If "`one screen`" is selected, the sequence of mutant files will be loaded onto one composite screen, as shown in Fig. A1.1. Otherwise the "mutant" files will be loaded to the whole screen in sequence, with an optional pause between successive screens.

When the composite screen has been loaded, prompts appear in the top *right*-hand corner,

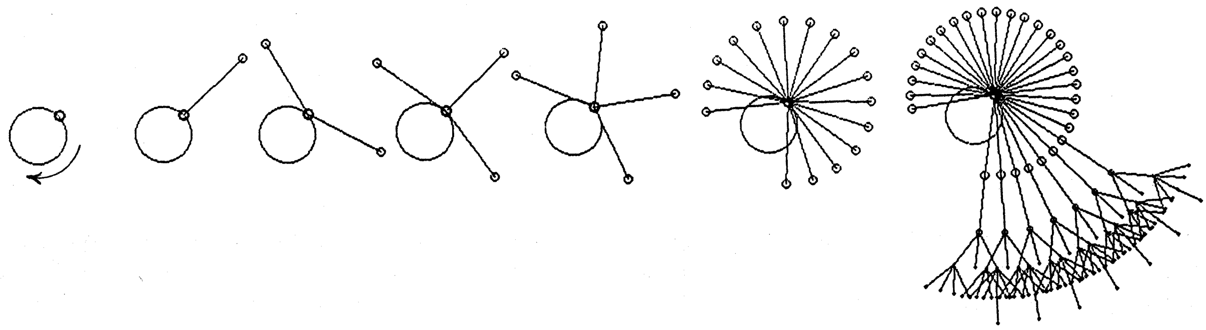
to dump to the printer...	`99 to dump:`
to save the composite screen to disc...	`99 to save:`
if save is selected...	`filename (default d:myfile-M):`
enter any legal filename ending with `-M`,	`d:193-M`

A filename ending with `-M` will be recognised as a composite screen filename. To load a composite screen file, proceed as for loading a single screen image file (see A1.2.18).

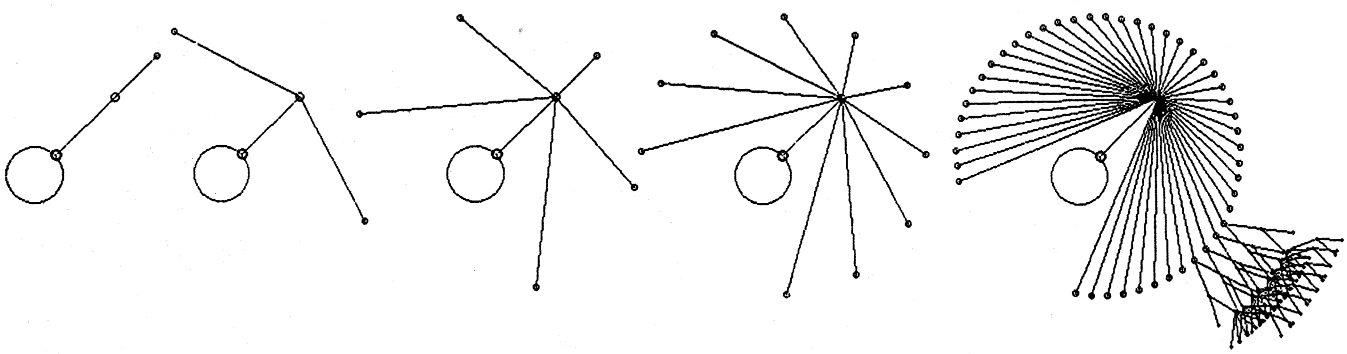
A1.3 Graphic Conventions

A1.3.1 Point Attractors

A point attractor is represented as a node cycling to itself. The centreline of the pre-image fan (level 1) is set at a default angle of 45 degrees, and the fan is spread out as illustrated below, to minimise overlap to transient branches. The centre line of subsequent pre-image fans (level 2+) are radial to the point attractor node.



The pre-image fan (level 2) of a single pre-image (at level 1) of a point attractor is spread out on a circle centred at the point attractor node, and radial to the level 1 pre-image. The centre lines of subsequent pre-image fans (level 3) are radial to the point attractor node.

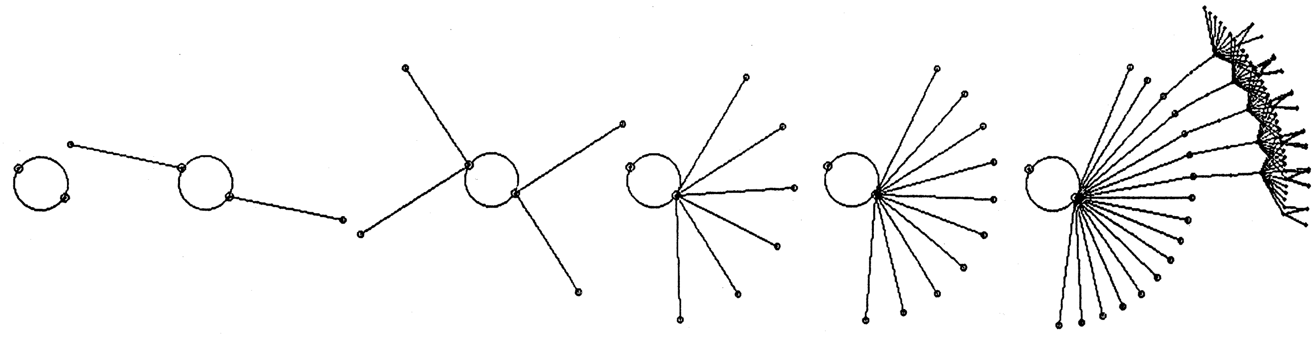


A1.3.2 Transient Branch Only

A NAT fragment representing an isolated transient branch from a given seed state is represented in a similar way to a point attractor, except that there is no indication of the seed node *cycling to itself*. The pre-image fan is evenly spaced around the seed node. (See the last example in A1.2.13.)

A1.3.3 Period-2 Attractors

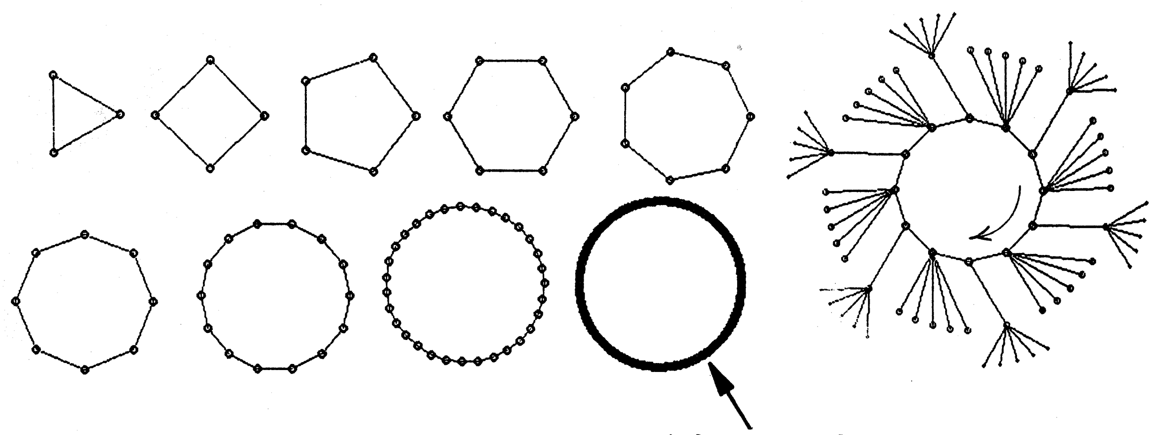
A period-2 attractor has a default orientation and spread-out pre-image fans as illustrated below. pre-image fans at level 1 are angled relative to the attractor cycle to indicate that evolution proceeds clockwise around the attractor. The centre lines of subsequent pre-image fans (level 2+) are radial to the notional centre point of the attractor cycle.



A1.3.4 Period-3+ Attractors

Attractor cycles with periods of 3 and above are represented as polygons. The diameter of the polygon asymptotically approaches an upper limit with increasing period, so that attractor cycles with larger periods are drawn with approximately the same diameter irrespective of the number of nodes in the attractor, as illustrated below.

Level 1 of each pre-image fan is angled relative to the centre of the polygon, to indicate that evolution proceeds clockwise around the cyclic attractor. By default, the first transient tree (if any) is attached to the node lying due east (0 degrees). The centre lines of subsequent pre-image fans (level 2+) are radial to the notional centre point of the attractor cycle.



period = 2047, rule 150,
$L = 23$, singleton seed

The default orientation of NATs may be rotated (see A1.2.9).

In transient trees, the gap between successive levels (representing time steps) decreases by a logarithmic function of the distance (in levels) from the attractor.

A1.3.5 Colours

Four colours—red, blue, green and yellow—are used for lines representing transitions (and four contrasting colours for nodes). Equivalent transients trees (and equivalent branches from uniform states) are coloured identically.

For attractor periods of 6 and above, successive nonequivalent transients trees (and branches) will be assigned the next available colour, thus cycling through the four colours. For attractor periods of 5 and below, successive fans will be assigned the next available colour.

Garden-of-Eden nodes and attractor cycle nodes without transients are coloured white.

A1.4 SPACE1: Space-Time Patterns

A1.4.1 Running the Program; the Set-up Screen

To run: insert the floppy disk into a drive, or save onto hard disk; if in drive A, at the DOS prompt, enter `A:SPACE1`. The setup screen (grey background) will appear, with this message:

```
free date bytes 503696 (or similar), at least 180000 required.
to quite Space1 enter 99 + RETURN, else RETURN:
```

The setup choices, as they would appear on screen, are summarised below, with a brief commentary on each choice.

In general, 99 selects an option, and RETURN the default. A series of prompts are presented to specify the pattern mode, rule parameters, and the seed line.

A1.4.2 Select Space-Time Pattern Mode

```
1. text mode (fast)..................max L -  80
2. graphics mode.....................max L -  79
3. graphics pixel mode (default)......max L - 640
enter 1, 2 or 3:
```

MODES 1 AND 2. The operation of modes 1 and 2 are basically the same, with space-time patterns presented as text on a scrolling screen. The differences between the two modes are as follows:

- Mode 1 is much faster than mode 2 (too fast for some applications).
- Mode 1 has 25 lines of scrolling text, whereas mode 2 has 29 lines.
- Mode 2 can be dumped to printer; mode 1 cannot.

RUNNING "FORWARDS" OR "BACKWARDS." In modes 1 and 2 (running "forward"), a repeat state, and thus the attractor cycle, may be identified. The CA may also be run "backwards"; pre-images (and pre-images of pre-images) may be generated from any seed state, thus potentially disclosing the seed's entire upstream configuration. If the seed state is on an attractor cycle, this configuration is the basin of attraction. If the seed is on a transient tree, the configuration is the transient branch joining the seed.

When running "forward" in modes 1 and 2, the decimal equivalent of the CA's global state will be given alongside the space-time pattern, up to $L = 53$.

MODE 3. In mode 3, space-time patterns are presented in high resolution on the basis of single screen pixels. The line length may be set to a maximum of 640. Patterns do not scroll; after 480 time steps, which fill the screen vertically, evolution of the CA is continued from the top of the screen by pressing RETURN.

Mode 3 does not allow for identifying repeat states, or running "backwards."

A1.4.3 Select the Type of Rule and Notation

The rule type may be selected as an $n = 5$ rule (in decimal, or binary), an $n = 3$ rule, or an $n = 5$ totalistic code (the specific rule will be chosen in A1.4.4). The default is an $n = 5$ decimal rule.

```
Select notation to specify 5-rule or subset
1) 5-rule Decimal(default)
2) 5-rule Binary
3) 3-rule
4) 5-rule Totalistic code
enter 1, 2, 3, 4, 5 or RETURN:
```

A1.4.4 Select the Rule

According to the choice of rule type and notation in section A1.4.3, the rule is selected in decimal or binary (the default is a random rule). For example,

```
select 3-rule notation 1) decimal (default), 2) binary
enter 1, 2 or RETURN:

enter 3-rule number 0 to 255 (default random):
enter 0s and 1s to make up an 8-bit binary number:
```

The rule table, the λ parameter, the λ ratio, and the Z parameter will be given for the chosen rule.

A1.4.5 Select the Line Length

The minimum line length is 1; the maximum line length (and default) depends on the mode:

- mode 1 - max L and default = 80
- mode 2 - max L and default = 79
- mode 3 - max L = 640, default = 150

For example,

```
select first line length (max 640) default 150:
```

In modes 1 and 2 only, the number of time steps for which the repeat check is active may be set. This allows attractor cycles to be identified. The default is 200. If 0 is set, the repeat check is deactivated.

```
extent of repeat check (default 200):
```

A1.4.6 Mutation and Automatic Mutation

While the CA space-time pattern is being generated on screen, a one-bit mutation may be made at a random position in the $n = 5$ rule table by the pressing ***key F1***. The rule just changed is restored by pressing ***key F2***. This may be done repeatedly to select interesting patterns. At each mutation, the colour of the space-time pattern changes from yellow to blue, or vice versa. In ***mode 3***, ***complex rules*** may be easily found using this method.

If *automatic mutation* is selected, the rule table will randomly mutate by one bit at every time step. During the run, ***key F1*** will toggle automatic mutation on and off. This will also change the colour of the space-time pattern between yellow and blue.

```
set automatic mutation enter 99:
```

To summarise, by default,	**key F1** - one-bit random mutation
	key F2 - restores rule just changed
If automatic mutation is set,	**key F1** - toggles automatic mutation on and off

A1.4.7 Select the Seed State

Various options are offered for specifying the seed state. The default is random. In mode 3, the binary option allows a central zone of the seed line (max 79 cells) to be defined, with other cells set to 0.

```
enter 0s and 1s to make up a line  78  characters long
------------------------------------------------------------------------------
100000000000010000000000001000000000000100000000000010000000000001000000000000
■············■············■············■············■············■············
111111111111110010010010010100100100100111111111111110010010010010100100100100
010010010010011111111111111001001001001010010010010011111111111111001001001001
001001001001000100100100100010010010010001001001001000100100100100010010010010
100100100100101001001001001111111111111110010010010010100100100100111111111111
alternative  1
■■■■■■■■■■■■■■··■··■··■··■·■··■··■··■··■■■■■■■■■■■■■■··■··■··■··■·■··■··■··■··
110110110110110011110011110001010001010000000000000000101000101000111100111100
011011011011011110011110011100111100111101101101101101000101000101010001010001
000000000000000101000101000111100111100110110110110110011110011110001010001010
101101101101101000101000101010001010001011011011011011110011110011100111100111
alternative  2
·■··■··■··■··■■■■■■■■■■■■■■··■··■··■··■·■··■··■··■··■■■■■■■■■■■■■■··■··■··■··■
011110011110011011011011011001111001111000101000101000000000000000101000101000
110011110011110110110110110100010100010101000101000101101101101101111001111001
101000101000101101101101101111001111001110011110011110110110110110100010100010
000101000101000000000000000101000101000111100111100110110110110110011110011110
alternative  3
```

FIGURE A1.2 Mode 2: "backwards." Example of running "backwards" from a given seed line, length 78, $n = 3$ rule 105. Mode 1 is similar but faster, and may not be printed.

```
select seed line
1) Singleton
2) Random (default)
3) Random-symmetric
4) Decimal
5) Binary
enter 1, 2, 3, 4 or 5:
```

A1.4.8 Pre-images

As described in A1.4.2, in modes 1 and 2 only, the pre-images (and pre-images of pre-images) will be generated if running "backwards" is selected:

```
Run CA forwards(default)-1 or backwards-2, enter 1 or 2:
```

If "backwards" is selected, a further option is offered, to start running backwards from a seed at a given number of steps forward in time from the selected seed state.

```
forward before running backward,
how many steps forward (default 0, max (varies)):
```

Running forward by at least one step before running backwards will ensure the existence of pre-images. A random state is very likely to have no pre-images (a garden-of-Eden state).

When run "backwards," "`error - 14 - out of memory`" may occur, if the configuration of rule and seed, combined with an excessive line length, results in too many *partial pre-images* or *pre-images*.

```
enter 0s and 1s to make up a line  78  characters long
------------------------------------------------------------------------------
110110110110110011110011110001010001010000000000000000101000101000111100111100
```

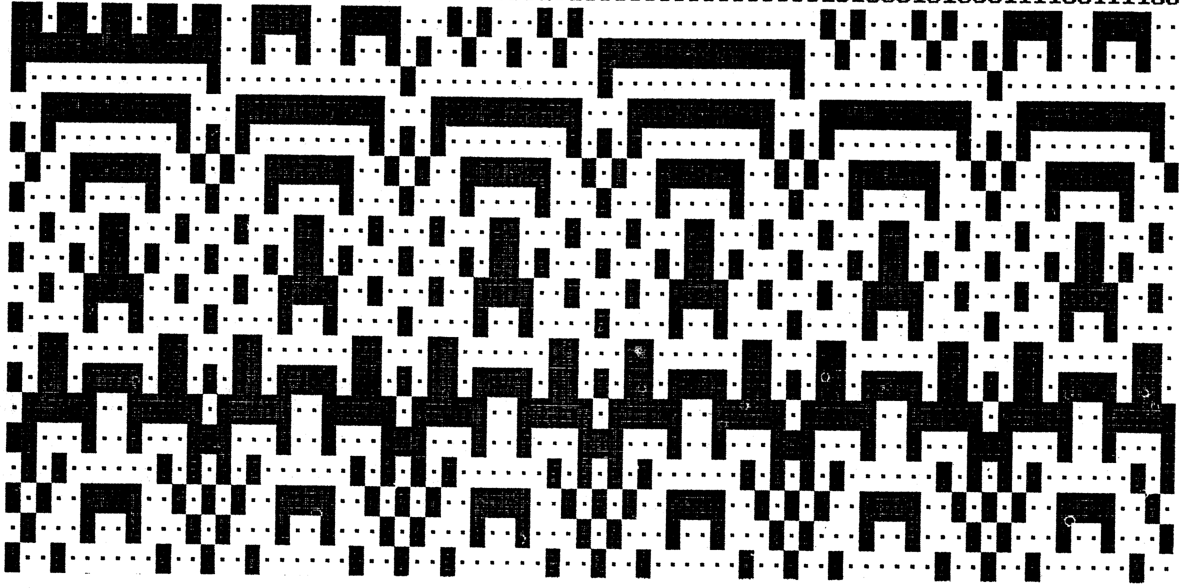


FIGURE A1.3 Mode 2: "forwards." Example of running "forwards" from a given seed line, length 78, $n = 3$ rule 105. Mode 1 is similar but faster, and may not be printed.

A1.4.9 The Graphics Screen and Examples

When the setup options have been selected, the space-time pattern will be generated on a black background. Modes 1 and 2 will scroll the space-time pattern; mode 3 will present successive screens. See Figures A1.2–5.

A1.4.10 The Interrupt Message, the Revision Menu, and Printing

Except when printing, **key F12 + RETURN** will interrupt operation and the interrupt message will appear in the bottom right-hand corner of the graphics screen, the dialogue area; in the setup screen the message appears in the text:

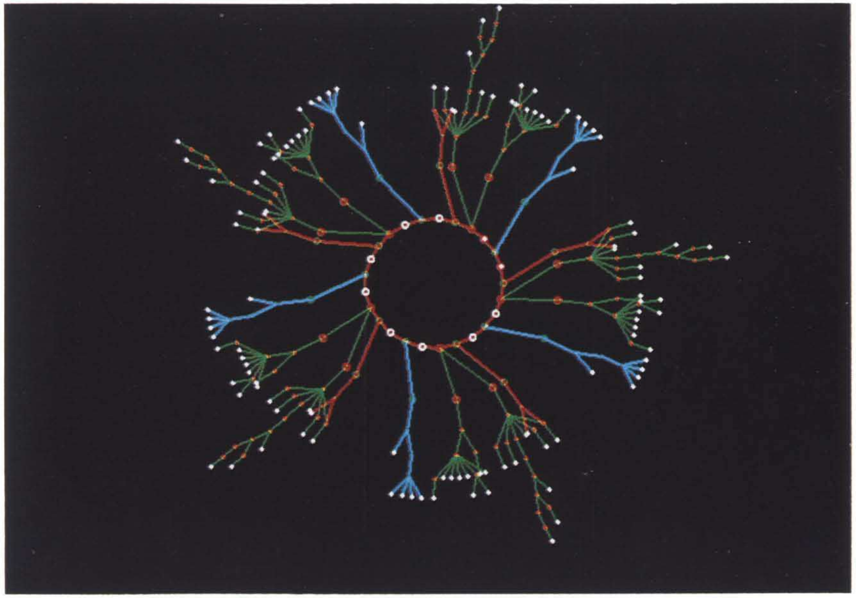
`continue-RETURN`	▪ Continues from the point of interrupt.
`printer dump-77` (not in mode 1)	▪ Prints the screen image to the printer. The latest (mutated) rule number (and λ, λ ratio and Z) will be given. To continue after printer dump, select 88 then RETURN twice. **Key F11** interrupts printing "error - 57 - printer not ready", may occur
`revision menu-88`	▪ See A1.4.11 below
`start menu-99`	▪ Returns to the beginning of the setup screen.

There is no facility to save and load screen images in Space1. The graphics are typically generated as quickly in the program as loading a file.

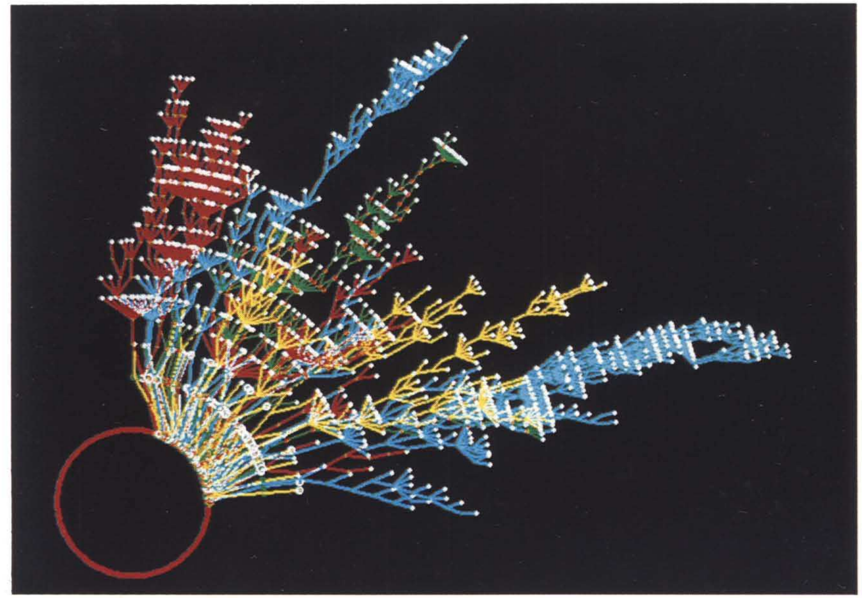
A1.4.11 The Revision Menu

The revision menu is presented after being selected from the interrupt options, or, if a repeat state has been identified, in modes 1 and 2. The revision menu allow a new seed state and new line length to be selected, for the current rule.

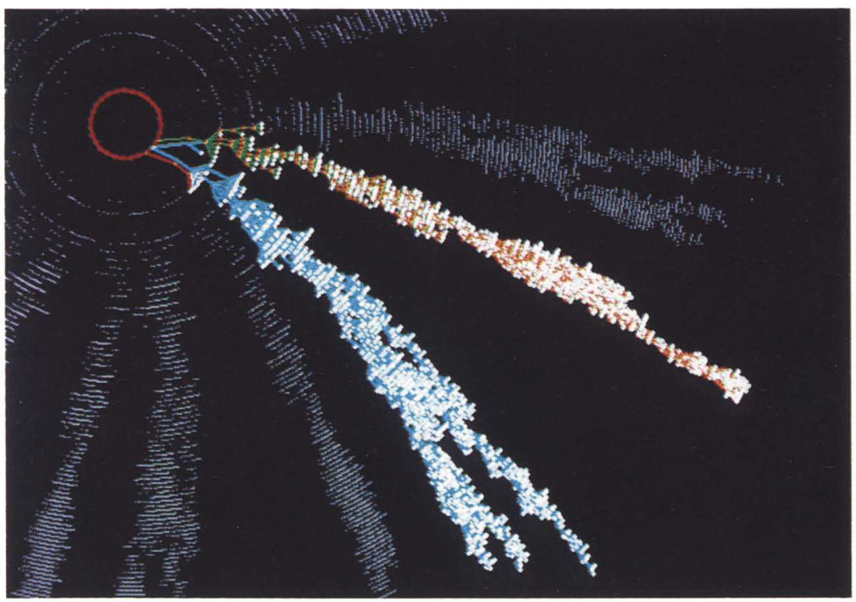
Color Plates



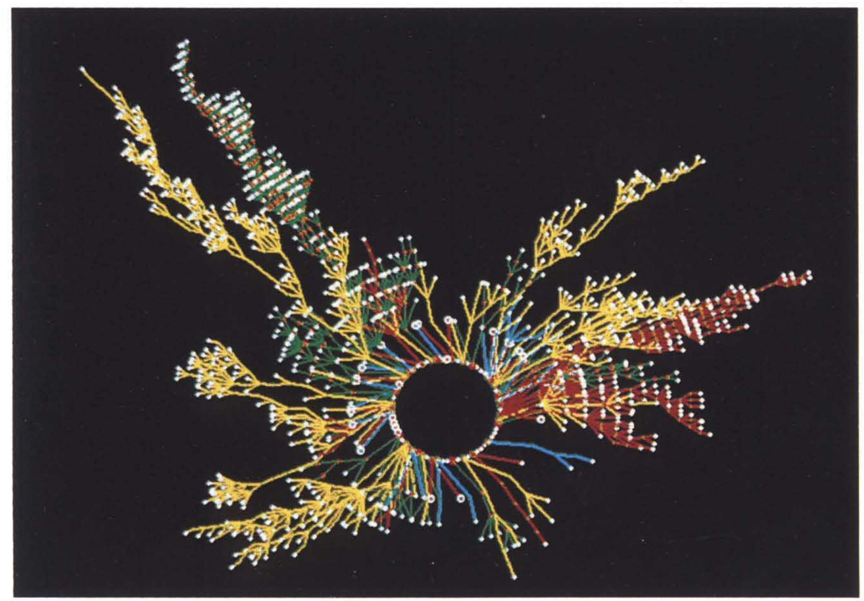
n=3 rule 193, L=10, seed singleton



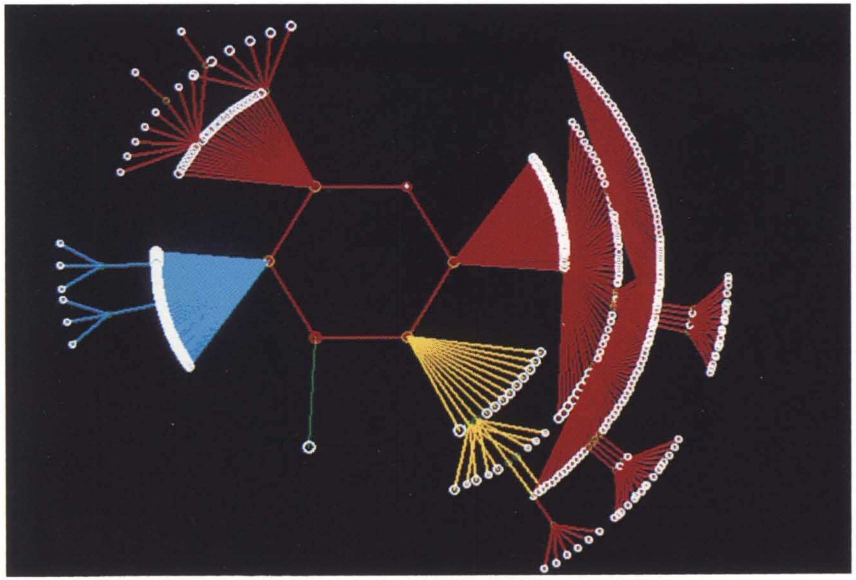
n=3 rule 193, L=15, seed 110011000001101
Equivalent transient trees suppressed.



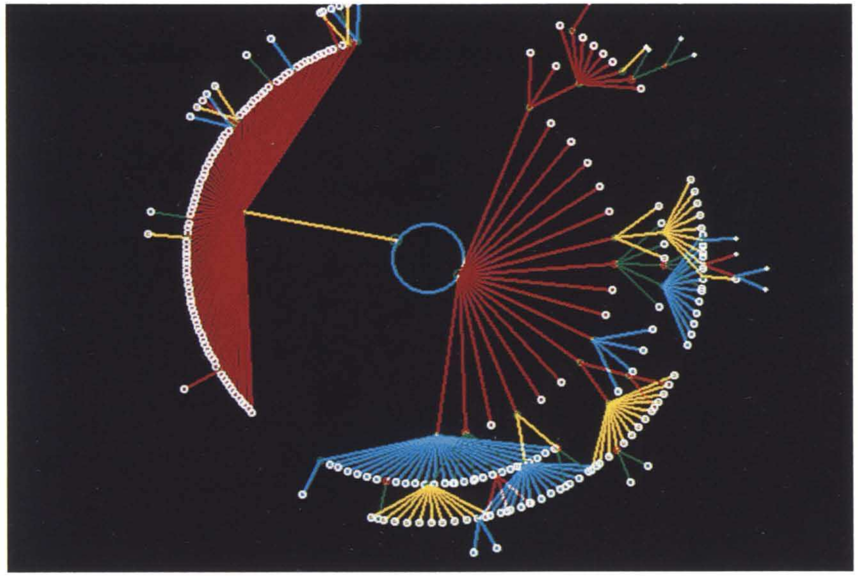
n=3 rule 193, L=18, seed 011010001110000010



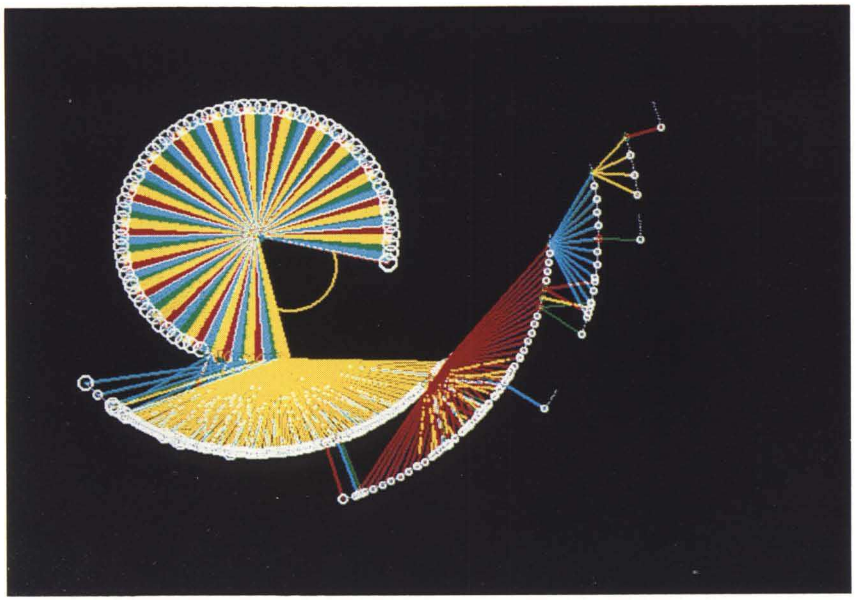
n=3 rule 41 (mutant 1), L=16, seed 0001010001000101



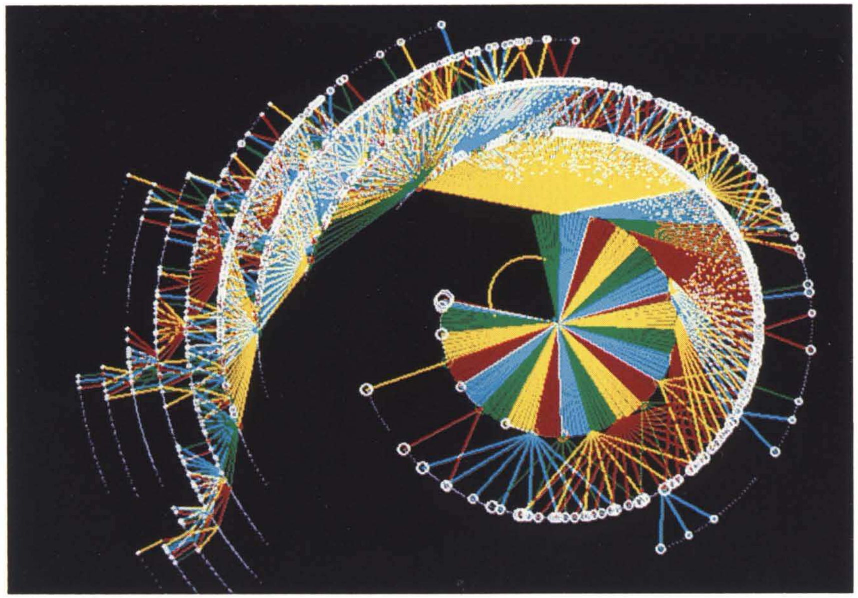
n=3 rule 18, L=18, seed 101000000101000000



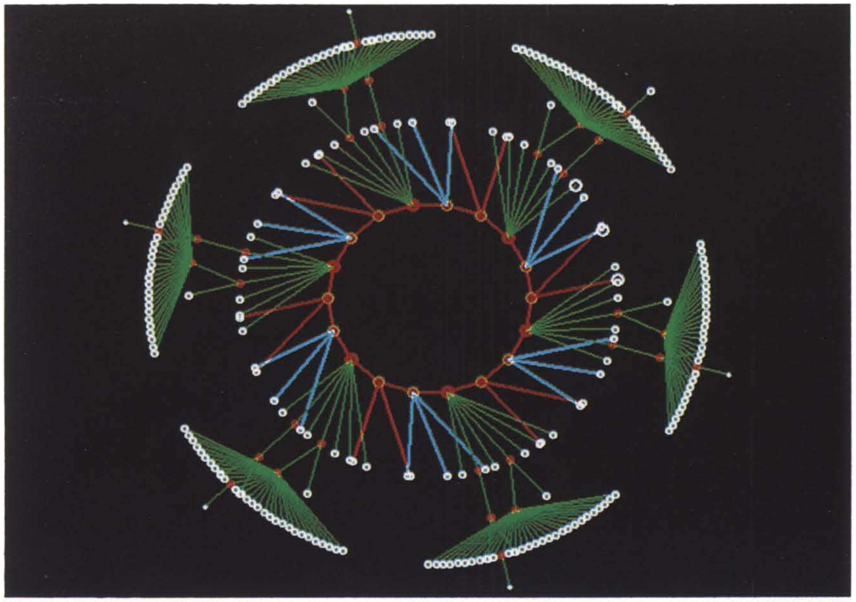
n=3 rule 33, L=16, seed 0111111111100000



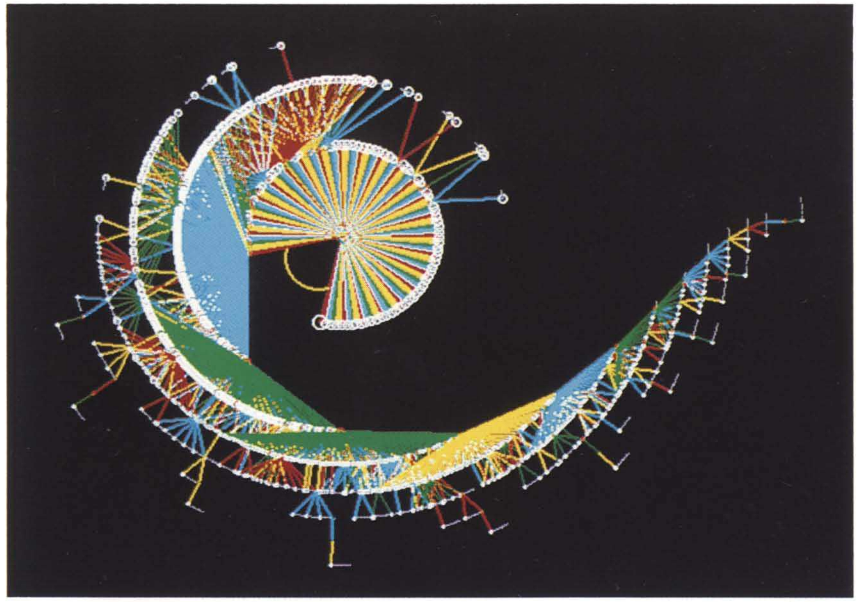
n=3 rule 251, L=12, seed singleton
Equivalent transient branches suppressed.



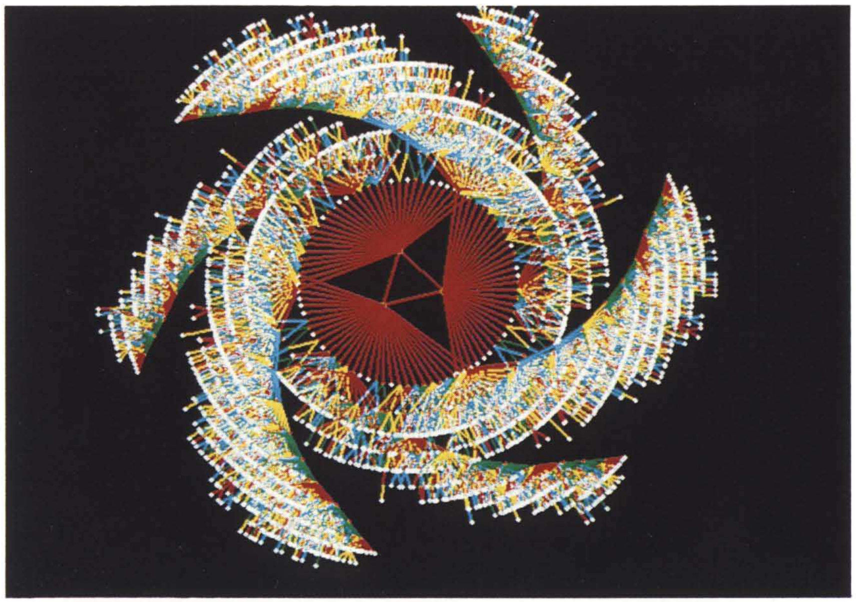
n=3 rule 249, L=15, seed all 0s
Equivalent transient branches suppressed.



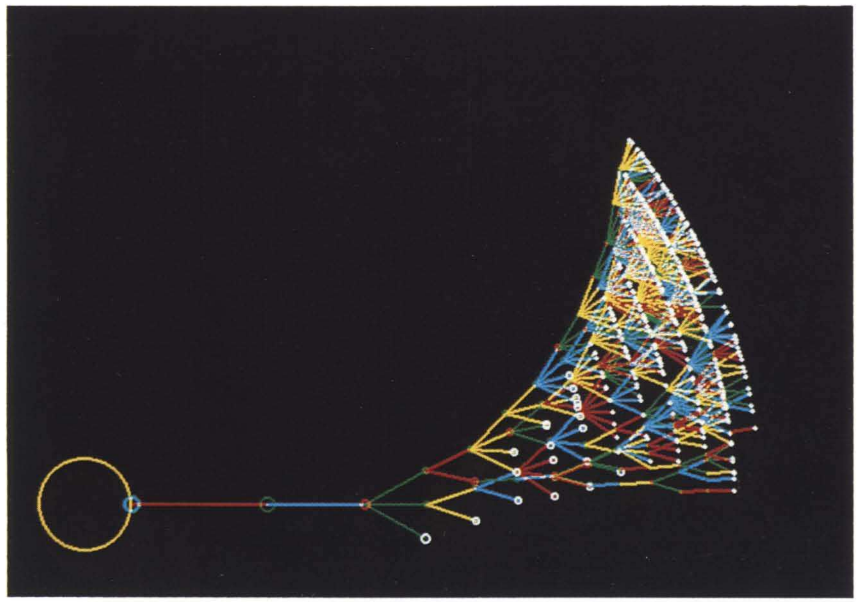
n=3 rule 9, L=12, seed 010000010001



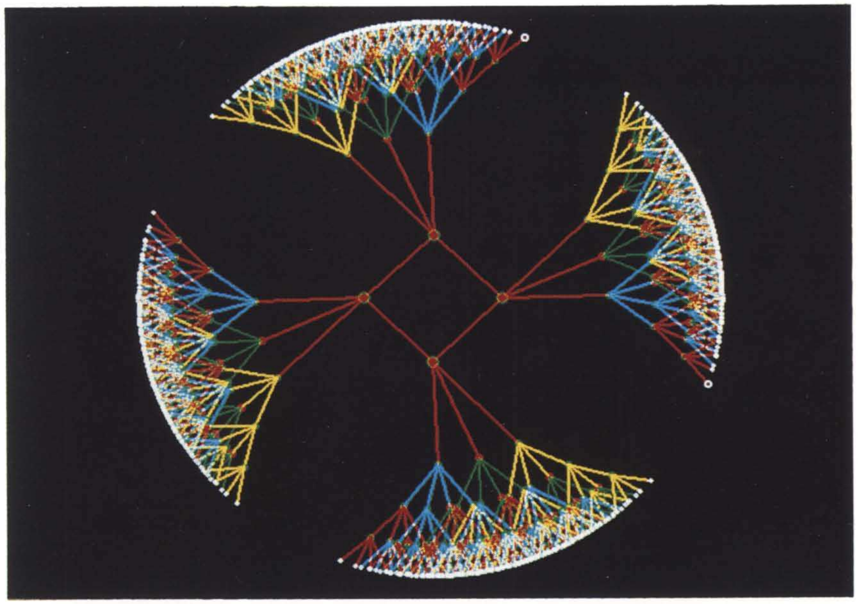
n=3 rule 250, L=15, seed all 1s
Equivalent transient branches suppressed.



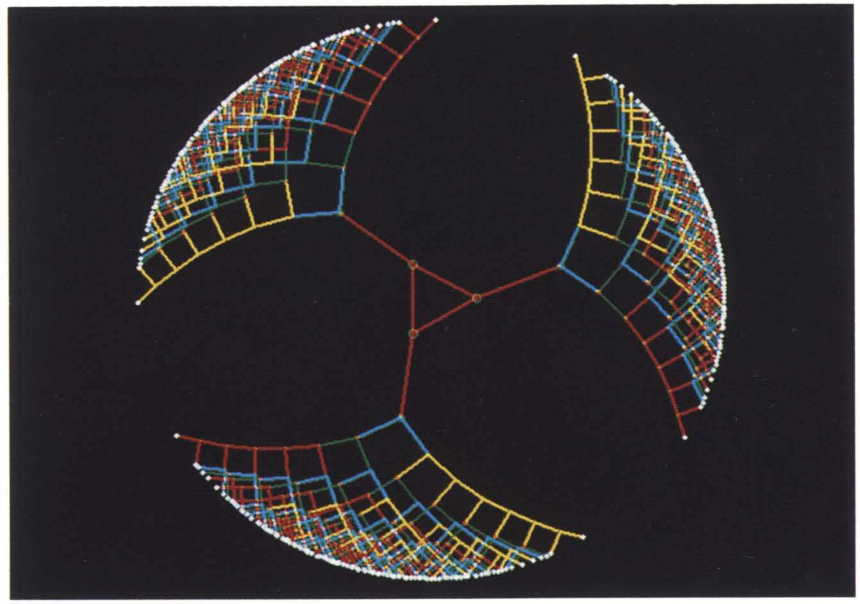
n=3 rule 58, L=15, seed 011011011011011



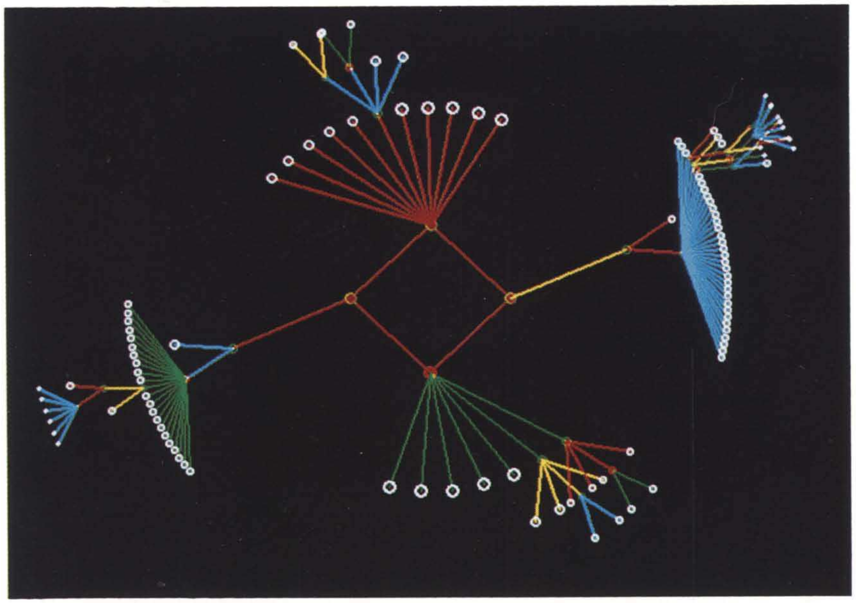
n=3 rule 228, L=15, seed singleton



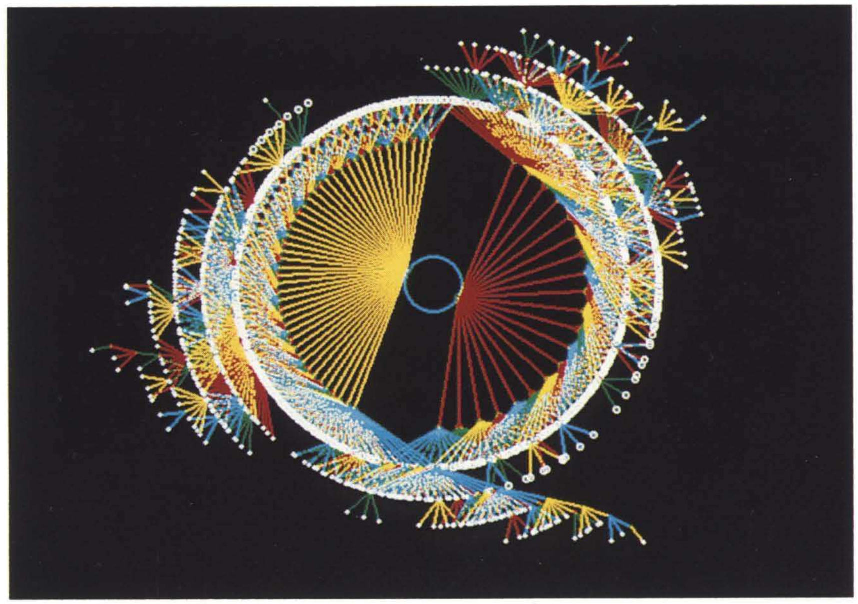
n=3 rule 90, L=24, seed 100000000000100000000000



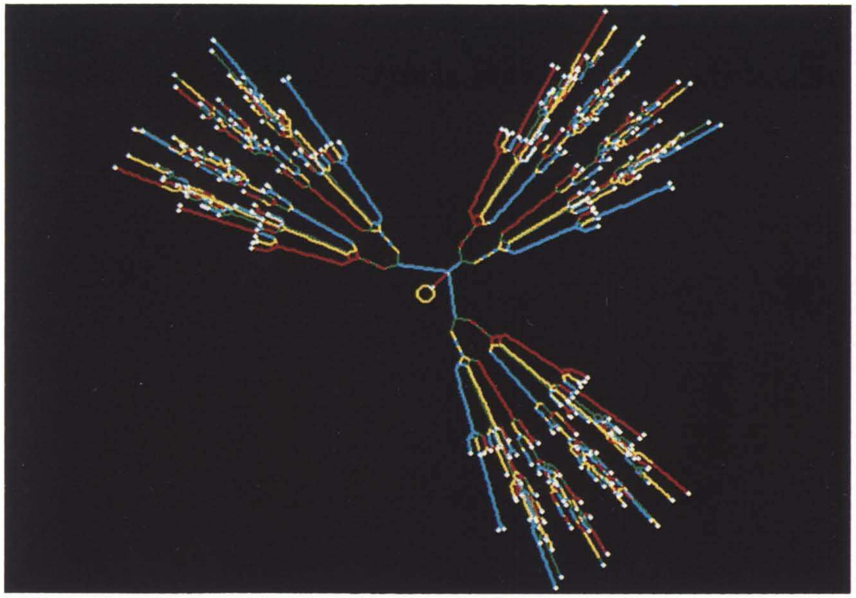
n=3 rule 60, L=24, seed 011011011011011011011011



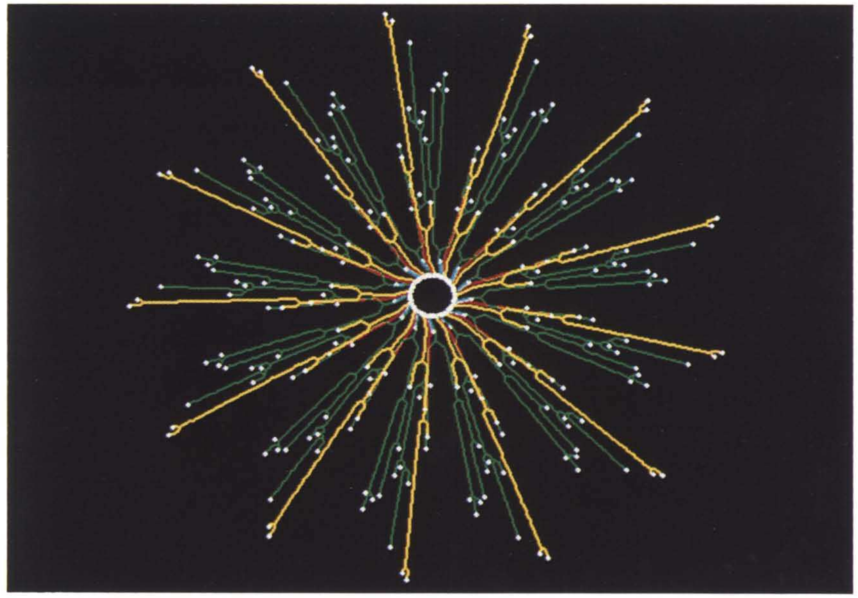
n=3 rule 126, L=11, seed singleton



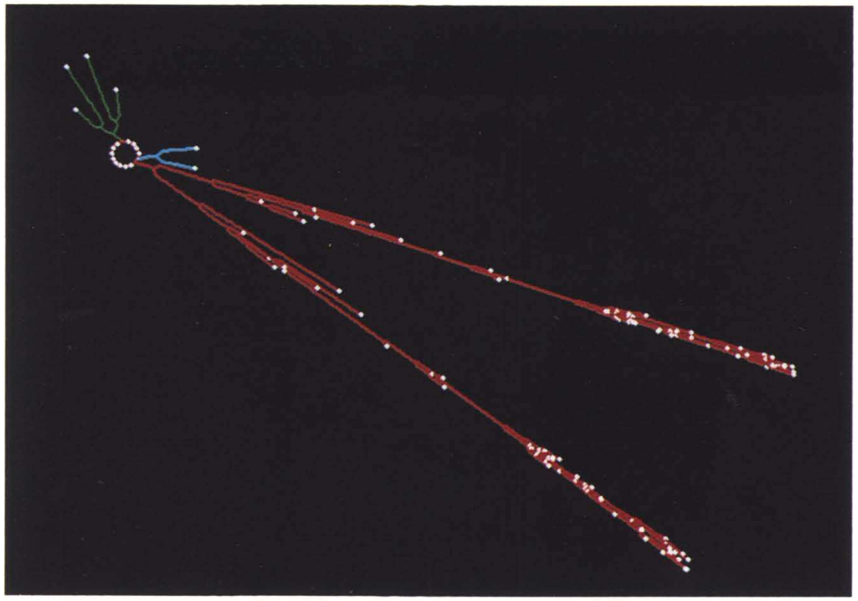
n=3 rule 51 (mutant 1), L=31, seed singleton



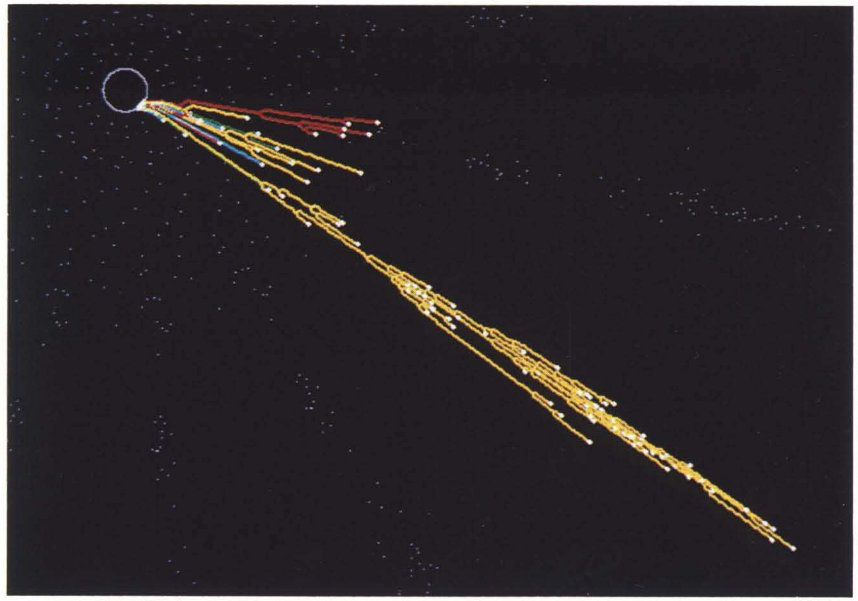
n=3 rule 225, L=12, seed all 0s



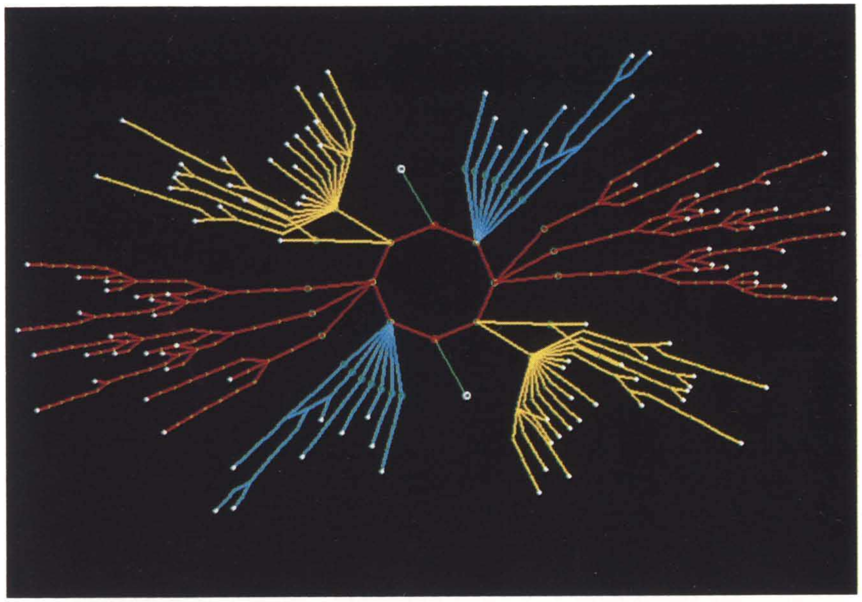
n=3 rule 225, L=13, seed 2769



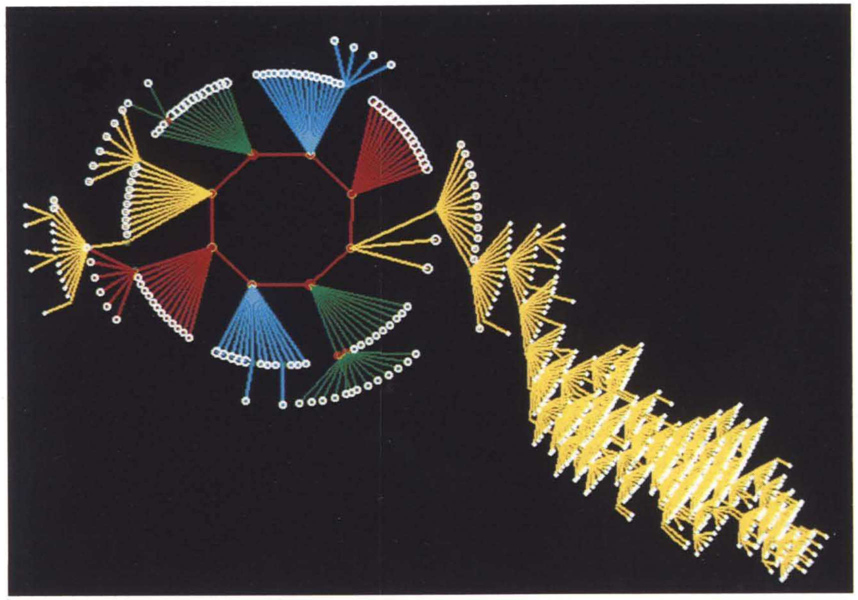
n=3 rule 225, L=16, seed 1000000010000000



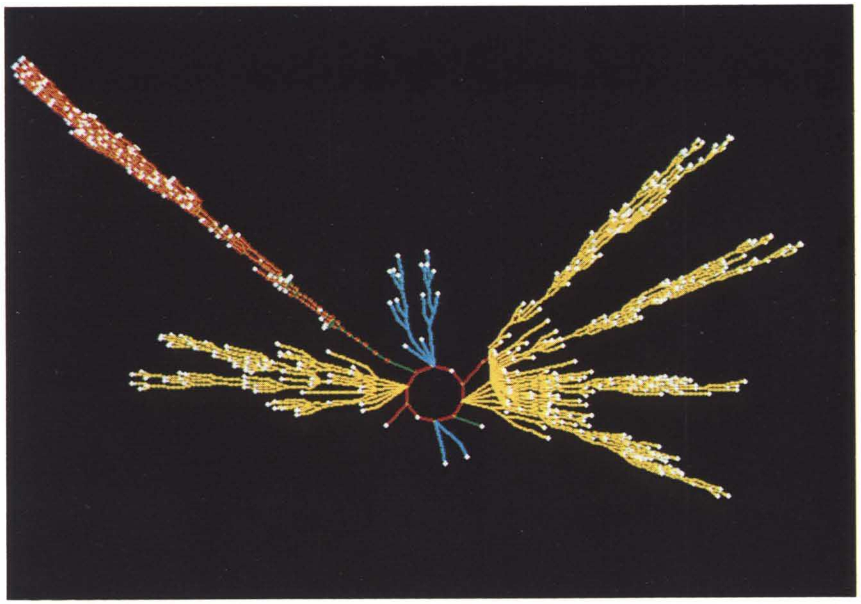
n=3 rule 30, L=15, seed 28096
Equivalent transient trees suppressed.



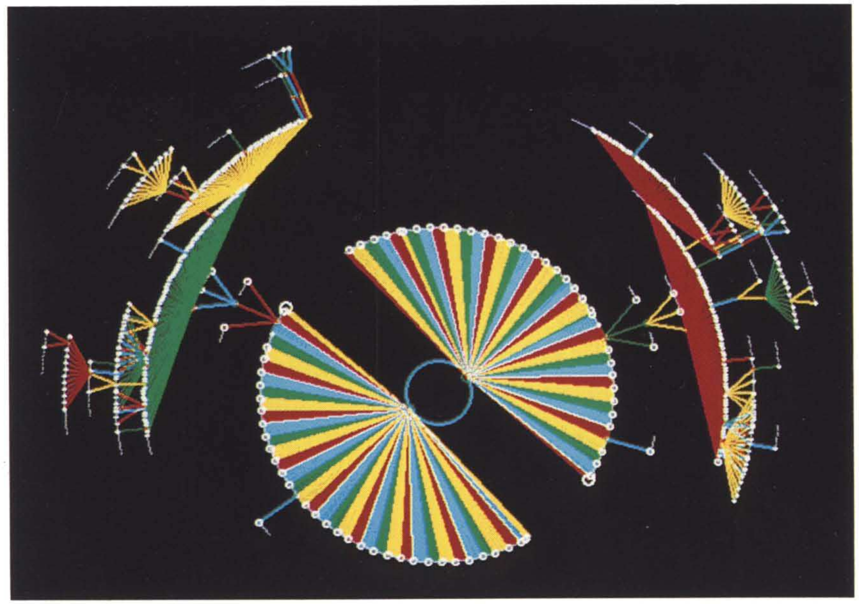
n=5 code 10, L=16, seed singleton



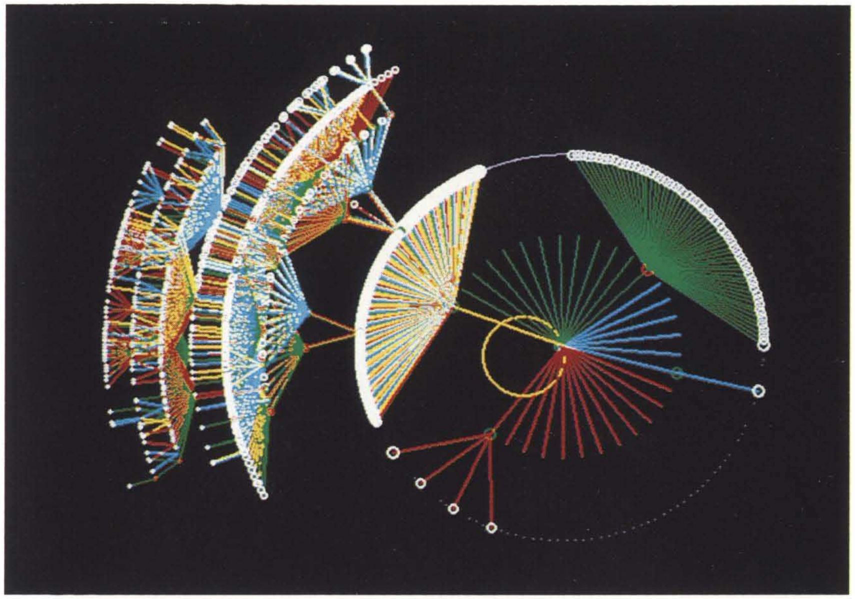
n=5 code 10, L=15, seed singleton



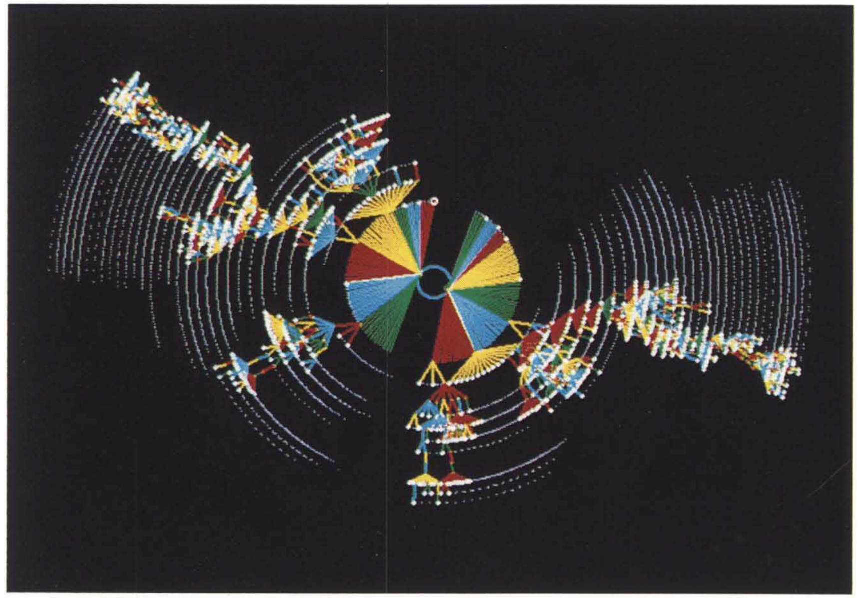
n=5 code 10, L=19, seed singleton



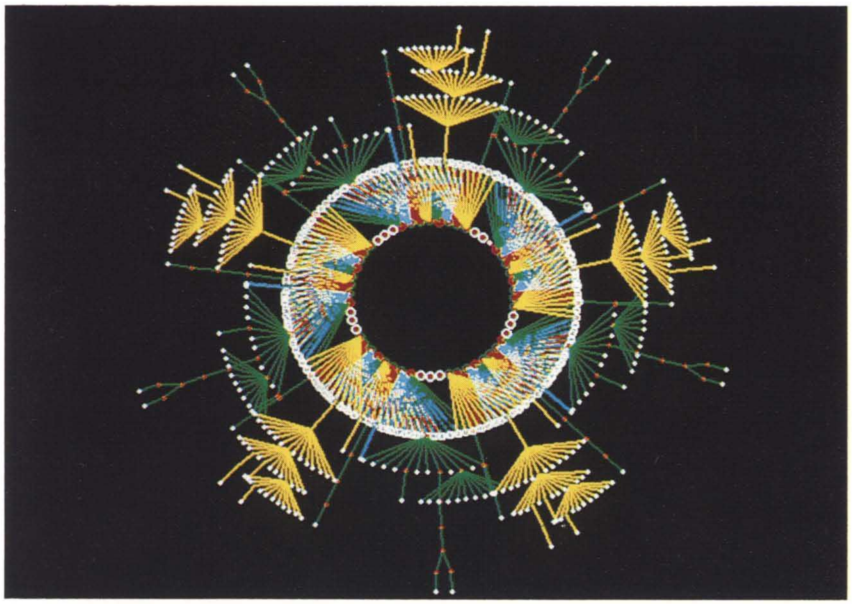
n=5 code 25, L=15, seed all 0s
Equivalent transient branches suppressed.



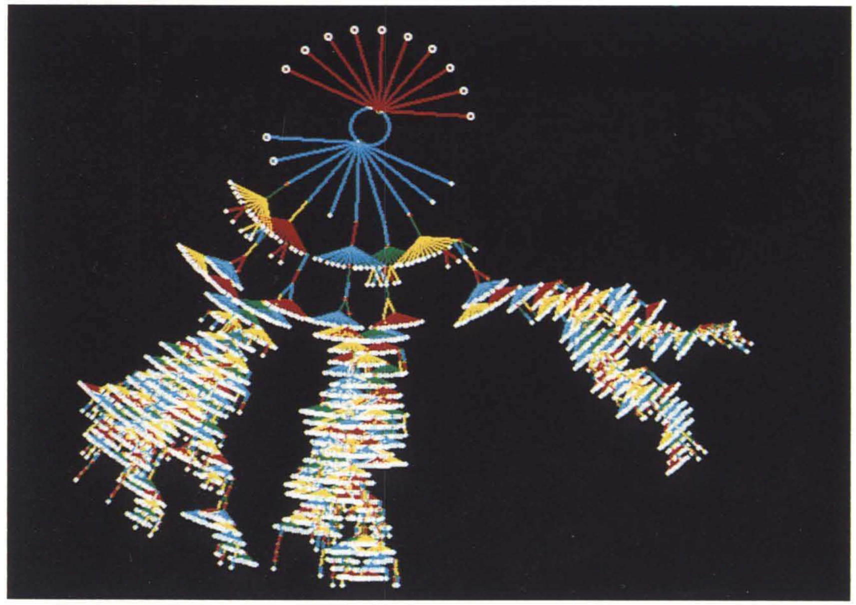
n=5 code 14, L=12, seed all 0s
Equivalent transient branches suppressed.



n=5 code 11, L=15, seed all 0s
Equivalent transient branches suppressed.



n=5 code 53, L=15, seed 101111101101100



n=5 code 53, L=15, seed singleton

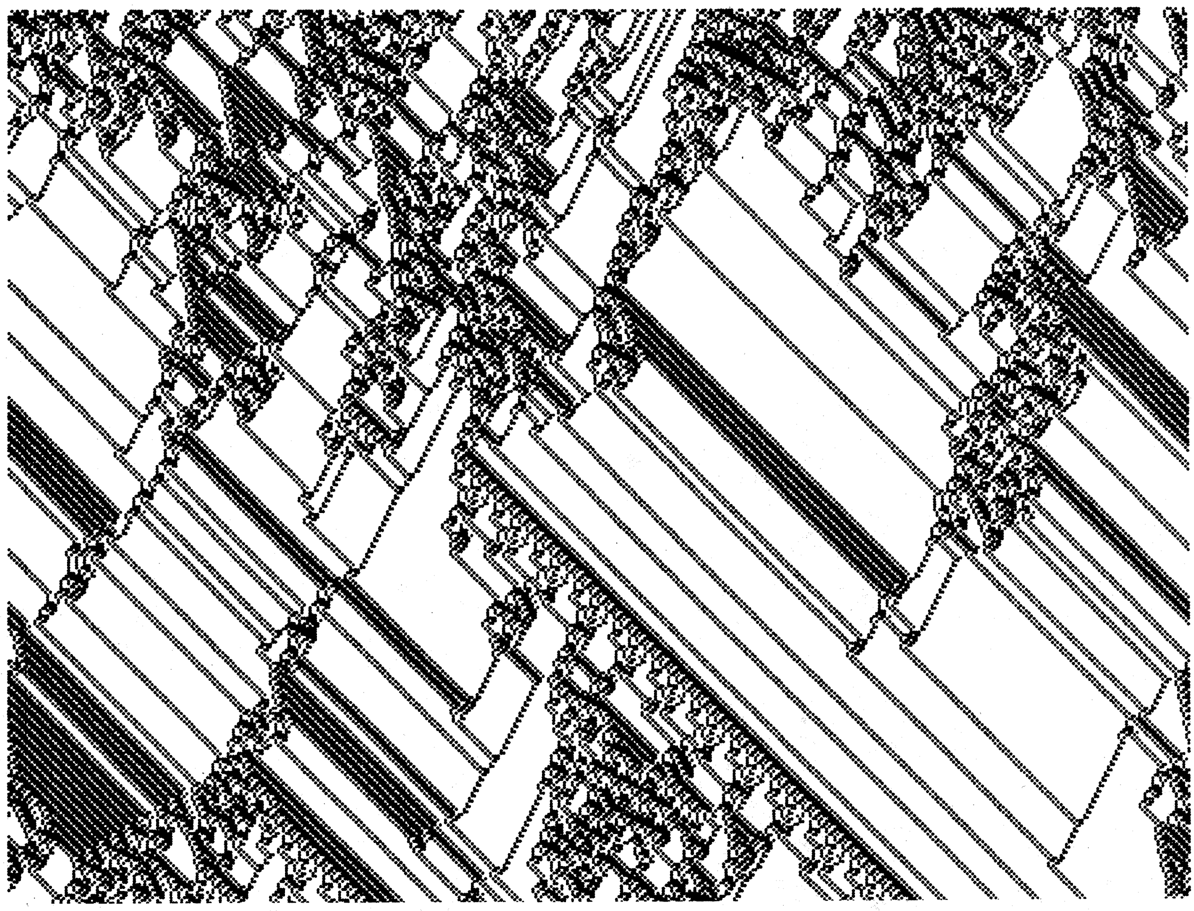


FIGURE A1.4 Mode 3. $n = 5$ rule 4234825112 ($Z = .6171875$, λ ratio $= .9375$), $L = 450$, 330 time steps. The rule "evolved" as illustrated in Fig. A1.5(a).

The latest (mutated) rule number (in decimal and binary), and the λ parameter, λ ratio, and the Z parameter will be given.

When running forward using *modes 1 and 2 only*, if a repeat state is found, the space-time patterns will stop. Information regarding the attractor cycle will be given.

The revision menu will then be presented, with the added options of selecting the *repeat state* as the new seed, and running forwards or backwards. A repeat state must be on the attractor cycle and thus have at least one pre-image.

A1.4.12 Color Cells According to Neighbourhoods

Each cell may be assigned a color depending on the neighbourhood which determined the cell's value. Colors are assigned to the rule table from a palette of 16 available colors. Black represents the neighbourhood all zeros, and white all ones. The color option is selected in the set-up screen, or toggled on and off during the run with key F3.

(a)

(b)

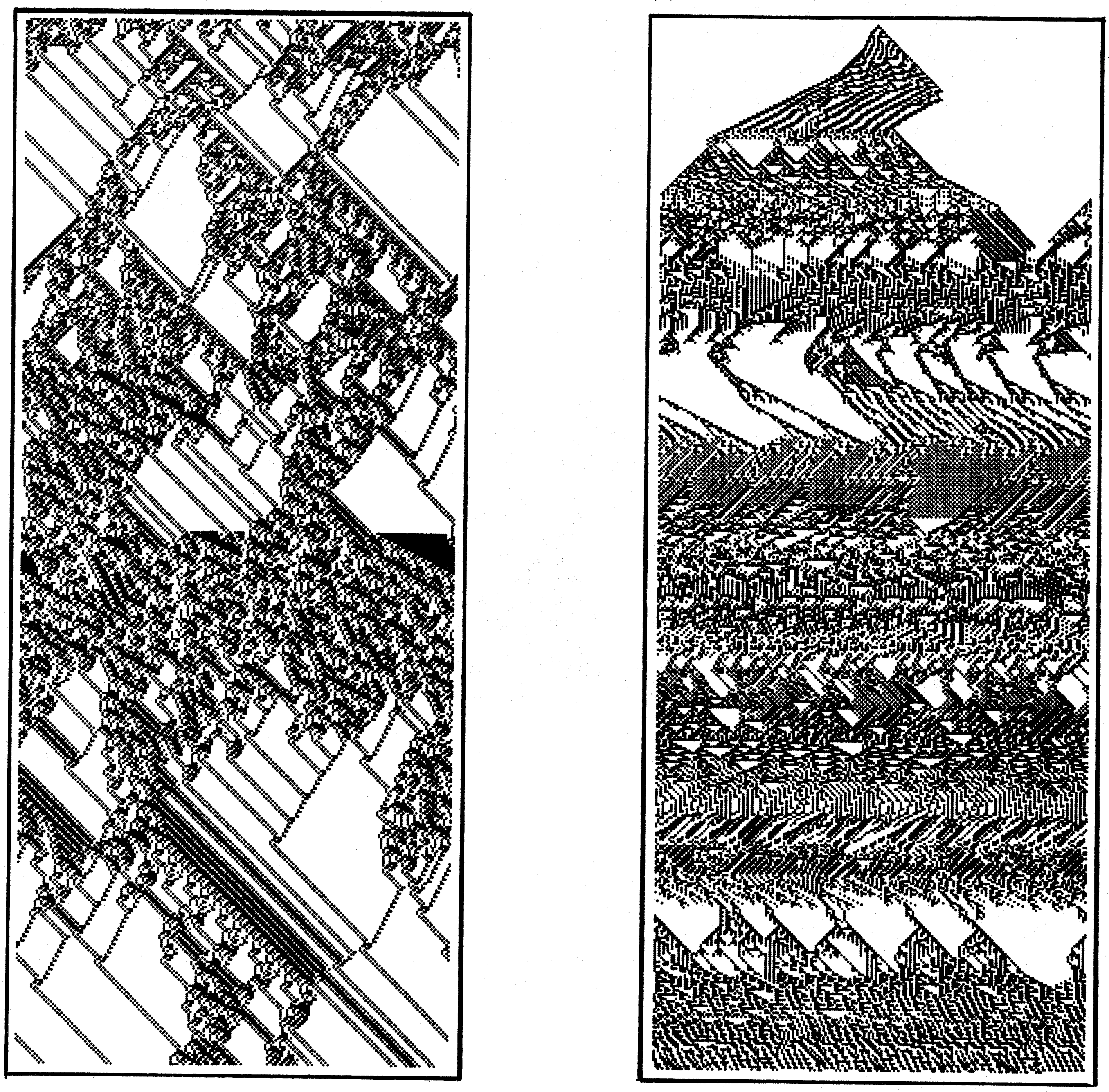


FIGURE A1.5 (a) Mode 3: one-bit mutations. 5-rule 1550470552 (see chapter 4, Fig. 4.1(b)) with a number of one-bit mutations during the run to "evolve" the $n = 5$ rule 4234825112, (see Fig. A1.4). 480 time steps. (b) Mode 3: automatic mutations. $n = 3$ rule 30, with a one-bit random mutation at each time step. $L = 200$. 480 time steps.

APPENDIX 2

Atlas of Basin of Attraction Fields

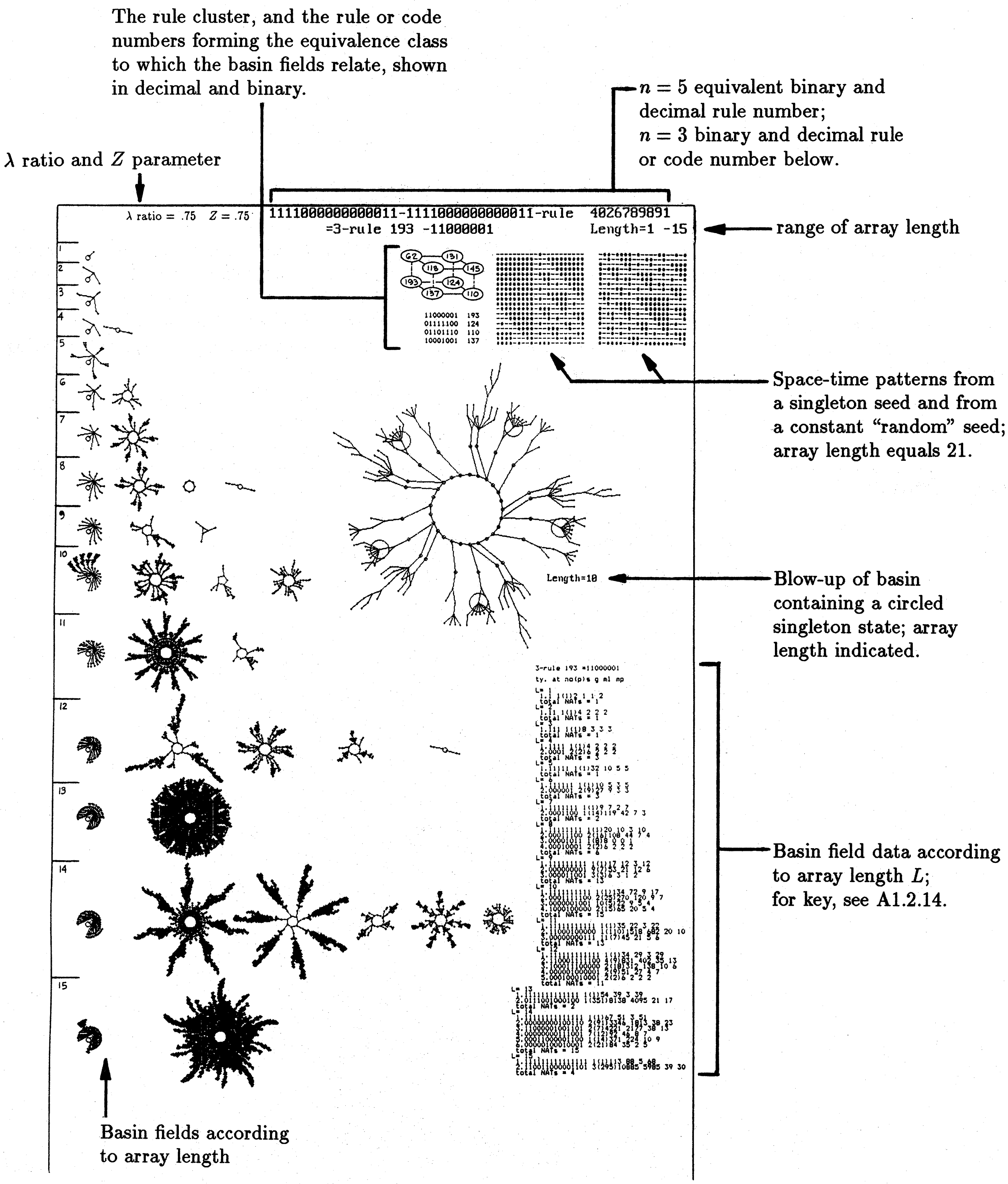
CA rules are presented in sequence according to the rule in the top left hand corner of the rule cluster. Complementary equivalence classes are presented on facing pages. Subject to available space, basin of attraction fields and data are shown for a range of array length L of 1 to 15 for $n = 3$ rules, and 3 to 16 for $n = 5$ totalistic codes. All basin of attraction fields are drawn to the same scale. The scale of the blow up of a typical basin varies.

Note that some pages are left intentionally blank in order for rules and their complements to appear on facing pages.

A2.1 Contents

A2.2 Key to Basin Field Presentation

A typical page from the Atlas of basin of attraction fields is annotated below.



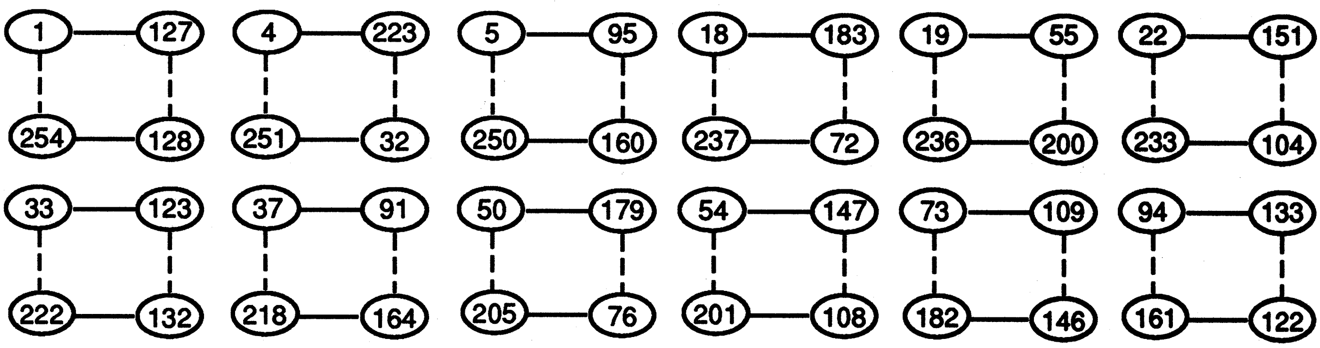
A2.3 n=3 Rules

A2.3.1 Index

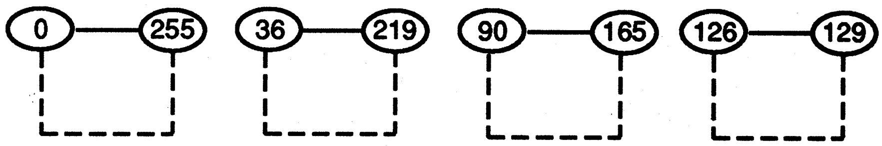
Key: [decimal rule number],*[hex rule number]*-[page number].

A2.3.2 Symmetric Rule Clusters (see section 3.3.8)

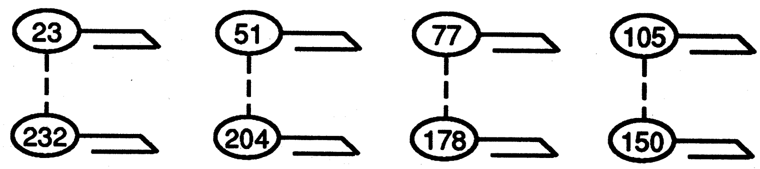
By definition, $R = R_r$, so the reflection links will collapse.



The cluster will collapse further, if for a given rule R, $R_c = R_n$



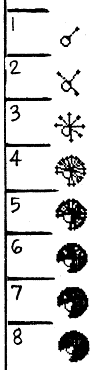
and also if $R = R_n$

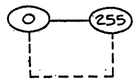


0				4				1				5			
		18				22				19				23	
	72				76				73				77		
			90				94				91				95
32				36				33				37			
		50				54				51				55	
	104				108				105				109		
			122				126				123				127
128				132				129				133			
		146				150				147				151	
	200				204				201				205		
			218				222				219				223
160				164				161				165			
		178				182				179				183	
	232				236				233				237		
			250				254				251				255

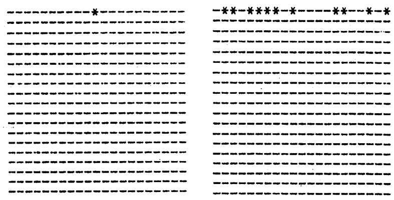
The rule-space matrix (see appendix 4).

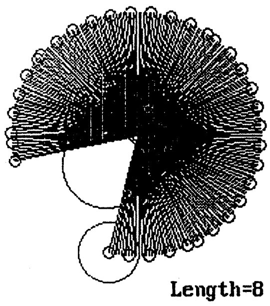
λ ratio = 0 $Z = 0$ 0000000000000000-0000000000000000-rule 0
=3-rule 0 -00000000 Length=1 -8





00000000 0
11111111 255

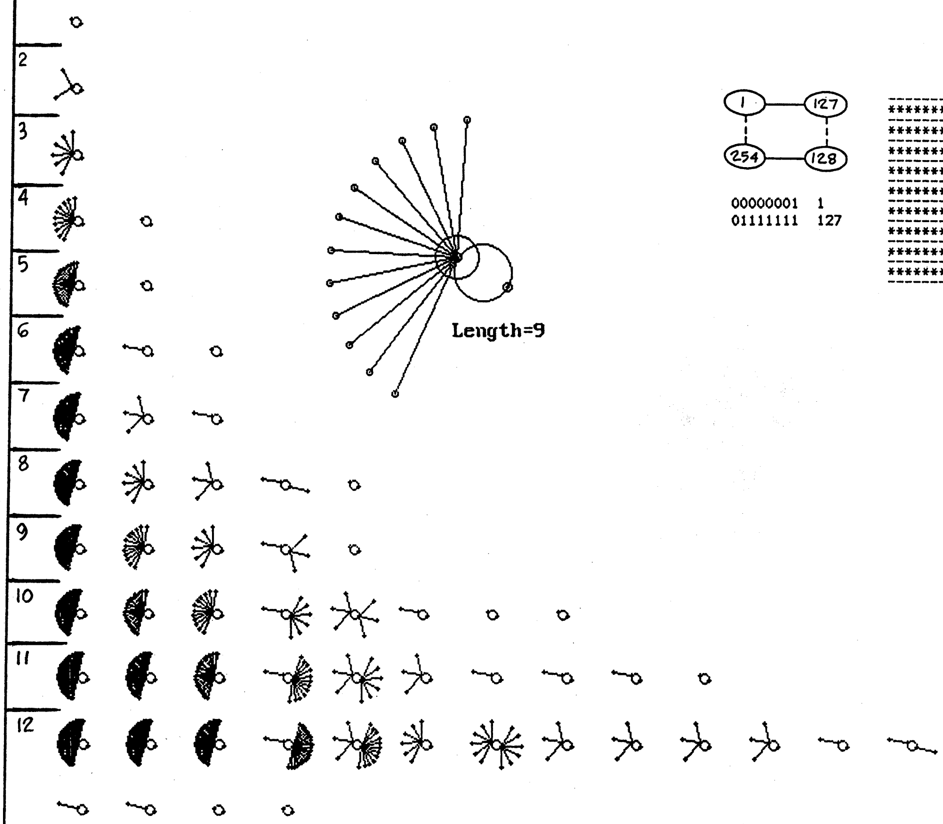




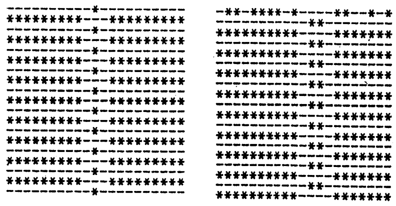
Length=8

```
3-rule 0 =00000000

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 3 1 4
 total NATs = 1
L= 3
 1.000 1(1)8 7 1 8
 total NATs = 1
L= 4
 1.0000 1(1)16 15 1 16
 total NATs = 1
L= 5
 1.00000 1(1)32 31 1 32
 total NATs = 1
L= 6
 1.000000 1(1)64 63 1 64
 total NATs = 1
L= 7
 1.0000000 1(1)128 127 1 128
 total NATs = 1
L= 8
 1.00000000 1(1)256 255 1 256
 total NATs = 1
```

λ ratio = .25 Z = .25 0000000000000011-0000000000000011-rule 196611
=3-rule 1 -00000001 Length=1 -12



00000001 1
01111111 127



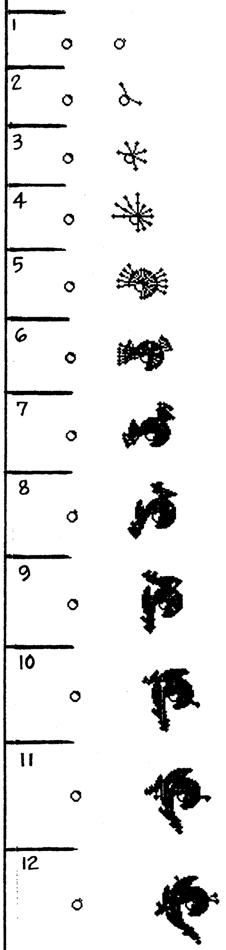
3-rule 1 =00000001

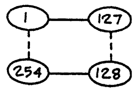
ty. at no(p)s g ml mp

L= 1
1.0 1(2)2 0 0 1
total NATs = 1
L= 2
1.00 1(2)4 2 1 3
total NATs = 1
L= 3
1.000 1(2)8 6 1 7
total NATs = 1
L= 4
1.0000 1(2)12 10 1 11
2.0001 2(2)2 0 0 1
total NATs = 3
L= 5
1.00000 1(2)22 20 1 21
2.00001 5(2)2 0 0 1
total NATs = 6
L= 6
1.000000 1(2)40 38 1 39
2.000001 6(2)3 1 1 2
3.000011 3(2)2 0 0 1
total NATs = 10
L= 7
1.0000000 1(2)72 70 1 71
2.0000001 7(2)5 3 1 4
3.0000011 7(2)3 1 1 2
total NATs = 15
L= 8
1.00000000 1(2)132 130 1 131
2.00000001 8(2)8 6 1 7
3.00000011 8(2)5 3 1 4
4.01110000 4(2)4 2 1 2
5.00010001 2(2)2 0 0 1
total NATs = 23
L= 9
1.000000000 1(2)242 240 1 241
2.000000001 9(2)14 12 1 13
3.000000011 9(2)8 6 1 7
4.011110000 9(2)6 4 1 4
5.000010001 9(2)2 0 0 1
total NATs = 37

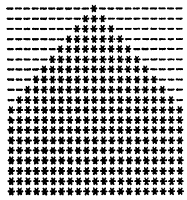
λ ratio = .25 Z = .25

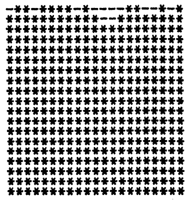
11111111111111100-11111111111111100-rule 4294770684
=3-rule 254 -11111110 Length=1 -12

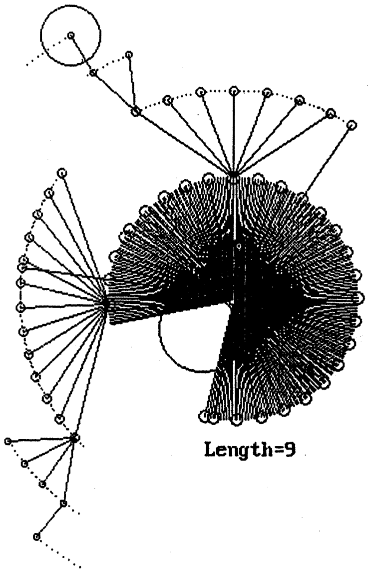




11111110 254
10000000 128



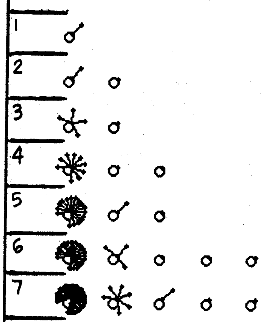


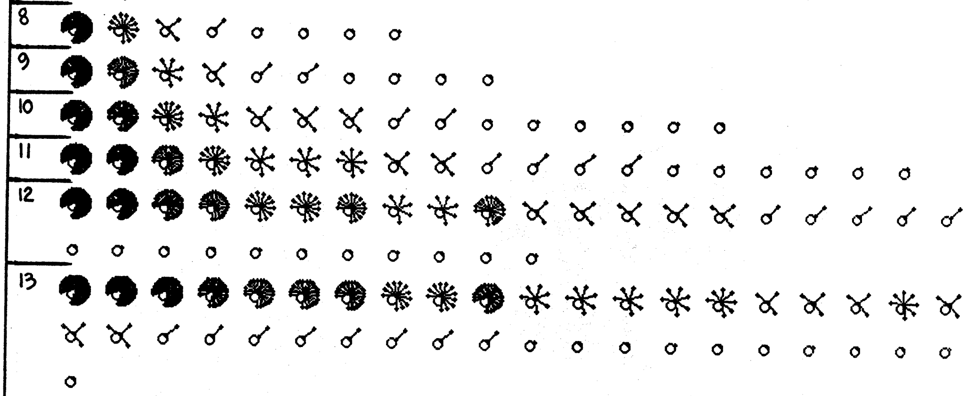


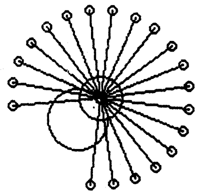
Length=9

```
3-rule 254 =11111110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.11 1(1)3 2 1 3
 total NATs = 2
L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)7 6 1 7
 total NATs = 2
L= 4
 1.0000 1(1)1 0 0 1
 2.1111 1(1)15 10 2 11
 total NATs = 2
L= 5
 1.00000 1(1)1 0 0 1
 2.11111 1(1)31 20 2 21
 total NATs = 2
L= 6
 1.000000 1(1)1 0 0 1
 2.111111 1(1)63 44 3 39
 total NATs = 2
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111111 1(1)127 98 3 71
 total NATs = 2
L= 8
 1.00000000 1(1)1 0 0 1
 2.11111111 1(1)255 210 4 131
 total NATs = 2
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111111 1(1)511 438 4 241
 total NATs = 2
L= 10
 1.0000000000 1(1)1 0 0 1
 2.1111111111 1(1)1023 902 5 443
 total NATs = 2
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111111 1(1)2047 1848 5 815
 total NATs = 2
L= 12
 1.000000000000 1(1)1 0 0 1
 2.111111111111 1(1)4095 3772 6 1499
 total NATs = 2
```

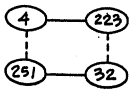
λ ratio = .25 Z = .25 0000000000110000-0000000000110000-rule 3145776
=3-rule 4 -00000100 Length=1 -13



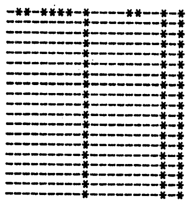


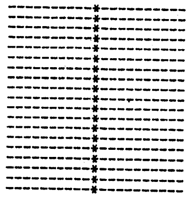


Length=9



00000100 4
11011111 223



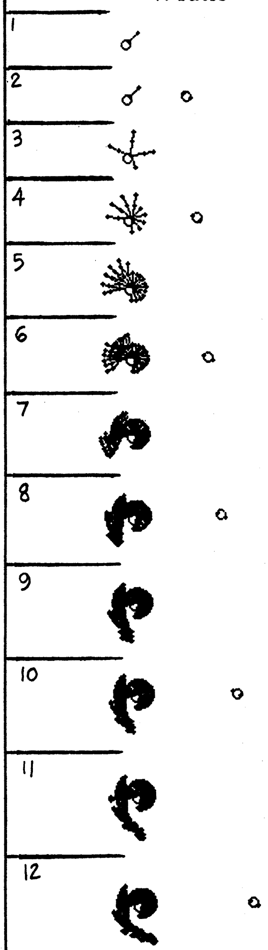


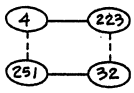
3-rule 4 =00000100

ty. at no(p)s g ml mp

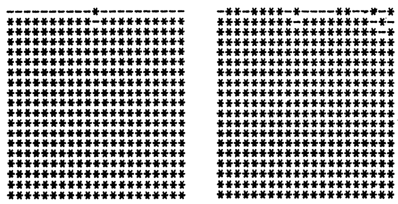
L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)2 1 1 2
2.01 2(1)1 0 0 1
total NATs = 3
L= 3
1.000 1(1)5 4 1 5
2.001 3(1)1 0 0 1
total NATs = 4
L= 4
1.0000 1(1)10 9 1 10
2.0001 4(1)1 0 0 1
3.0101 2(1)1 0 0 1
total NATs = 7
L= 5
1.00000 1(1)17 16 1 17
2.00001 5(1)2 1 1 2
3.00101 5(1)1 0 0 1
total NATs = 11
L= 6
1.000000 1(1)29 28 1 29
2.000001 6(1)4 3 1 4
3.000101 6(1)1 0 0 1
4.001001 3(1)1 0 0 1
5.010101 2(1)1 0 0 1
total NATs = 18
L= 7
1.0000000 1(1)51 50 1 51
2.0000001 7(1)7 6 1 7
3.0000101 7(1)2 1 1 2
4.0001001 7(1)1 0 0 1
5.0010101 7(1)1 0 0 1
total NATs = 29
L= 8
1.00000000 1(1)90 89 1 90
2.00000001 8(1)12 11 1 12
3.00000101 8(1)4 3 1 4
4.00001001 8(1)2 1 1 2
5.00010001 4(1)1 0 0 1
6.00010101 8(1)1 0 0 1
7.00100101 8(1)1 0 0 1
8.01010101 2(1)1 0 0 1
total NATs = 47

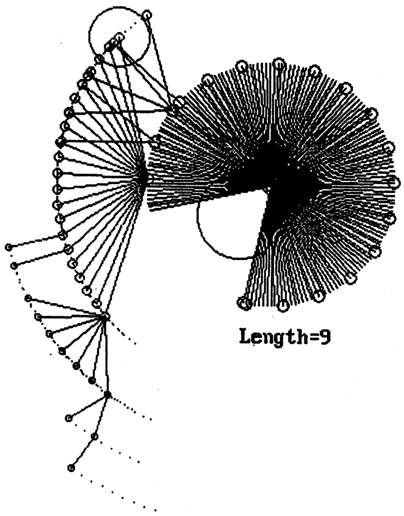
λ ratio = .25 $Z = .25$ 1111111111001111-1111111111001111-rule   4291821519
=3-rule 251 -11111011   Length=1 -12





11111011 251
00100000 32



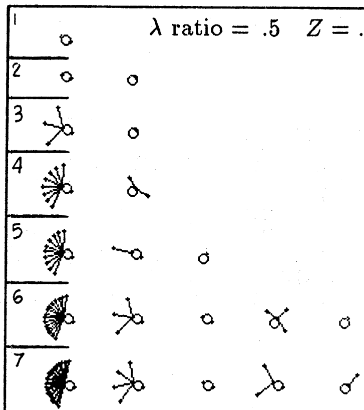
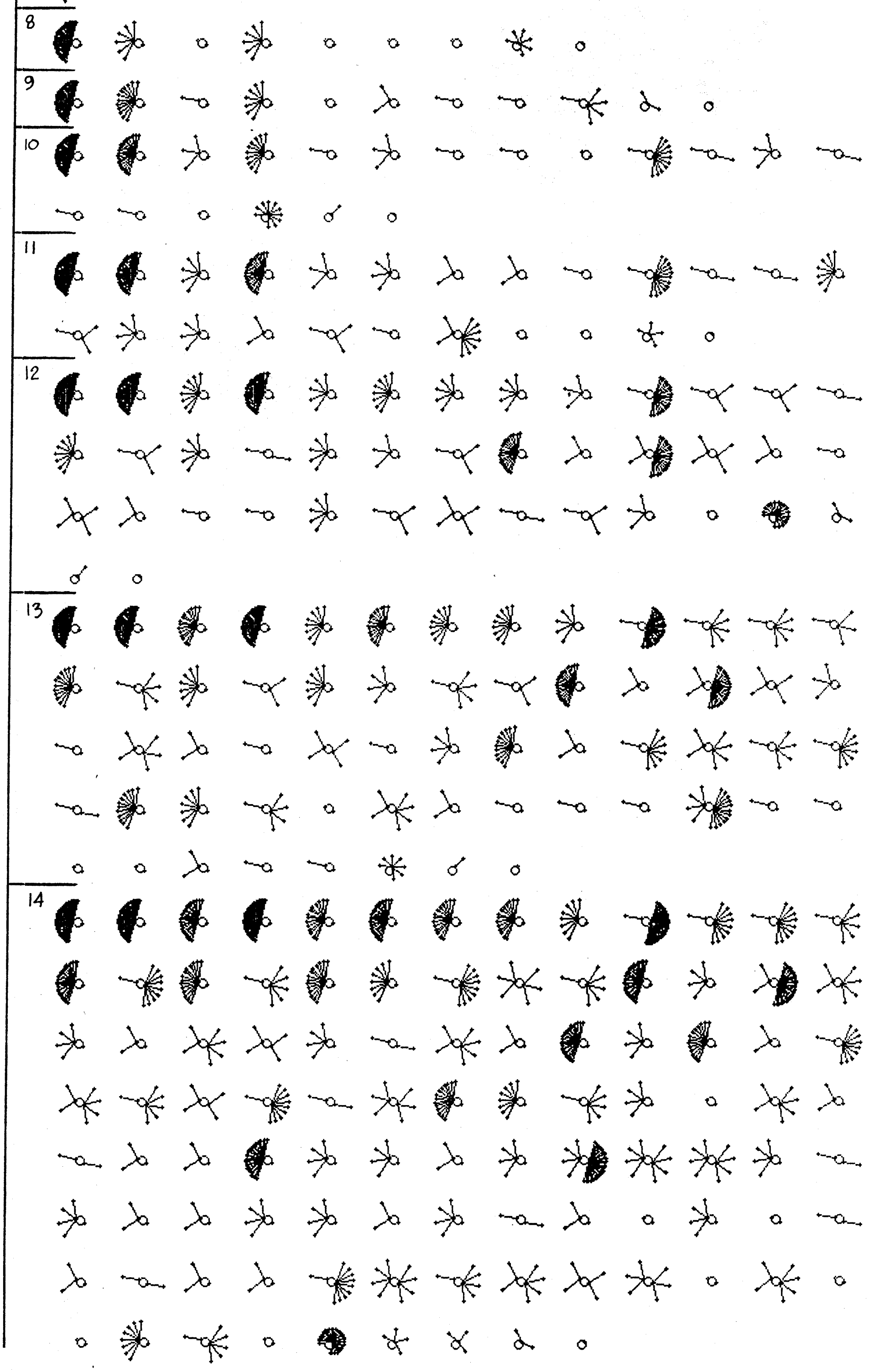


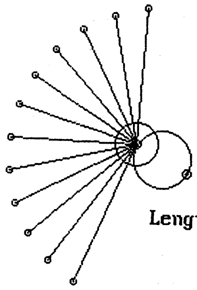
Length=9

```
3-rùle 251 =11111011

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.111 1(1)8 4 2 5
 total NATs = 1
L= 4
 1.1111 1(1)14 9 2 10
 2.0101 1(2)2 0 0 1
 total NATs = 2
L= 5
 1.11111 1(1)32 21 3 17
 total NATs = 1
L= 6
 1.111111 1(1)62 46 3 29
 2.010101 1(2)2 0 0 1
 total NATs = 2
L= 7
 1.1111111 1(1)128 99 4 51
 total NATs = 1
L= 8
 1.11111111 1(1)254 209 4 90
 2.01010101 1(2)2 0 0 1
 total NATs = 2
L= 9
 1.111111111 1(1)512 436 5 158
 total NATs = 1
L= 10
 1.1111111111 1(1)1022 901 5 277
 2.0101010101 1(2)2 0 0 1
 total NATs = 2
L= 11
 1.11111111111 1(1)2048 1849 6 486
 total NATs = 1
L= 12
 1.111111111111 1(1)4094 3774 6 853
 2.010101010101 1(2)2 0 0 1
 total NATs = 2
```

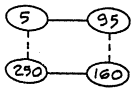
λ ratio = .5 Z = .5

0000000000110011-0000000000110011-rule 3342387
=3-rule 5 -00000101
Length=1 -14

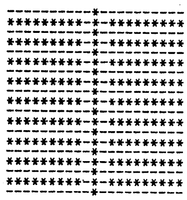



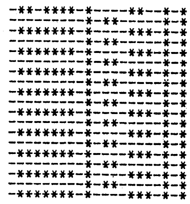


Length=9



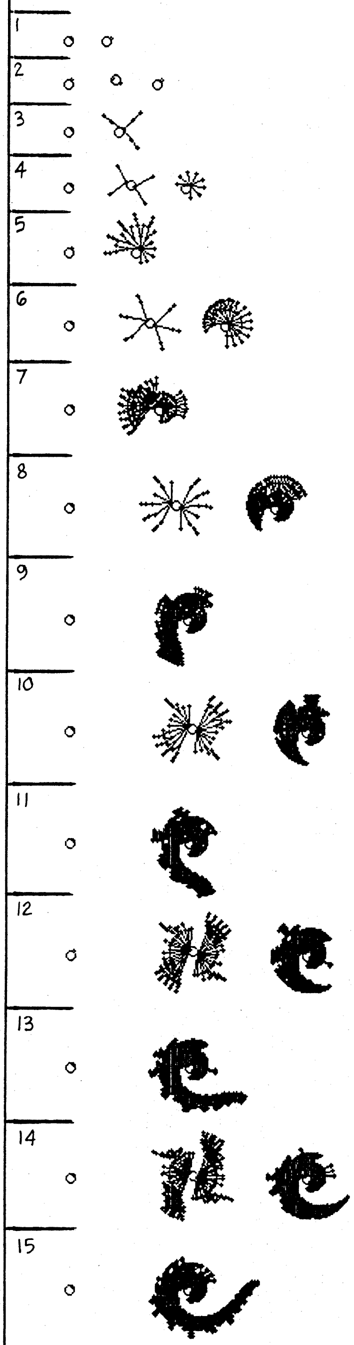
00000101 5
01011111 95

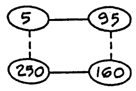




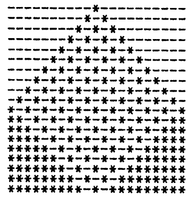
```
3-rule 5 =00000101

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(2)5 3 1 4
 2.001 3(1)1 0 0 1
 total NATs = 4
L= 4
 1.0000 1(2)10 8 1 9
 2.0101 2(1)3 2 1 3
 total NATs = 3
L= 5
 1.00000 1(2)12 10 1 11
 2.00001 5(2)3 1 1 2
 3.00101 5(1)1 0 0 1
 total NATs = 11
L= 6
 1.000000 1(2)17 15 1 16
 2.000001 6(2)5 3 1 4
 3.000011 3(2)2 0 0 1
 4.010101 2(1)4 3 1 4
 5.001001 3(1)1 0 0 1
 total NATs = 15
L= 7
 1.0000000 1(2)30 28 1 29
 2.0000001 7(2)6 4 1 5
 3.0000011 7(2)2 0 0 1
 4.0000101 7(2)4 2 1 3
 5.0101001 7(1)2 1 1 2
 total NATs = 29
L= 8
 1.00000000 1(2)50 48 1 49
 2.00000001 8(2)8 6 1 7
 3.00000011 8(2)2 0 0 1
 4.00000101 8(2)8 6 1 7
 5.00000111 4(2)2 0 0 1
 6.00001001 8(2)2 0 0 1
 7.00001011 8(2)2 0 0 1
 8.01010101 2(1)7 6 1 7
 9.00100101 8(1)1 0 0 1
 total NATs = 55
L= 9
 1.000000000 1(2)77 75 1 76
 2.000000001 9(2)14 12 1 13
 3.000000011 9(2)3 1 1 2
 4.000000101 9(2)9 7 1 8
 5.000000111 9(2)2 0 0 1
 6.000001001 9(2)4 2 1 3
 7.000001011 9(2)3 1 1 2
 8.000001101 9(2)3 1 1 2
 9.011010101 9(2)7 5 1 5
 10.010100101 9(1)3 2 1 3
 11.001001001 3(1)1 0 0 1
 total NATs = 85
L= 10
 1.0000000000 1(2)122 120 1 121
 2.0000000001 10(2)23 21 1 22
 3.0000000011 10(2)5 3 1 4
 4.0000000101 10(2)12 10 1 11
 5.0000000111 10(2)3 1 1 2
 6.0000001001 10(2)5 3 1 4
 7.0000001011 10(2)3 1 1 2
 8.0000001101 10(2)3 1 1 2
 9.0000001111 5(2)2 0 0 1
 10.0111010101 10(2)13 11 1 11
 11.0111010000 10(2)4 2 1 2
 12.0000100001 5(2)5 3 1 4
 13.0110101000 10(2)4 2 1 2
 14.0000100101 10(2)3 1 1 2
 15.0000101001 10(2)3 1 1 2
 16.0000101101 5(2)2 0 0 1
 17.0101010101 2(1)11 10 1 11
 18.0101001001 10(1)2 1 1 2
 19.0010100101 5(1)1 0 0 1
 total NATs = 153
L= 11
 1.00000000000 1(2)200 198 1 199
 2.00000000001 11(2)35 33 1 34
 3.00000000011 11(2)7 5 1 6
 4.00000000101 11(2)22 20 1 21
 5.00000000111 11(2)5 3 1 4
 6.00000001001 11(2)6 4 1 5
 7.00000001011 11(2)4 2 1 3
 8.00000001101 11(2)4 2 1 3
 9.00000001111 11(2)3 1 1 2
 10.01111010101 11(2)15 13 1 13
 11.01111010000 11(2)4 2 1 2
 12.01111000001 11(2)4 2 1 2
 13.00000100001 11(2)9 7 1 8
 14.01110101000 11(2)5 3 1 3
 15.00000100101 11(2)6 4 1 5
 16.00000101001 11(2)6 4 1 5
 17.00000101101 11(2)4 2 1 3
 18.01110000101 11(2)5 3 1 3
 19.00001000011 11(2)3 1 1 2
 20.01101010101 11(2)11 9 1 8
 21.00001001001 11(2)2 0 0 1
 22.00001001011 11(2)2 0 0 1
 23.01010101001 11(1)5 4 1 5
 24.00100100101 11(1)1 0 0 1
 total NATs = 254
```

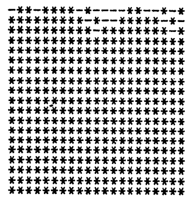
λ ratio = .5   Z = .5   1111111111001100-1111111111001100-rule 4291624908
=3-rule 250 -11111010   Length=1 -15

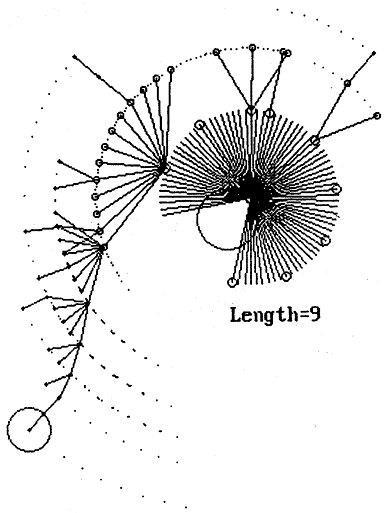




11111010 250
10100000 160



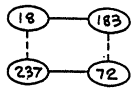




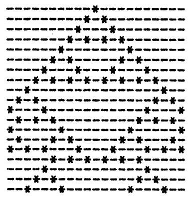
Length=9

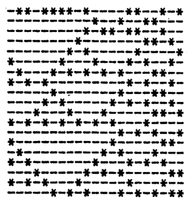
```
3-rule 250 =11111010

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 1(2)2 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)7 3 2 4
 total NATs = 2
L= 4
 1.0000 1(1)1 0 0 1
 2.1010 1(2)6 4 1 3
 3.1111 1(1)9 8 1 9
 total NATs = 3
L= 5
 1.00000 1(1)1 0 0 1
 2.11111 1(1)31 15 4 11
 total NATs = 2
L= 6
 1.000000 1(1)1 0 0 1
 2.010101 1(2)14 6 2 4
 3.111111 1(1)49 33 2 16
 total NATs = 3
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111111 1(1)127 77 6 29
 total NATs = 2
L= 8
 1.00000000 1(1)1 0 0 1
 2.10101010 1(2)30 12 3 7
 3.11111111 1(1)225 144 3 49
 total NATs = 3
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111111 1(1)511 354 8 76
 total NATs = 2
L= 10
 1.0000000000 1(1)1 0 0 1
 2.0101010101 1(2)62 30 4 11
 3.1111111111 1(1)961 705 4 121
 total NATs = 3
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111111 1(1)2047 1562 10 199
 total NATs = 2
L= 12
 1.000000000000 1(1)1 0 0 1
 2.101010101010 1(2)126 70 5 18
 3.111111111111 1(1)3969 3185 5 324
 total NATs = 3
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.1111111111111 1(1)8191 6695 12 521
 total NATs = 2
L= 14
 1.00000000000000 1(1)1 0 0 1
 2.01010101010101 1(2)254 154 6 29
 3.11111111111111 1(1)16129 13629 6 841
 total NATs = 3
L= 15
 1.000000000000000 1(1)1 0 0 1
 2.111111111111111 1(1)32767 28158 14 1364
 total NATs = 2
```

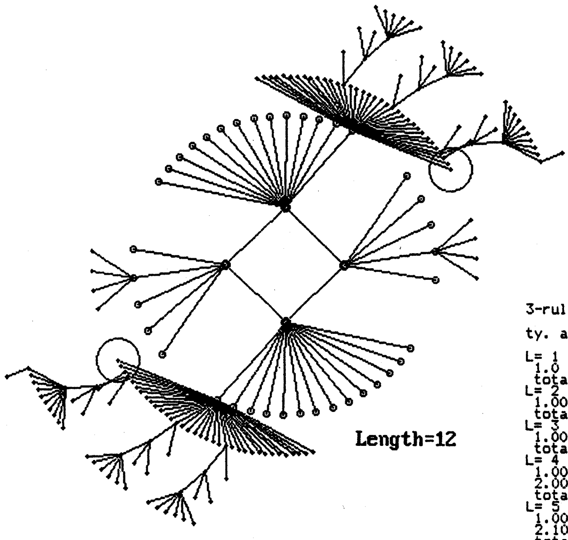
λ ratio = .5   Z = .5   **0000001100001100-0000001100001100-rule 51118860**
=3-rule 18 -00010010   **Length=1 -15**



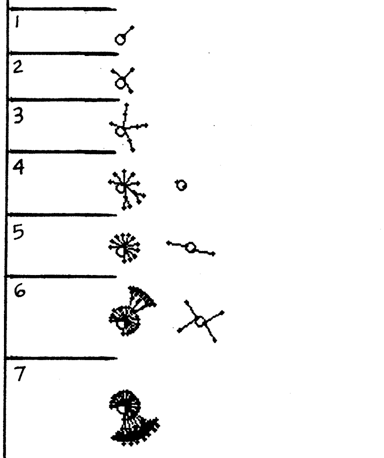
00010010 18
10110111 183

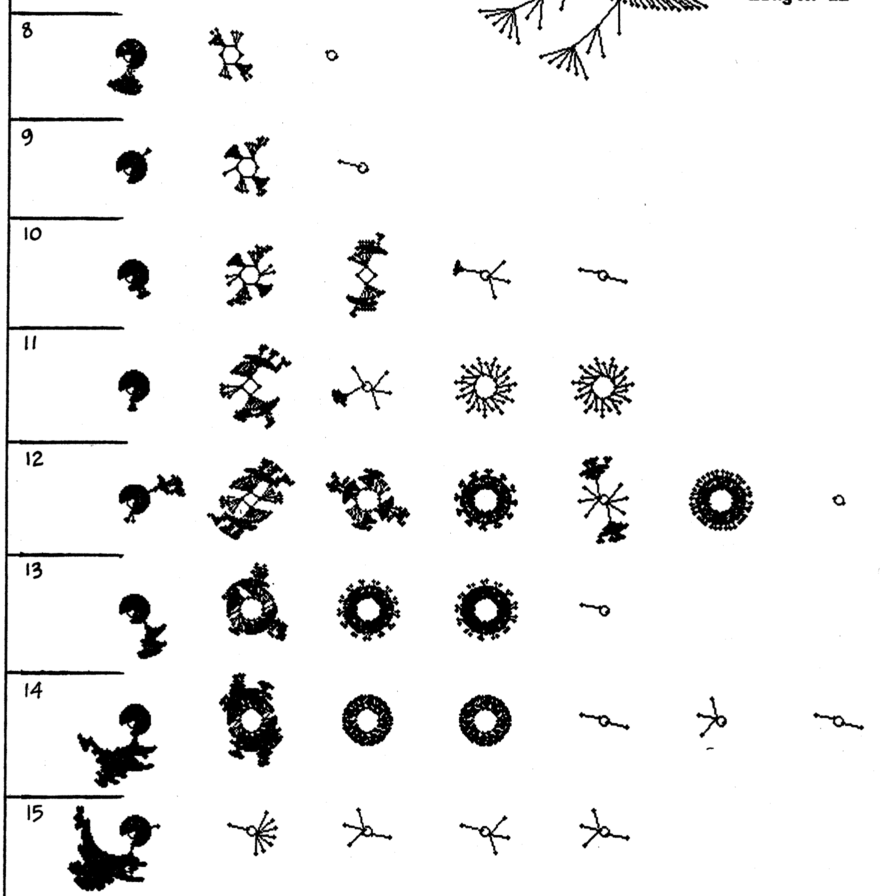






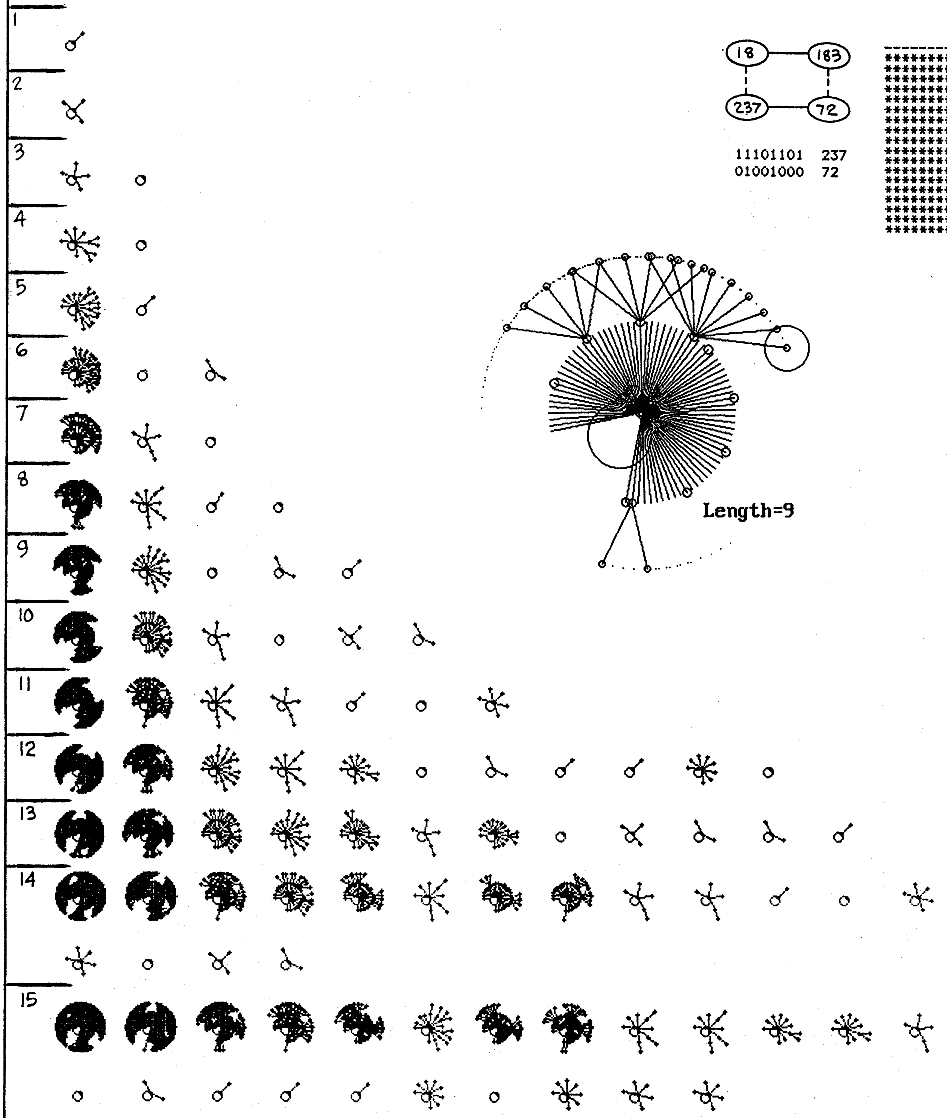
Length=12

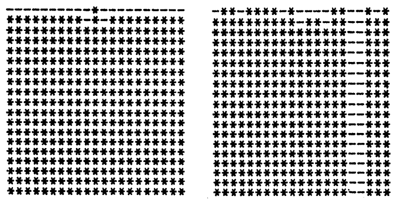





```
3-rule 18 =00010010

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 3 1 4
 total NATs = 1
L= 3
 1.000 1(1)8 4 2 5
 total NATs = 1
L= 4
 1.0000 1(1)12 9 2 8
 2.0011 2(2)2 0 0 1
 total NATs = 3
L= 5
 1.00000 1(1)12 11 1 12
 2.10010 5(2)4 2 1 2
 total NATs = 6
L= 6
 1.000000 1(1)46 33 4 19
 2.100010 3(2)6 4 1 3
 total NATs = 4
L= 7
 1.0000000 1(1)128 92 5 30
 total NATs = 1
L= 8
 1.00000000 1(1)132 109 5 48
 2.00000011 4(6)30 20 2 8
 3.00110011 2(2)2 0 0 1
 total NATs = 7
L= 9
 1.000000000 1(1)80 76 2 77
 2.100000010 9(6)45 34 3 13
 3.000110011 9(2)3 1 1 2
 total NATs = 19
L= 10
 1.0000000000 1(1)134 123 2 124
 2.1000000010 5(6)62 52 3 14
 3.1000000100 5(4)96 72 5 21
 4.0100101001 5(2)12 9 2 6
 5.0001010011 10(2)4 2 1 2
 total NATs = 26
L= 11
 1.00000000000 1(1)211 199 2 200
 2.01000000100 11(4)136 110 6 34
 3.01000101001 11(2)23 19 3 8
 4.01001000011 1(11)44 22 2 3
 5.01001011000 1(11)44 22 2 3
 total NATs = 25
L= 12
 1.000000000000 1(1)534 459 8 323
 2.010000000100 6(4)198 168 6 35
 3.000000000011 6(14)278 238 5 55
 4.010001000011 1(36)180 120 2 5
 5.010001010001 3(2)114 90 5 13
 6.010001011000 1(36)180 120 2 5
 7.001100110011 2(2)2 0 0 1
 total NATs = 20
L= 13
 1.0000000000000 1(1)2121 1730 8 522
 2.1000000000010 13(14)428 373 6 89
 3.1000001010100 1(39)234 182 2 6
 4.0100010101000 1(39)234 182 2 6
 5.0001100110011 13(2)3 1 1 2
 total NATs = 29
L= 14
 1.00000000000000 1(1)11960 10300 17 844
 2.10000000000010 7(14)582 518 6 90
 3.10000010101000 1(14)98 84 1 7
 4.10000010001010 1(14)98 84 1 7
 5.00010100110011 14(2)4 2 1 2
 6.00011000110011 14(2)5 3 1 4
 7.00011001010011 7(2)4 2 1 2
 total NATs = 45.
L= 15
 1.000000000000000 1(1)32448 28619 19 1365
 2.100101001010010 5(2)10 8 1 8
 3.000101000110011 15(2)6 4 1 4
 4.000101001010011 15(2)6 4 1 4
 5.000101001100011 15(2)6 4 1 4
 total NATs = 51
```

λ ratio = .5 Z = .5   1111110011110011-1111110011110011-rule 4243848435
=3-rule 237 -11101101   Length=1 -15

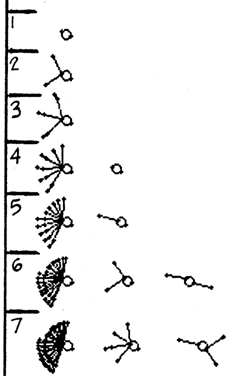


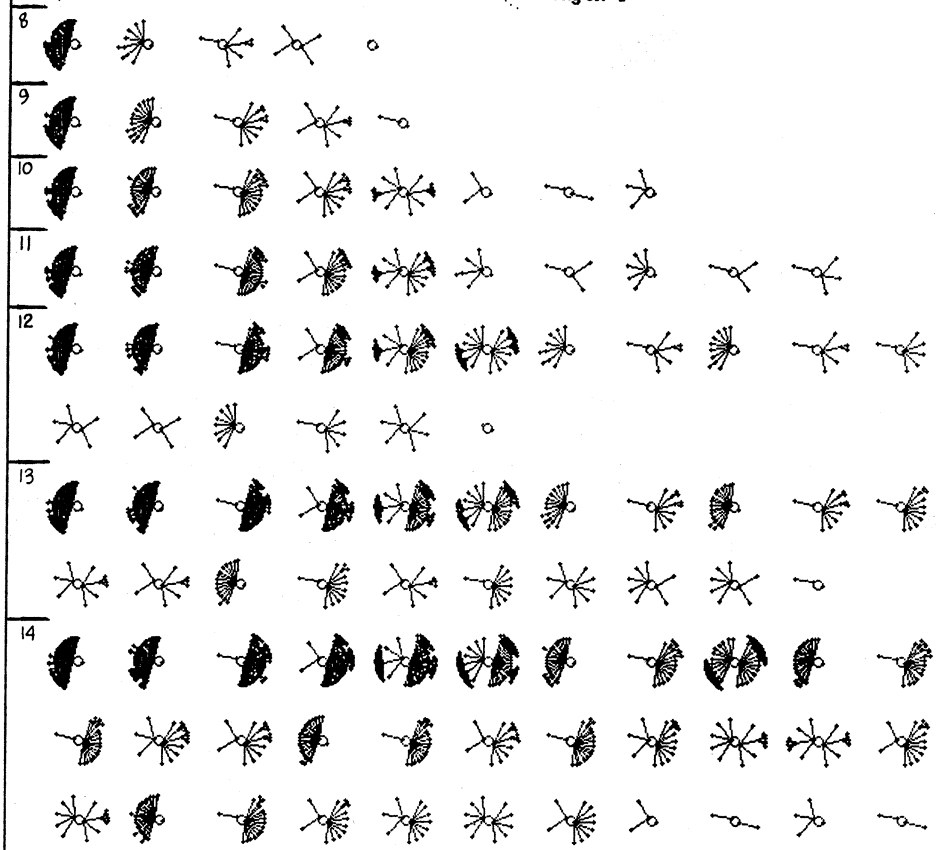


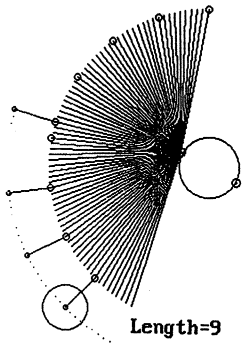
```
3-rule 237 =11101101

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)4 3 1 4
 total NATs = 1
L= 3
 1.111 1(1)5 4 1 5
 2.001 3(1)1 0 0 1
 total NATs = 4
L= 4
 1.1111 1(1)12 9 2 8
 2.0011 4(1)1 0 0 1
 total NATs = 5
L= 5
 1.11111 1(1)22 16 2 12
 2.00111 5(1)2 1 1 2
 total NATs = 6
L= 6
 1.111111 1(1)43 33 2 19
 2.001001 3(1)1 0 0 1
 3.001111 6(1)3 2 1 3
 total NATs = 10
L= 7
 1.1111111 1(1)79 64 2 30
 2.1111001 7(1)6 4 2 5
 3.0010011 7(1)1 0 0 1
 total NATs = 15
L= 8
 1.11111111 1(1)148 125 2 48
 2.11111001 8(1)11 7 2 8
 3.00100111 8(1)2 1 1 2
 4.00110011 4(1)1 0 0 1
 total NATs = 21
L= 9
 1.111111111 1(1)275 238 2 77
 2.111111001 9(1)21 14 2 13
 3.001001001 3(1)1 0 0 1
 4.001001111 9(1)3 2 1 3
 5.001110011 9(1)2 1 1 2
 total NATs = 31
L= 10
 1.1111111111 1(1)514 453 2 124
 2.1111111001 10(1)39 28 2 21
 3.1111001001 10(1)6 4 2 5
 4.0010010011 10(1)1 0 0 1
 5.0011100111 5(1)4 3 1 4
 6.0011110011 10(1)3 2 1 3
 total NATs = 46
L= 11
 1.11111111111 1(1)959 859 2 200
 2.11111111001 11(1)73 56 2 34
 3.11111001001 11(1)11 7 2 8
 4.11110011001 11(1)6 4 2 5
 5.00100100111 11(1)2 1 1 2
 6.00100110011 11(1)1 0 0 1
 7.00111001111 11(1)6 5 1 6
 total NATs = 67
L= 12
 1.111111111111 1(1)1791 1629 2 323
 2.111111111001 12(1)136 109 2 55
 3.111111001001 12(1)21 14 2 13
 4.111110011001 12(1)11 7 2 8
 5.111100111001 12(1)12 10 2 10
 6.001001001001 3(1)1 0 0 1
 7.001001001111 12(1)3 2 1 3
 8.001001110011 12(1)2 1 1 2
 9.001001100111 12(1)2 1 1 2
 10.001111001111 6(1)9 8 1 9
 11.001100110011 4(1)1 0 0 1
 total NATs = 98
L= 13
 1.1111111111111 1(1)3343 3082 2 522
 2.1111111111001 13(1)254 210 2 89
 3.1111111001001 13(1)39 28 2 21
 4.1111110011001 13(1)21 14 2 13
 5.1111100111001 13(1)22 18 2 16
 6.1111001001001 13(1)6 4 2 5
 7.1111001111001 13(1)18 16 2 15
 8.0010010010011 13(1)1 0 0 1
 9.0010011100111 13(1)4 3 1 4
 10.0010011110011 13(1)3 2 1 3
 11.0010011001111 13(1)3 2 1 3
 12.0011100110011 13(1)2 1 1 2
 total NATs = 144
L= 14
 1.11111111111111 1(1)6241 5820 2 844
 2.11111111111001 14(1)474 402 2 144
 3.11111111001001 14(1)73 56 2 34
 4.11111110011001 14(1)39 28 2 21
 5.11111100111001 14(1)42 35 2 26
 6.11111001001001 14(1)11 7 2 8
 7.11111001111001 14(1)33 29 2 24
 8.11110011111001 7(1)36 32 2 25
 9.11110010011001 14(1)6 4 2 5
 10.11110011001001 14(1)6 4 2 5
 11.00100100100111 14(1)2 1 1 2
 12.00100100110011 14(1)1 0 0 1
 13.00100111001111 14(1)6 5 1 6
 14.00100111100111 14(1)6 5 1 6
 15.00100110010011 7(1)1 0 0 1
 16.00111001110011 14(1)4 3 1 4
 17.00111100110011 14(1)3 2 1 3
 total NATs = 211
```

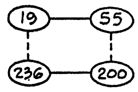
λ ratio = .75 Z = .625

0000001100001111-0000001100001111-rule 51315471
=3-rule 19 -00010011
Length=1 -14

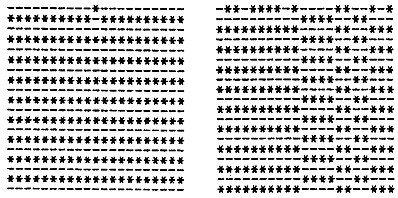








00010011 19
00110111 55



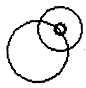
3-rule 19 =00010011

ty. at no(p)s g ml mp

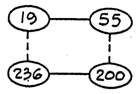
L= 1
1.0 1(2)2 0 0 1
total NATs = 1
L= 2
1.00 1(2)4 2 1 3
total NATs = 1
L= 3
1.000 1(2)8 3 2 4
total NATs = 1
L= 4
1.0000 1(2)12 6 2 7
2.0011 2(2)2 0 0 1
total NATs = 3
L= 5
1.00000 1(2)17 10 2 11
2.00011 5(2)3 1 1 2
total NATs = 6
L= 6
1.000000 1(2)28 17 2 18
2.000011 6(2)4 2 1 3
3.111000 3(2)4 2 1 2
total NATs = 10
L= 7
1.0000000 1(2)44 28 2 29
2.0000011 7(2)7 4 2 5
3.1111000 7(2)5 3 1 3
total NATs = 15
L= 8
1.00000000 1(2)68 46 2 47
2.00000011 8(2)11 7 2 8
3.11111000 8(2)9 6 2 5
4.11110000 4(2)6 4 1 3
5.00110011 2(2)2 0 0 1
total NATs = 23
L= 9
1.000000000 1(2)107 75 2 76
2.000000011 9(2)17 12 2 13
3.111111000 9(2)14 10 2 8
4.111110000 9(2)11 8 2 5
5.000110011 9(2)3 1 1 2
total NATs = 37
L= 10
1.0000000000 1(2)169 122 2 123
2.0000000011 10(2)27 20 2 21
3.1111111000 10(2)21 16 2 13
4.1111110000 10(2)17 13 2 8
5.1111100000 5(2)20 16 2 5
6.0000110011 10(2)4 2 1 3
7.1110001100 10(2)4 2 1 2
8.0001100011 5(2)5 3 1 4
total NATs = 61
L= 11
1.00000000000 1(2)266 198 2 199
2.00000000011 11(2)43 33 2 34
3.11111111000 11(2)33 26 2 21
4.11111110000 11(2)25 20 2 13
5.11111100000 11(2)31 26 2 8
6.00000110011 11(2)7 4 2 5
7.11110001100 11(2)5 3 1 3
8.00001100011 11(2)7 5 1 6
9.11110011000 11(2)5 3 1 3
10.11100011100 11(2)6 4 1 4
total NATs = 100
L= 12
1.000000000000 1(2)420 321 2 322
2.000000000011 12(2)68 54 2 55
3.111111111000 12(2)52 42 2 34
4.111111110000 12(2)39 32 2 21
5.111111100000 12(2)46 40 2 13
6.111111000000 6(2)48 42 2 8
7.000000110011 12(2)11 7 2 8
8.111110001100 12(2)9 6 2 5
9.000001100011 12(2)12 9 2 10
10.111110011000 12(2)9 6 2 5
11.111100011100 12(2)8 6 1 6
12.111100011000 12(2)7 5 1 4
13.111100001100 12(2)6 4 1 3
14.000011000011 6(2)10 8 1 9
15.111100111000 12(2)8 6 1 6
16.111000111000 3(2)8 6 1 4
17.001100110011 2(2)2 0 0 1
total NATs = 162
L= 13
1.0000000000000 1(2)665 520 2 521
2.0000000000011 13(2)108 88 2 89
3.1111111111000 13(2)81 67 2 55
4.1111111110000 13(2)61 51 2 34
5.1111111100000 13(2)72 64 2 21
6.1111111000000 13(2)71 64 2 13
7.0000000110011 13(2)17 12 2 13
8.1111110001100 13(2)14 10 2 8
9.0000001100011 13(2)19 15 2 16
10.1111110011000 13(2)14 10 2 8
11.1111100011100 13(2)14 11 2 10
12.1111100011000 13(2)13 10 2 5
13.1111100001100 13(2)11 8 2 5
14.0000011000011 13(2)17 14 2 15
15.1111100111000 13(2)14 11 2 10
16.1111100110000 13(2)11 8 2 5
17.1111000111100 13(2)11 9 1 9
18.1111000111000 13(2)10 8 1 6
19.1111000110000 13(2)9 7 1 6
20.1111000011000 13(2)9 7 1 6
21.0001100110011 13(2)3 1 1 2
total NATs = 261

λ ratio = .75 Z = .625

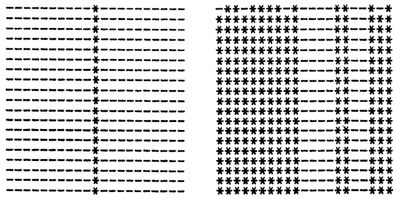
1111110011110000-1111110011110000-rule 4243651824
=3-rule 236 -11101100 Length=1 -14

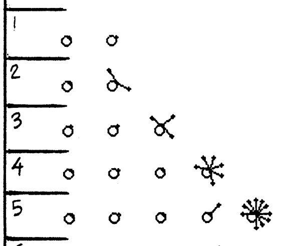


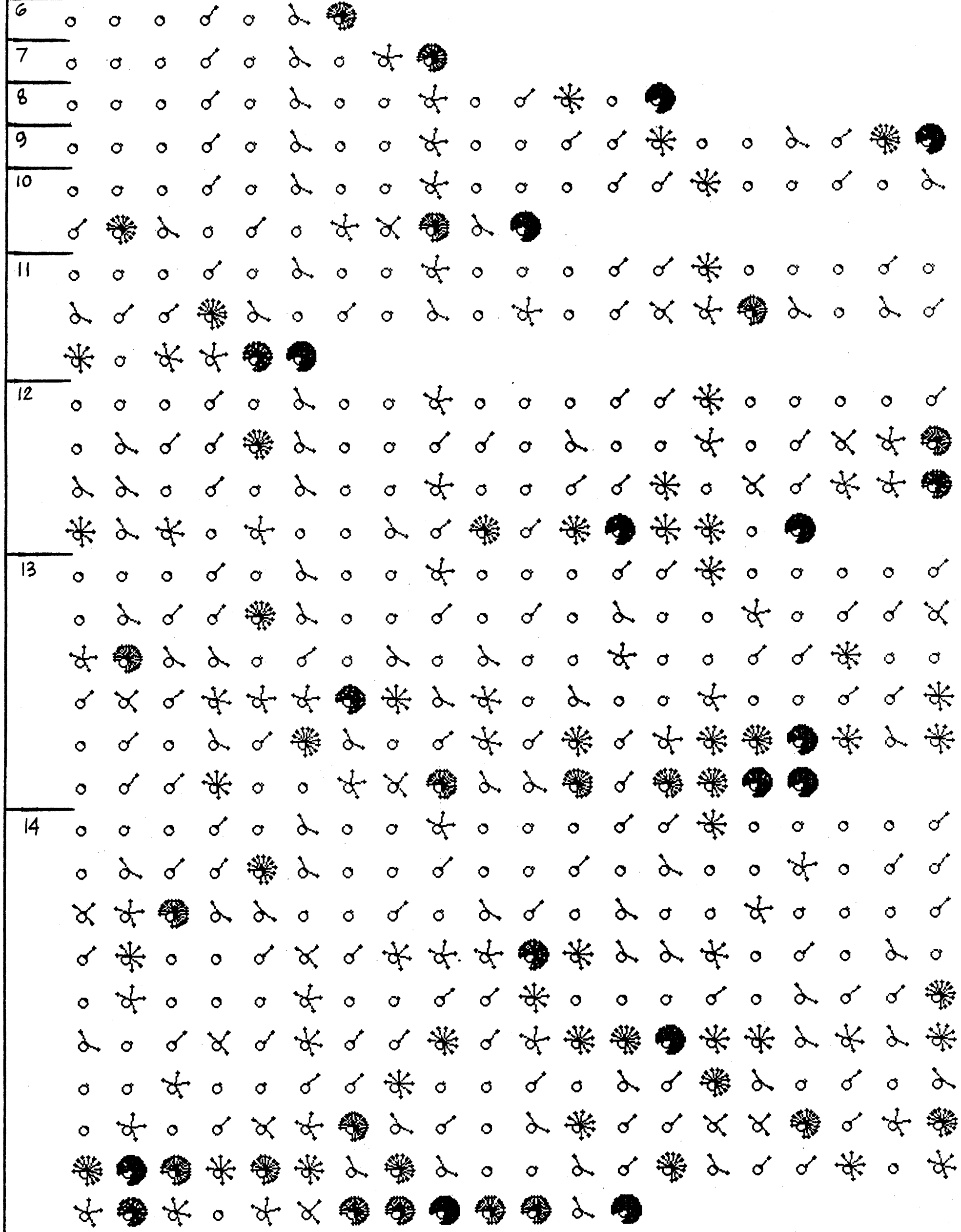
all Lengths



11101100 236
11001000 200

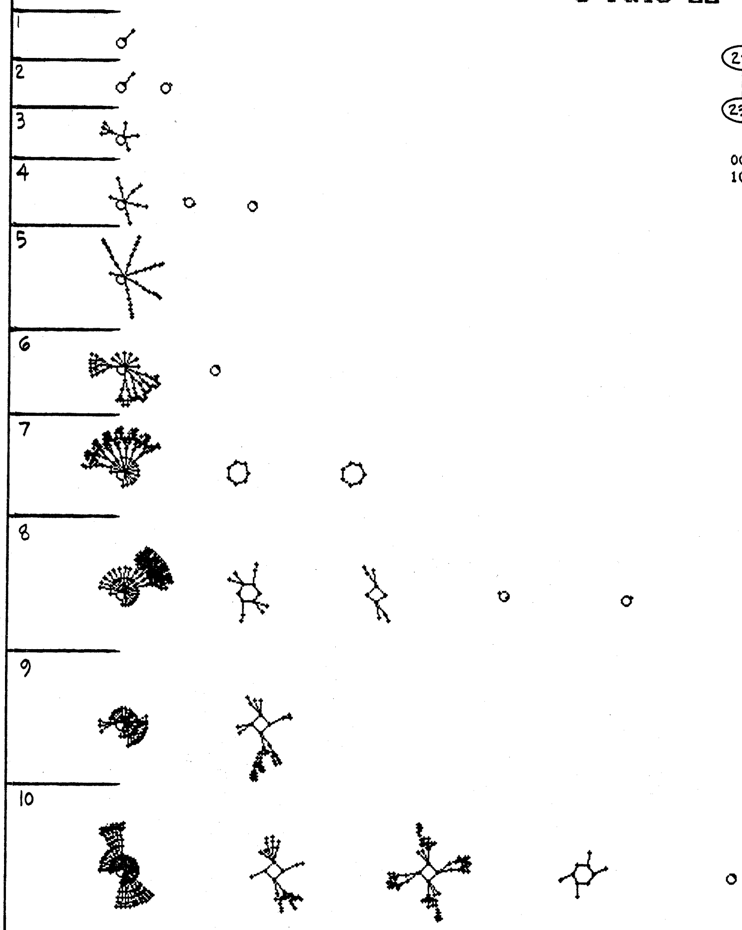


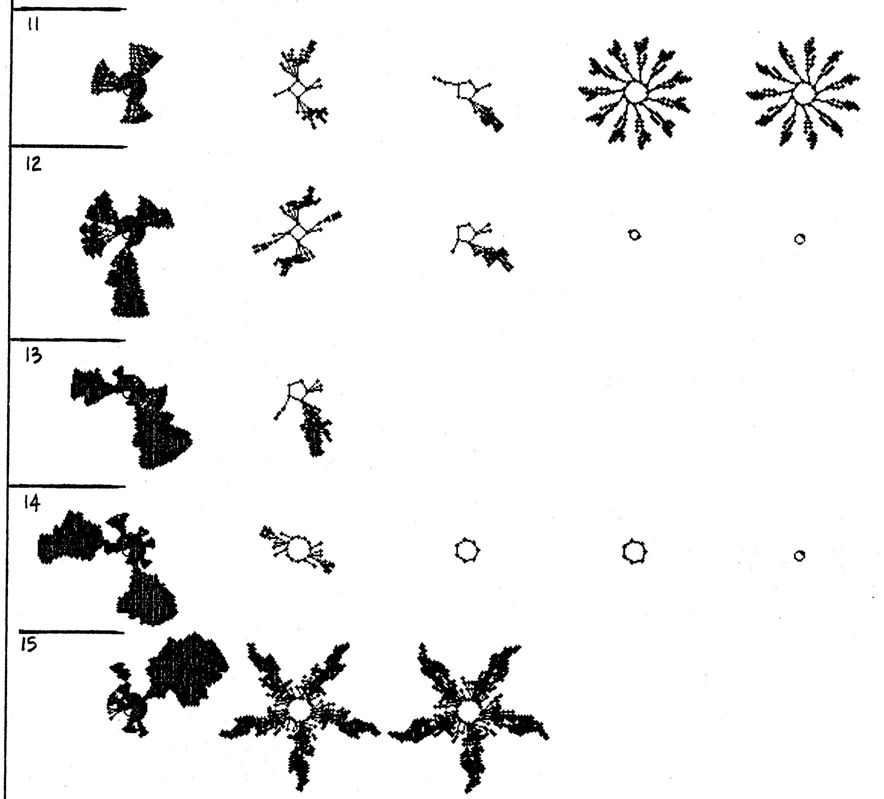


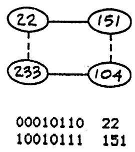


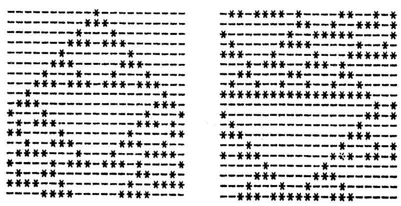

```
3-rule 236 =11101100

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.11 1(1)3 2 1 3
 total NATs = 2
L= 3
 1.000 1(1)1 0 0 1
 2.001 3(1)1 0 0 1
 3.111 1(1)4 3 1 4
 total NATs = 5
L= 4
 1.0000 1(1)1 0 0 1
 2.0001 4(1)1 0 0 1
 3.0011 4(1)1 0 0 1
 4.1111 1(1)7 6 1 7
 total NATs = 10
L= 5
 1.00000 1(1)1 0 0 1
 2.00001 5(1)1 0 0 1
 3.00011 5(1)1 0 0 1
 4.00111 5(1)2 1 1 2
 5.11111 1(1)11 10 1 11
 total NATs = 17
L= 6
 1.000000 1(1)1 0 0 1
 2.000001 6(1)1 0 0 1
 3.000011 6(1)1 0 0 1
 4.000111 6(1)2 1 1 2
 5.001001 3(1)1 0 0 1
 6.001111 6(1)3 2 1 3
 7.111111 1(1)18 17 1 18
 total NATs = 29
L= 7
 1.0000000 1(1)1 0 0 1
 2.0000001 7(1)1 0 0 1
 3.0000011 7(1)1 0 0 1
 4.0000111 7(1)2 1 1 2
 5.0001001 7(1)1 0 0 1
 6.0001111 7(1)3 2 1 3
 7.0010011 7(1)1 0 0 1
 8.0011111 7(1)5 4 1 5
 9.1111111 1(1)29 28 1 29
 total NATs = 51
L= 8
 1.00000000 1(1)1 0 0 1
 2.00000001 8(1)1 0 0 1
 3.00000011 8(1)1 0 0 1
 4.00000111 8(1)2 1 1 2
 5.00001001 8(1)1 0 0 1
 6.00001111 8(1)3 2 1 3
 7.00010001 4(1)1 0 0 1
 8.00010011 8(1)1 0 0 1
 9.00011111 8(1)5 4 1 5
 10.00011001 8(1)1 0 0 1
 11.00100111 8(1)2 1 1 2
 12.00111111 8(1)8 7 1 8
 13.00110011 4(1)1 0 0 1
 14.11111111 1(1)47 46 1 47
 total NATs = 90
L= 9
 1.000000000 1(1)1 0 0 1
 2.000000001 9(1)1 0 0 1
 3.000000011 9(1)1 0 0 1
 4.000000111 9(1)2 1 1 2
 5.000001001 9(1)1 0 0 1
 6.000001111 9(1)3 2 1 3
 7.000010001 9(1)1 0 0 1
 8.000010011 9(1)1 0 0 1
 9.000011111 9(1)5 4 1 5
 10.000011001 9(1)1 0 0 1
 11.000100011 9(1)1 0 0 1
 12.000100111 9(1)2 1 1 2
 13.000111001 9(1)2 1 1 2
 14.000111111 9(1)8 7 1 8
 15.000110011 9(1)1 0 0 1
 16.001001001 3(1)1 0 0 1
 17.001001111 9(1)3 2 1 3
 18.001110011 9(1)2 1 1 2
 19.001111111 9(1)13 12 1 13
 20.111111111 1(1)76 75 1 76
 total NATs = 158
L= 10
 1.0000000000 1(1)1 0 0 1
 2.0000000001 10(1)1 0 0 1
 3.0000000011 10(1)1 0 0 1
 4.0000000111 10(1)2 1 1 2
 5.0000001001 10(1)1 0 0 1
 6.0000001111 10(1)3 2 1 3
 7.0000010001 10(1)1 0 0 1
 8.0000010011 10(1)1 0 0 1
 9.0000011111 10(1)5 4 1 5
 10.0000011001 10(1)1 0 0 1
 11.0000100001 5(1)1 0 0 1
 12.0000100011 10(1)1 0 0 1
 13.0000100111 10(1)2 1 1 2
 14.0000111001 10(1)2 1 1 2
 15.0000111111 10(1)8 7 1 8
 16.0000110001 10(1)1 0 0 1
 17.0000110011 10(1)1 0 0 1
 18.0001000111 10(1)2 1 1 2
 19.0001001001 10(1)1 0 0 1
 20.0001001111 10(1)3 2 1 3
 21.0001110011 10(1)2 1 1 2
 22.0001111111 10(1)13 12 1 13
 23.0001111001 10(1)3 2 1 3
 24.0001100011 5(1)1 0 0 1
 25.0001100111 10(1)2 1 1 2
 26.0010010011 10(1)1 0 0 1
 27.0010011111 10(1)5 4 1 5
 28.0011100111 5(1)4 3 1 4
 29.0011111111 10(1)21 20 1 21
 30.0011110011 10(1)3 2 1 3
 31.1111111111 1(1)123 122 1 123
 total NATs = 277
```

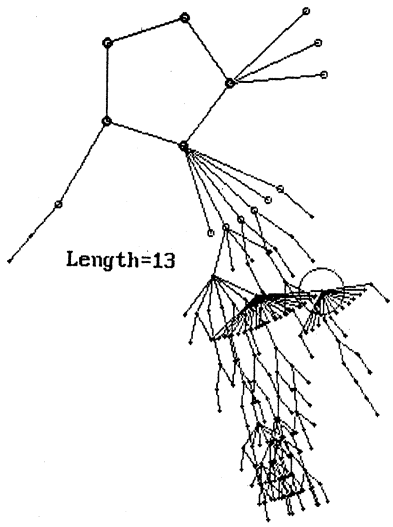
λ ratio = .75 Z = .75   0000001100111100-0000001100111100-rule 54264636
=3-rule 22 -00010110   Length=1 -15





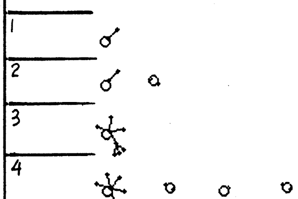


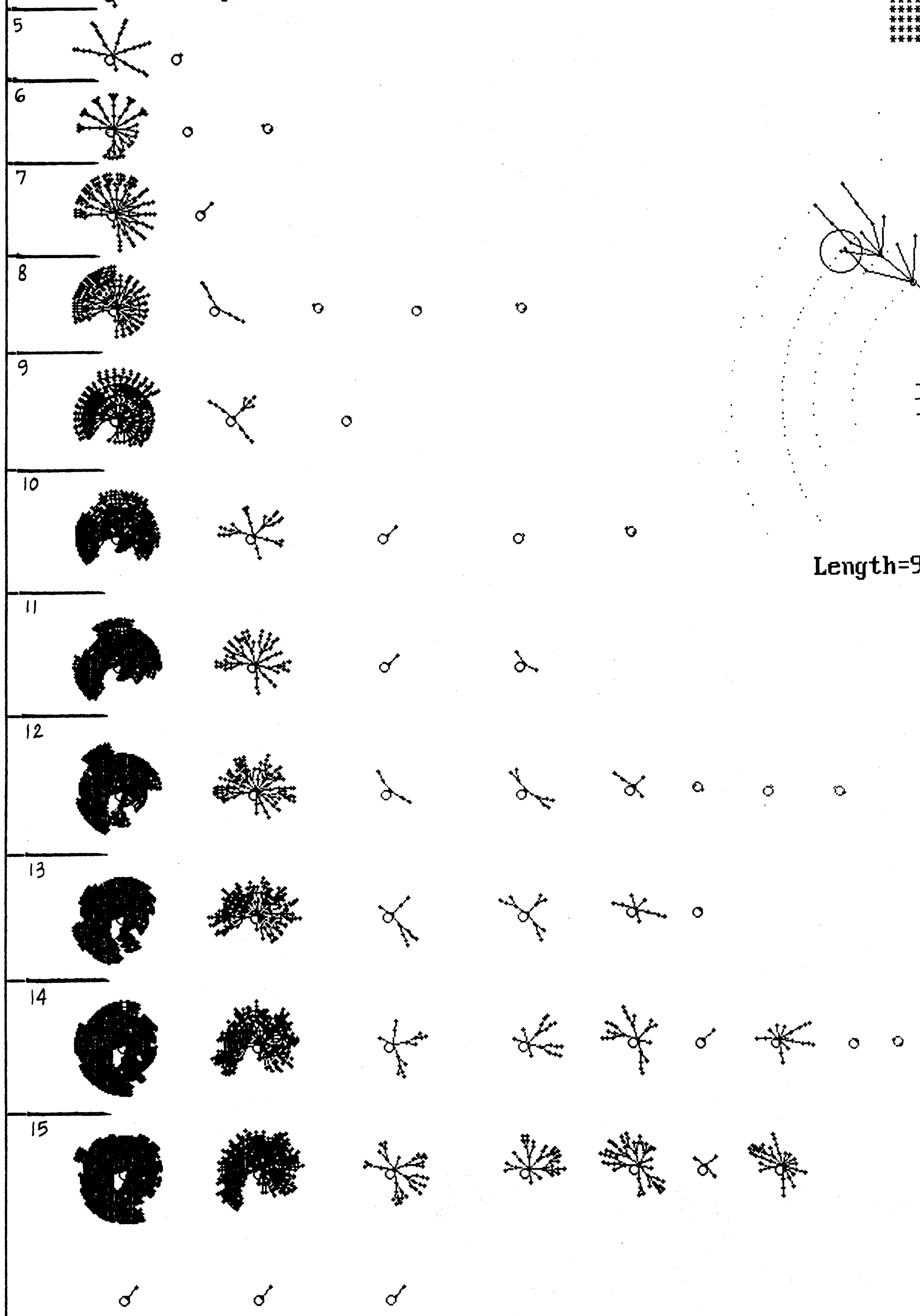


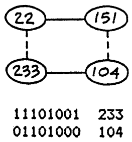


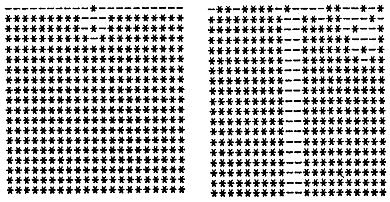

```
3-rule 22 =00010110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)2 1 1 2
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)8 6 2 5
 total NATs = 1
L= 4
 1.0000 1(1)10 5 2 6
 2.0011 2(2)2 0 0 1
 3.0101 2(1)1 0 0 1
 total NATs = 5
L= 5
 1.00000 1(1)32 6 6 7
 total NATs = 1
L= 6
 1.000000 1(1)62 30 5 11
 2.010101 2(1)1 0 0 1
 total NATs = 3
L= 7
 1.0000000 1(1)114 50 6 16
 2.0001011 1(7)7 0 0 1
 3.0001101 1(7)7 0 0 1
 total NATs = 3
L= 8
 1.00000000 1(1)154 49 8 22
 2.00000011 4(6)14 6 2 3
 3.00000101 4(4)10 4 2 3
 4.00110011 2(2)2 0 0 1
 5.01010101 2(1)1 0 0 1
 total NATs = 13
L= 9
 1.000000000 1(1)53 42 2 32
 2.001001000 9(4)51 23 9 5
 total NATs = 10
L= 10
 1.0000000000 1(1)142 56 7 47
 2.0100000100 10(4)39 19 5 7
 3.1000000100 5(4)78 36 7 6
 4.0000010011 10(6)10 4 1 2
 5.0101010101 2(1)1 0 0 1
 total NATs = 28
L= 11
 1.00000000000 1(1)233 100 6 68
 2.01000000100 11(4)67 34 7 9
 3.01000010000 11(5)50 20 7 9
 4.10010000111 1(11)264 88 9 3
 5.00010011110 1(11)264 88 9 3
 total NATs = 25
L= 12
 1.000000000000 1(1)2686 1278 23 99
 2.010000000100 6(4)104 54 6 13
 3.001000010000 12(5)65 37 9 13
 4.001100110011 2(2)2 0 0 1
 5.010101010101 2(1)1 0 0 1
 total NATs = 23
L= 13
 1.0000000000000 1(1)5540 2757 20 145
 2.0001000010000 13(5)204 104 13 19
 total NATs = 14
L= 14
 1.00000000000000 1(1)15752 7827 25 212
 2.10000001111000 14(12)44 18 4 4
 3.00010110001011 1(7)7 0 0 1
 4.00011010001101 1(7)7 0 0 1
 5.01010101010101 2(1)1 0 0 1
 total NATs = 19
L= 15
 1.000000000000000 1(1)25928 14141 32 310
 2.000011000001000 3(20)1140 565 20 13
 3.000011000000010 3(20)1140 565 20 13
 total NATs = 7
```

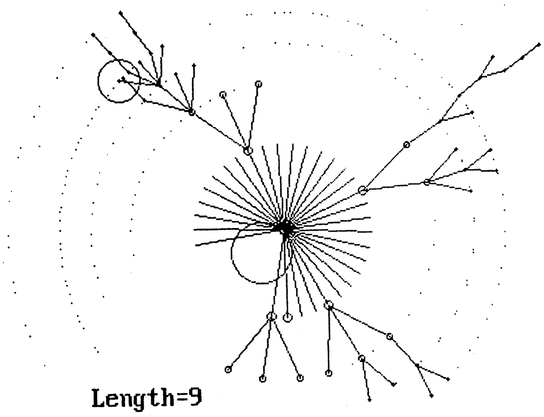
λ ratio = .75 Z = .75 1111110011000011-1111110011000011-rule 4240702659
=3-rule 233 -11101001 Length=1 -15









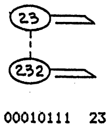


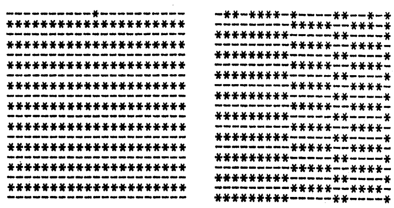
Length=9

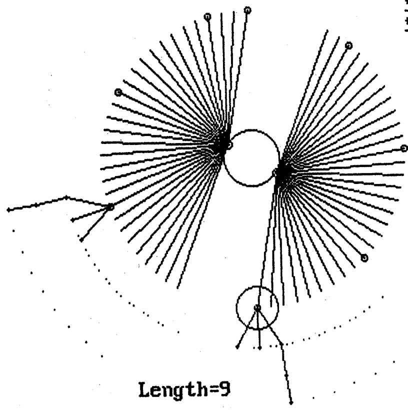
```
3-rule 233 =11101001

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.111 1(1)8 6 2 5
 total NATs = 1
L= 4
 1.1111 1(1)6 5 1 6
 2.0001 2(2)2 0 0 1
 3.0011 4(1)1 0 0 1
 4.0101 1(2)2 0 0 1
 total NATs = 8
L= 5
 1.11111 1(1)27 6 5 7
 2.00111 5(1)1 0 0 1
 total NATs = 6
L= 6
 1.111111 1(1)56 30 4 11
 2.001111 6(1)1 0 0 1
 3.010101 1(2)2 0 0 1
 total NATs = 8
L= 7
 1.1111111 1(1)114 43 5 16
 2.0011111 7(1)2 1 1 2
 total NATs = 8
L= 8
 1.11111111 1(1)190 73 6 22
 2.11100111 8(1)7 2 3 3
 3.00010001 2(2)2 0 0 1
 4.00110011 4(1)1 0 0 1
 5.01010101 1(2)2 0 0 1
 total NATs = 16
L= 9
 1.111111111 1(1)395 204 8 32
 2.111100111 9(1)12 5 3 4
 3.001100111 9(1)1 0 0 1
 total NATs = 19
L= 10
 1.1111111111 1(1)727 346 7 47
 2.1111100111 10(1)27 11 4 6
 3.0110011110 10(1)2 1 1 2
 4.0011100111 5(1)1 0 0 1
 5.0101010101 1(2)2 0 0 1
 total NATs = 27
L= 11
 1.11111111111 1(1)1355 595 7 68
 2.11111100111 11(1)58 22 5 9
 3.01110011110 11(1)2 1 1 2
 4.01100111110 11(1)3 2 1 3
 total NATs = 34
L= 12
 1.111111111111 1(1)2616 1374 9 99
 2.111111100111 12(1)106 48 6 13
 3.011110011110 6(1)5 2 2 3
 4.011100111110 12(1)9 4 3 3
 5.011001111110 12(1)5 3 2 4
 6.000100010001 2(2)2 0 0 1
 7.001100110011 4(1)1 0 0 1
 8.010101010101 1(2)2 0 0 1
 total NATs = 50
L= 13
 1.1111111111111 1(1)5020 2614 9 145
 2.1111111100111 13(1)211 100 8 19
 3.0111110011110 13(1)11 4 4 4
 4.1111001110011 13(1)12 6 3 4
 5.0110011111110 13(1)9 5 3 6
 6.0011001100111 13(1)1 0 0 1
 total NATs = 66
L= 14
 1.11111111111111 1(1)9662 4936 8 212
 2.11111111100111 14(1)411 194 8 28
 3.01111110011110 14(1)16 8 4 5
 4.01111100111110 7(1)18 7 4 6
 5.11111001110011 14(1)24 10 4 6
 6.01100111001110 14(1)2 1 1 2
 7.01100111111110 14(1)17 8 4 9
 8.00110011001111 14(1)1 0 0 1
 9.01010101010101 1(2)2 0 0 1
 total NATs = 93
```

λ ratio = 1 $Z = .5$
minority rule

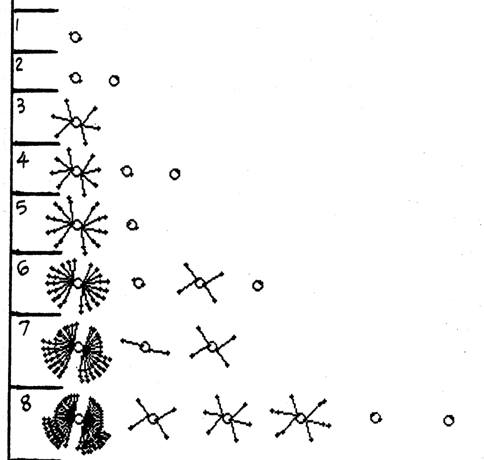
0000001100111111-0000001100111111-rule 54461247
=3-rule 23 -00010111
Length=1 -15

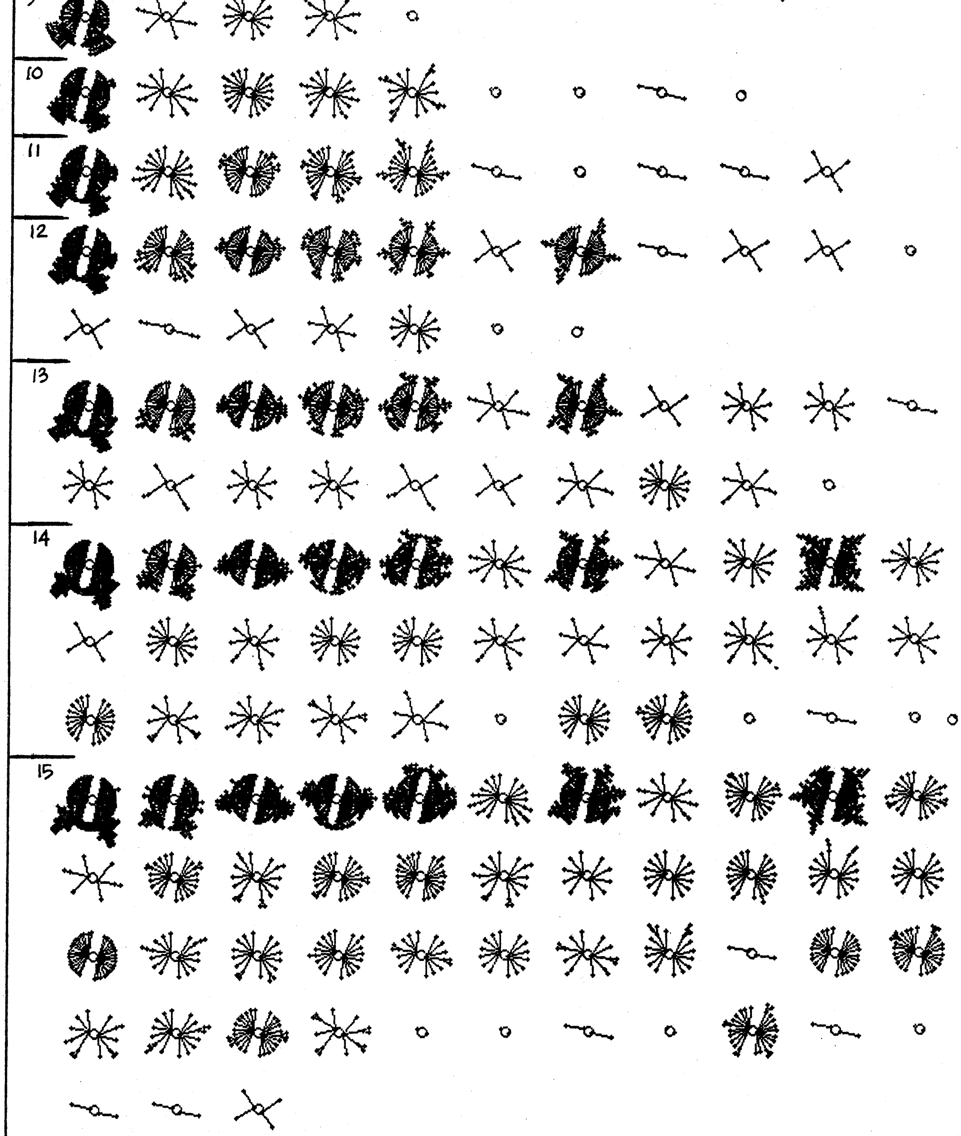






Length=9

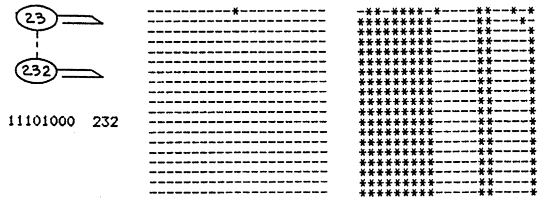


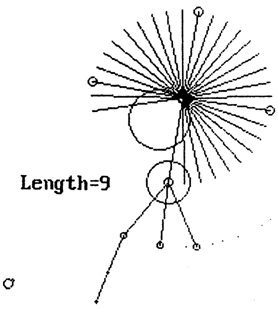


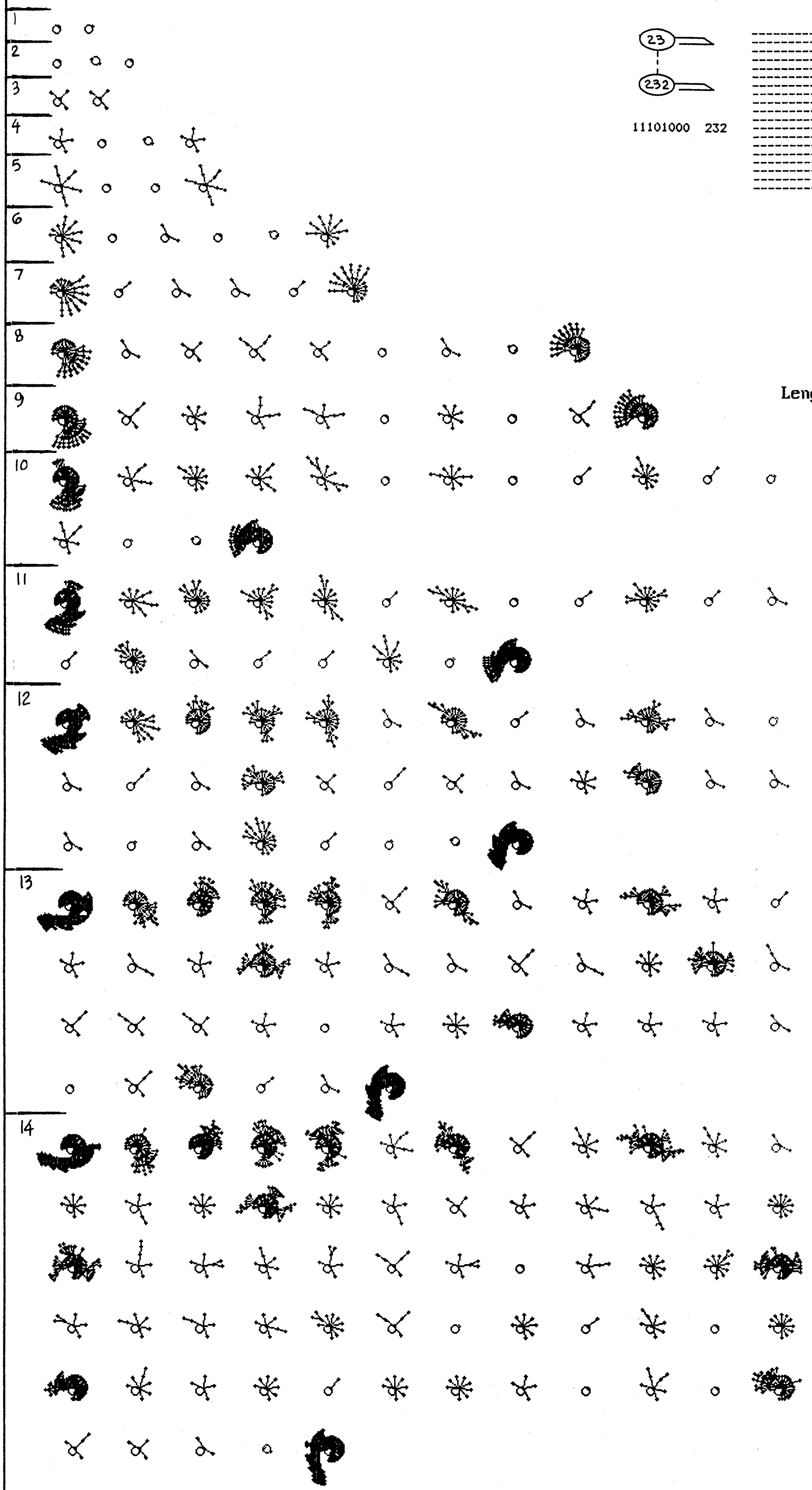

```
3-rule 23 =00010111

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(2)8 6 1 4
 total NATs = 1
L= 4
 1.0000 1(2)10 8 1 5
 2.0011 2(2)2 0 0 1
 3.0101 2(1)1 0 0 1
 total NATs = 5
L= 5
 1.00000 1(2)22 10 2 6
 2.00011 5(2)2 0 0 1
 total NATs = 6
L= 6
 1.000000 1(2)32 18 2 10
 2.000011 6(2)2 0 0 1
 3.000111 3(2)6 4 1 3
 4.010101 2(1)1 0 0 1
 total NATs = 12
L= 7
 1.0000000 1(2)58 28 3 15
 2.0000011 7(2)4 2 1 2
 3.0000111 7(2)6 4 1 3
 total NATs = 15
L= 8
 1.00000000 1(2)90 56 3 21
 2.00000011 8(2)6 4 1 3
 3.00000111 8(2)8 6 1 4
 4.00001111 4(2)12 6 2 4
 5.00110011 2(2)2 0 0 1
 6.01010101 2(1)1 0 0 1
 total NATs = 25
L= 9
 1.000000000 1(2)152 96 4 31
 2.000000011 9(2)10 6 2 4
 3.000000111 9(2)14 12 1 7
 4.000001111 9(2)14 8 2 5
 5.000110011 9(2)2 0 0 1
 total NATs = 37
L= 10
 1.0000000000 1(2)242 170 4 46
 2.0000000011 10(2)16 10 2 6
 3.0000000111 10(2)22 18 2 10
 4.0000001111 10(2)20 14 2 8
 5.1111100000 5(2)26 16 3 7
 6.0000110011 10(2)2 0 0 1
 7.0001100011 5(2)2 0 0 1
 8.0001100111 10(2)4 2 1 2
 9.0101010101 2(1)1 0 0 1
 total NATs = 63
L= 11
 1.00000000000 1(2)398 286 5 67
 2.00000000011 11(2)26 16 3 9
 3.00000000111 11(2)36 30 2 14
 4.00000001111 11(2)34 26 2 12
 5.11111100000 11(2)34 24 3 11
 6.00000110011 11(2)4 2 1 2
 7.00001100011 11(2)2 0 0 1
 8.00001100111 11(2)4 2 1 2
 9.00001110011 11(2)4 2 1 2
 10.00011000111 11(2)6 4 1 3
 total NATs = 100
L= 12
 1.000000000000 1(2)640 482 5 98
 2.000000000011 12(2)42 28 3 13
 3.000000000111 12(2)58 48 3 21
 4.000000001111 12(2)54 44 2 17
 5.111111100000 12(2)52 40 3 16
 6.000000110011 12(2)6 4 1 3
 7.111111000000 6(2)58 42 4 18
 8.000001100011 12(2)4 2 1 2
 9.000001100111 12(2)6 4 1 3
 10.000001110011 12(2)6 4 1 3
 11.000011000011 6(2)2 0 0 1
 12.000011000111 12(2)6 4 1 3
 13.000011001111 12(2)6 2 2 2
 14.111100011100 12(2)6 4 1 3
 15.000011100111 12(2)8 6 1 4
 16.111000111000 3(2)14 12 1 7
 17.001100110011 2(2)2 0 0 1
 18.010101010101 2(1)1 0 0 1
 total NATs = 164
L= 13
 1.0000000000000 1(2)1042 806 6 144
 2.0000000000011 13(2)68 48 4 19
 3.0000000000111 13(2)94 80 3 31
 4.0000000001111 13(2)88 70 3 25
 5.1111111100000 13(2)86 72 3 23
 6.0000000110011 13(2)10 6 2 4
 7.1111111000000 13(2)84 66 4 26
 8.0000001100011 13(2)6 4 1 3
 9.0000001100111 13(2)10 8 1 5
 10.0000001110011 13(2)10 8 1 5
 11.0000011000011 13(2)4 2 1 2
 12.0000011000111 13(2)10 8 1 5
 13.0000011001111 13(2)8 4 2 3
 14.1111100011100 13(2)10 8 1 5
 15.0000011100111 13(2)10 8 1 5
 16.0000011110011 13(2)8 4 2 3
 17.0000110000111 13(2)6 4 1 3
 18.0000110001111 13(2)10 6 2 4
 19.1111000111000 13(2)16 14 1 8
 20.0000111001111 13(2)10 6 2 4
 21.0001100110011 13(2)2 0 0 1
 total NATs = 261
```

λ ratio = 1 $Z = .5$
majority rule

11111100011000000-11111100011000000-rule 4240506048
=3-rule 232 -11101000 Length=1 -14



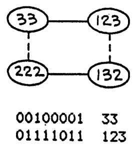


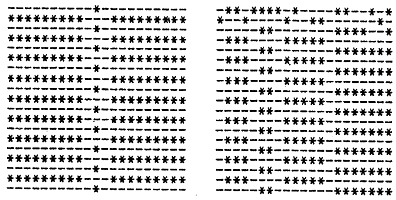


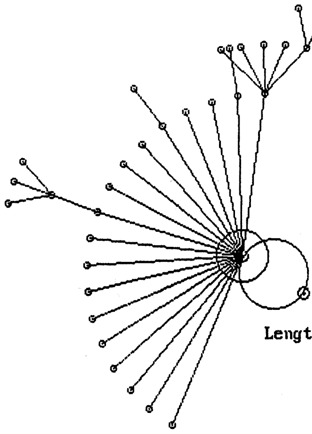

```
3-rule 232 =11101000

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 1(2)2 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)4 3 1 4
 2.111 1(1)4 3 1 4
 total NATs = 2
L= 4
 1.0000 1(1)5 4 1 5
 2.0011 4(1)1 0 0 1
 3.0101 1(2)2 0 0 1
 4.1111 1(1)5 4 1 5
 total NATs = 7
L= 5
 1.00000 1(1)11 5 2 6
 2.00011 5(1)1 0 0 1
 3.00111 5(1)1 0 0 1
 4.11111 1(1)11 5 2 6
 total NATs = 12
L= 6
 1.000000 1(1)16 9 2 10
 2.000011 6(1)1 0 0 1
 3.000111 6(1)3 2 1 3
 4.001111 6(1)1 0 0 1
 5.010101 1(2)2 0 0 1
 6.111111 1(1)16 9 2 10
 total NATs = 21
L= 7
 1.0000000 1(1)29 14 3 15
 2.0000011 7(1)2 1 1 2
 3.0000111 7(1)3 2 1 3
 4.0001111 7(1)3 2 1 3
 5.0011111 7(1)2 1 1 2
 6.1111111 1(1)29 14 3 15
 total NATs = 30
L= 8
 1.00000000 1(1)45 28 3 21
 2.00000011 8(1)3 2 1 3
 3.00000111 8(1)4 3 1 4
 4.00001111 8(1)6 3 2 4
 5.00011111 8(1)4 3 1 4
 6.00110011 4(1)1 0 0 1
 7.00111111 8(1)3 2 1 3
 8.01010101 1(2)2 0 0 1
 9.11111111 1(1)45 28 3 21
 total NATs = 47
L= 9
 1.000000000 1(1)76 48 4 31
 2.000000011 9(1)5 3 2 4
 3.000000111 9(1)7 6 1 7
 4.000001111 9(1)7 4 2 5
 5.000011111 9(1)7 4 2 5
 6.000110011 9(1)1 0 0 1
 7.000111111 9(1)7 6 1 7
 8.001100111 9(1)1 0 0 1
 9.001111111 9(1)5 3 2 4
 10.111111111 1(1)76 48 4 31
 total NATs = 74
L= 10
 1.0000000000 1(1)121 85 4 46
 2.0000000011 10(1)8 5 2 6
 3.0000000111 10(1)11 9 2 10
 4.0000001111 10(1)10 7 2 8
 5.0000011111 10(1)13 8 3 7
 6.0000110011 10(1)1 0 0 1
 7.0000111111 10(1)10 7 2 8
 8.0001100011 5(1)1 0 0 1
 9.0001100111 10(1)2 1 1 2
 10.0001111111 10(1)11 9 2 10
 11.0001110011 10(1)2 1 1 2
 12.0011001111 10(1)1 0 0 1
 13.0011111111 10(1)8 5 2 6
 14.0011100111 5(1)1 0 0 1
 15.0101010101 1(2)2 0 0 1
 16.1111111111 1(1)121 85 4 46
 total NATs = 123
```

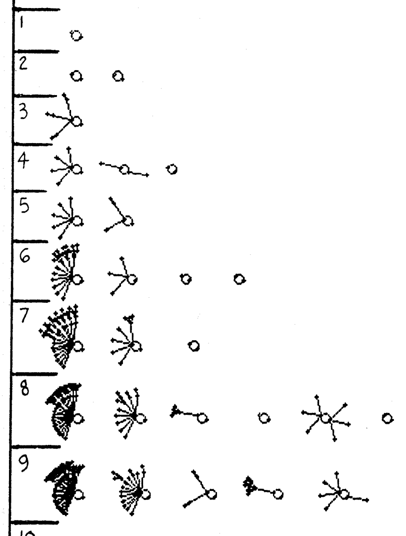
λ ratio = .5 Z = .5

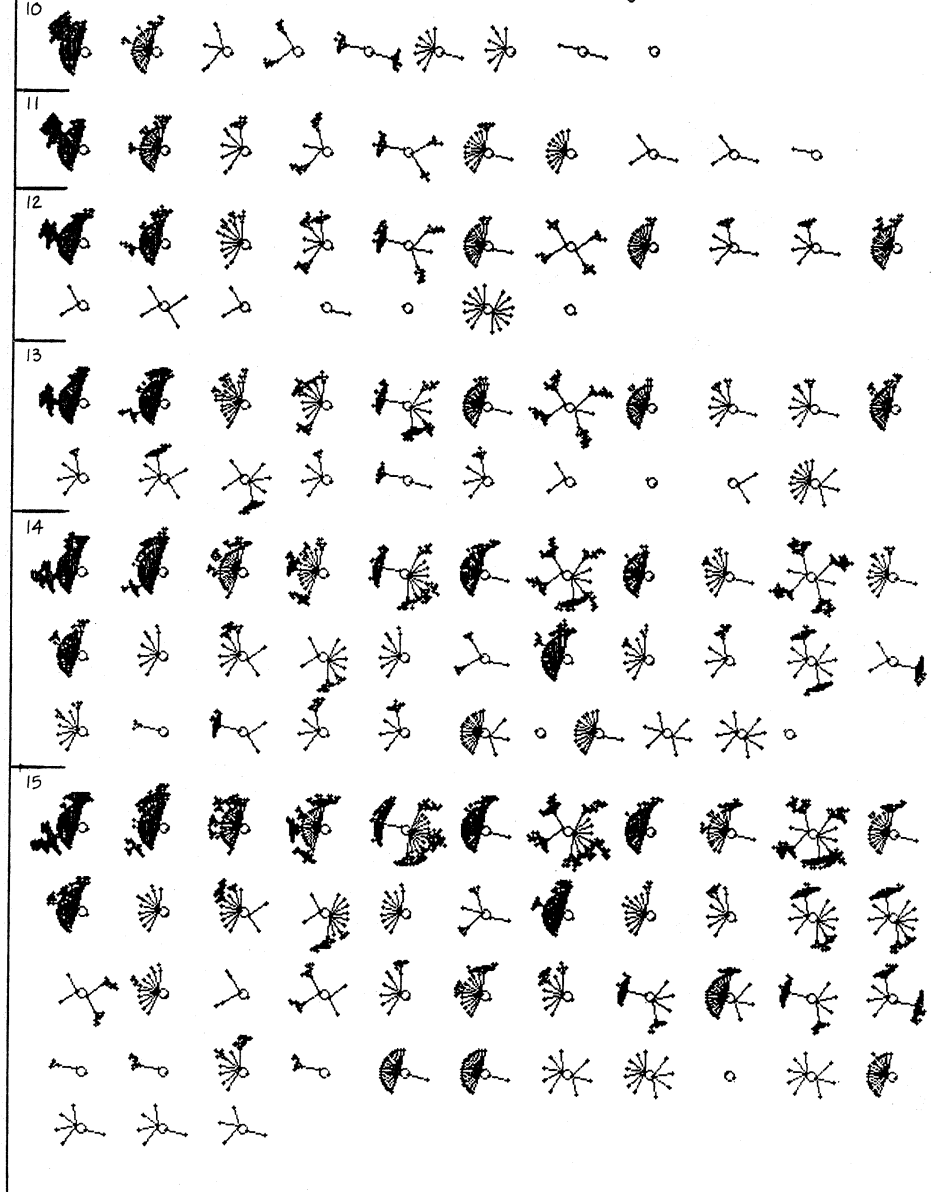
0000110000000011-0000110000000011-rule 201526275
=3-rule 33 -00100001 Length=1 -15







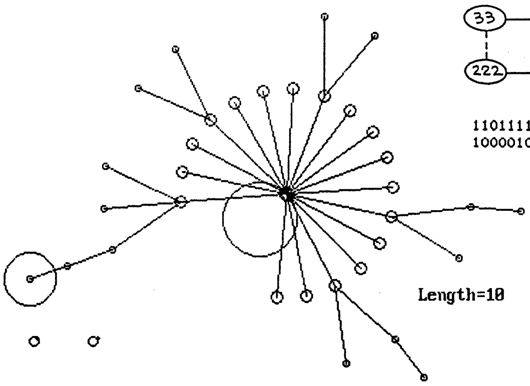


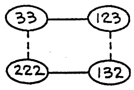



```
3-rule 33 =00100001

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.000 1(2)8 3 2 4
 total NATs = 1
L= 4
 1.0000 1(2)6 4 1 5
 2.0001 2(2)4 2 1 2
 3.0101 1(2)2 0 0 1
 total NATs = 4
L= 5
 1.00000 1(2)7 5 1 6
 2.00001 5(2)5 2 2 3
 total NATs = 6
L= 6
 1.000000 1(2)26 15 3 10
 2.000001 6(2)5 3 1 4
 3.000011 3(2)2 0 0 1
 4.010101 1(2)2 0 0 1
 total NATs = 11
L= 7
 1.0000000 1(2)44 21 4 15
 2.0000001 7(2)10 7 2 6
 3.0000011 7(2)2 0 0 1
 total NATs = 15
L= 8
 1.00000000 1(2)54 36 3 21
 2.00000001 8(2)15 9 3 9
 3.00000011 8(2)7 4 2 4
 4.00000111 4(2)2 0 0 1
 5.00010001 2(2)8 6 1 4
 6.01010101 1(2)2 0 0 1
 total NATs = 24
L= 9
 1.000000000 1(2)89 66 3 31
 2.000000001 9(2)20 13 4 13
 3.000000011 9(2)6 2 2 3
 4.000000111 9(2)14 9 3 7
 5.000010001 9(2)7 5 1 5
 total NATs = 37
L= 10
 1.0000000000 1(2)157 105 5 46
 2.0000000001 10(2)33 25 3 19
 3.0000000011 10(2)7 3 2 4
 4.0000000111 10(2)14 8 3 3
 5.0111100000 5(2)28 20 3 10
 6.0000010001 10(2)10 8 1 8
 7.0000100001 5(2)9 6 2 7
 8.0000100011 10(2)4 2 1 2
 9.0101010101 1(2)2 0 0 1
 total NATs = 62
L= 11
 1.00000000000 1(2)233 165 6 67
 2.00000000001 11(2)50 38 3 28
 3.00000000011 11(2)18 11 3 7
 4.00000000111 11(2)23 15 4 5
 5.01111100000 11(2)29 20 3 14
 6.00000010001 11(2)20 17 2 12
 7.00000100001 11(2)12 10 1 11
 8.00000100011 11(2)5 3 1 3
 9.00000110001 11(2)5 3 1 3
 10.00001000011 11(2)3 1 1 2
 total NATs = 100
L= 12
 1.000000000000 1(2)354 283 5 98
 2.000000000001 12(2)78 60 5 41
 3.000000000011 12(2)22 12 3 9
 4.000000000111 12(2)39 29 4 11
 5.011111100000 12(2)51 38 4 21
 6.000000010001 12(2)23 19 2 17
 7.011111000000 6(2)26 16 3 4
 8.000000100001 12(2)20 17 2 16
 9.000000100011 12(2)15 12 2 8
 10.000000110001 12(2)15 12 2 8
 11.000001000001 6(2)31 25 3 18
 12.000001000011 12(2)4 2 1 3
 13.000001000111 12(2)6 4 1 3
 14.000001100001 12(2)4 2 1 3
 15.000001100011 12(2)3 1 1 2
 16.000011000011 3(2)2 0 0 1
 17.000100010001 2(2)16 14 1 8
 18.010101010101 1(2)2 0 0 1
 total NATs = 163
L= 13
 1.0000000000000 1(2)574 468 5 144
 2.0000000000001 13(2)124 103 6 60
 3.0000000000011 13(2)31 19 3 13
 4.0000000000111 13(2)47 34 4 10
 5.0111111100000 13(2)79 61 4 31
 6.0000000010001 13(2)31 27 2 25
 7.0111111000000 13(2)59 41 5 6
 8.0000000100001 13(2)28 23 2 23
 9.0000000100011 13(2)14 10 2 7
 10.0000000110001 13(2)14 10 2 7
 11.0000001000001 13(2)40 33 3 26
 12.0000001000011 13(2)9 6 2 5
 13.0000001000111 13(2)25 21 3 13
 14.0111100000100 13(2)25 21 3 13
 15.0000001100001 13(2)9 6 2 5
 16.0000001100011 13(2)15 12 2 11
 17.0000010000011 13(2)12 8 3 5
 18.0000010000111 13(2)4 2 1 3
 19.0000011000011 13(2)2 0 0 1
 20.0000011000111 13(2)4 2 1 3
 21.0000100010001 13(2)14 12 1 10
 total NATs = 261
```

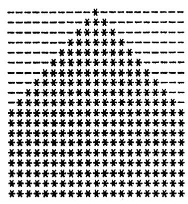
λ ratio = .5 Z = .5

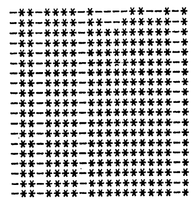
1111001111111100-1111001111111100-rule 4093441020
=3-rule 222 -11011110 Length=1 -13

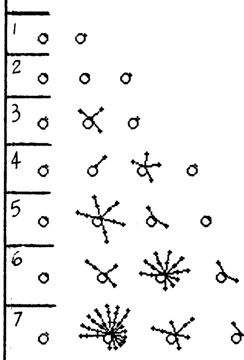


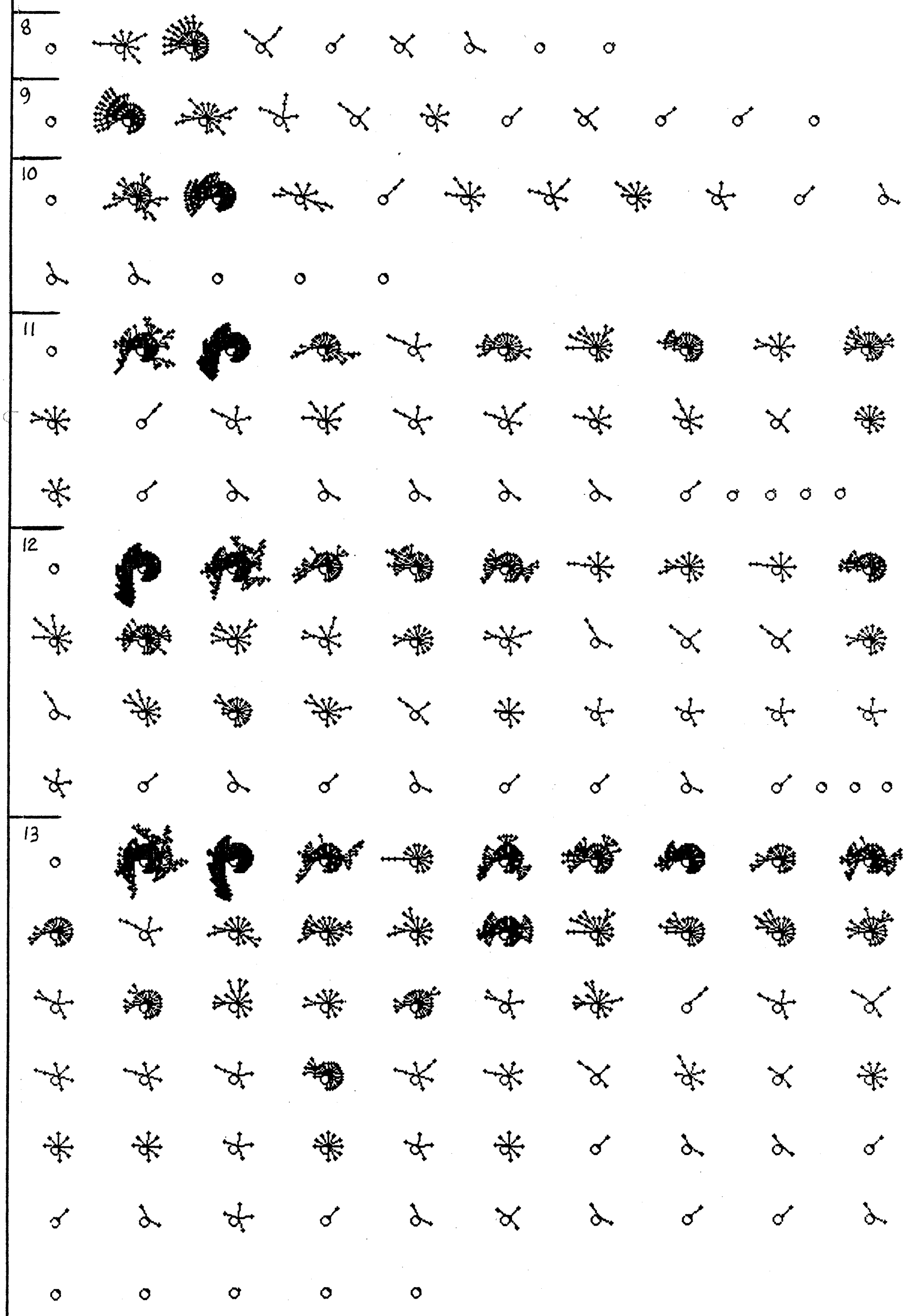


11011110 222
10000100 132







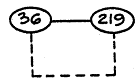



```
3-rule 222 =11011110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 2(1)1 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 4
L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)4 3 1 4
 3.011 3(1)1 0 0 1
 total NATs = 5
L= 4
 1.0000 1(1)1 0 0 1
 2.1011 4(1)2 1 1 2
 3.1111 1(1)5 4 1 5
 4.0101 2(1)1 0 0 1
 total NATs = 8
L= 5
 1.00000 1(1)1 0 0 1
 2.11111 1(1)11 5 2 6
 3.10111 5(1)3 2 1 3
 4.01011 5(1)1 0 0 1
 total NATs = 12
L= 6
 1.000000 1(1)1 0 0 1
 2.110111 6(1)5 3 2 4
 3.111111 1(1)16 9 2 10
 4.101101 3(1)3 2 1 3
 5.010101 2(1)1 0 0 1
 6.010111 6(1)1 0 0 1
 total NATs = 19
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111111 1(1)29 14 3 15
 3.1101111 7(1)8 5 2 6
 4.1011011 7(1)3 2 1 3
 5.1110101 7(1)2 1 1 2
 6.0101011 7(1)1 0 0 1
 total NATs = 30
L= 8
 1.00000000 1(1)1 0 0 1
 2.11101111 8(1)13 8 3 9
 3.11111111 1(1)45 28 3 21
 4.11011101 4(1)6 3 2 4
 5.10110101 8(1)2 1 1 2
 6.10110111 8(1)4 3 1 4
 7.11101011 8(1)3 2 1 3
 8.01010101 2(1)1 0 0 1
 9.01010111 8(1)1 0 0 1
 total NATs = 48
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111111 1(1)76 48 4 31
 3.111011111 9(1)21 14 3 13
 4.110111011 9(1)7 4 2 5
 5.111110101 9(1)5 3 2 4
 6.101111101 9(1)7 6 1 7
 7.101101011 9(1)2 1 1 2
 8.101101101 3(1)4 3 1 4
 9.101110101 9(1)2 1 1 2
 10.111010101 9(1)2 1 1 2
 11.010101011 9(1)1 0 0 1
 total NATs = 77
L= 10
 1.0000000000 1(1)1 0 0 1
 2.1111011111 10(1)34 24 4 19
 3.1111111111 1(1)121 85 4 46
 4.1110111101 5(1)13 8 3 7
 5.1101110101 10(1)3 1 2 2
 6.1101110111 10(1)10 7 2 8
 7.1111101011 10(1)8 5 2 6
 8.1111101101 10(1)11 9 2 10
 9.1011101101 10(1)5 4 1 5
 10.1011010101 10(1)2 1 1 2
 11.1011010111 10(1)3 2 1 3
 12.1011110101 10(1)3 2 1 3
 13.1110101011 10(1)3 2 1 3
 14.0101010101 2(1)1 0 0 1
 15.0101010111 10(1)1 0 0 1
 16.0101101011 5(1)1 0 0 1
 total NATs = 124
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111111 1(1)199 143 5 67
 3.11110111111 11(1)55 41 4 28
 4.11101111011 11(1)17 12 3 11
 5.11111110101 11(1)13 8 3 9
 6.11011111101 11(1)17 13 2 12
 7.11011101011 11(1)4 2 2 3
 8.11011101101 11(1)7 5 2 6
 9.11011110101 11(1)4 2 2 3
 10.11111101101 11(1)18 15 2 14
 11.11111010101 11(1)5 3 2 4
 12.10111110101 11(1)5 4 1 5
 13.10110110111 11(1)7 6 1 7
 14.10110101111 11(1)5 4 1 5
 15.10110101011 11(1)2 1 1 2
 16.10110101101 11(1)3 2 1 3
 17.10111010101 11(1)2 1 1 2
 18.11101010101 11(1)2 1 1 2
 19.01010101011 11(1)1 0 0 1
 20.01011010111 11(1)1 0 0 1
 total NATs = 200
L= 12
 1.000000000000 1(1)1 0 0 1
 2.111110111111 12(1)89 69 5 41
 3.111111111111 1(1)320 241 5 98
 4.111101111101 6(1)29 21 4 18
 5.111011110101 12(1)7 4 3 5
 6.111011110111 12(1)26 20 3 16
 7.111111101011 12(1)21 14 3 13
 8.111111101101 12(1)29 24 3 21
 9.110111101101 12(1)10 8 2 8

 26.101111010101 12(1)3 2 1 3
 27.111010101011 12(1)3 2 1 3
 28.111010110101 12(1)2 1 1 2
 29.010101010101 2(1)1 0 0 1
 30.010101010111 12(1)1 0 0 1
 31.010101101011 12(1)1 0 0 1
 32.010111010111 6(1)1 0 0 1
 total NATs = 323
```

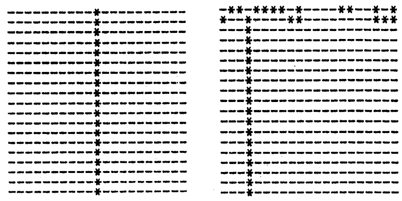
intentionally blank

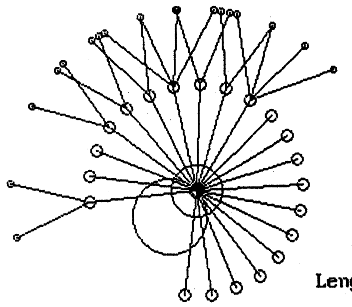
λ ratio = .5 Z = .5

00001100000110000-00001100000110000-rule 204475440
=3-rule 36 -00100100
Length=1 -15

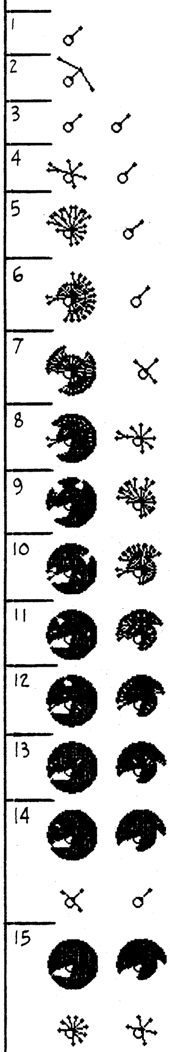


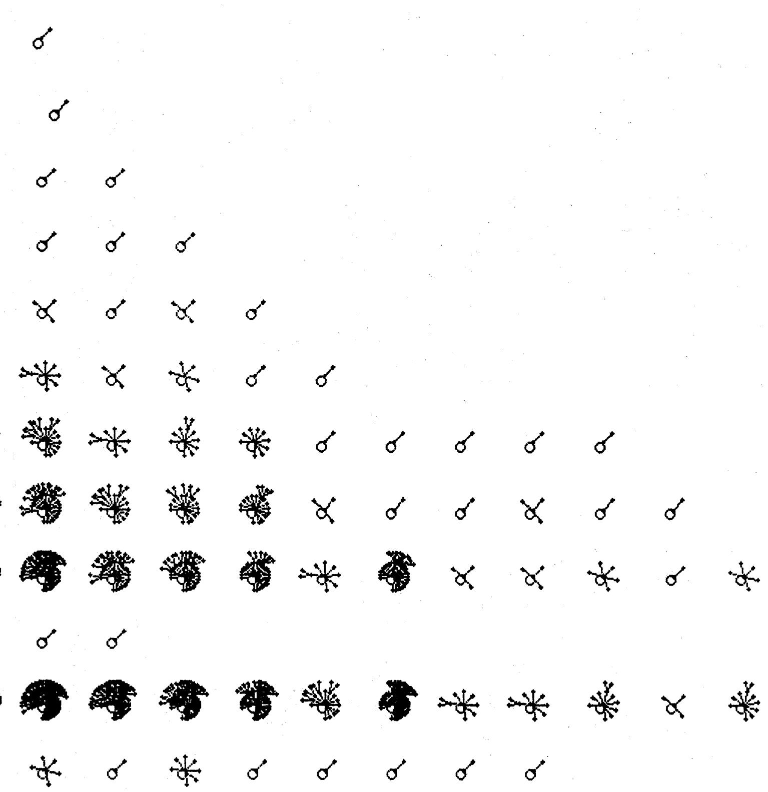
00100100 36
11011011 219





Length=10



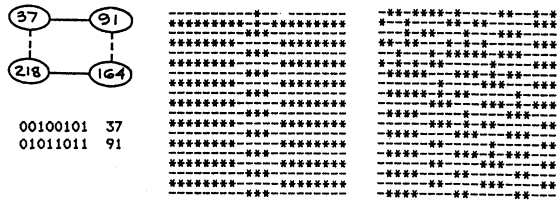


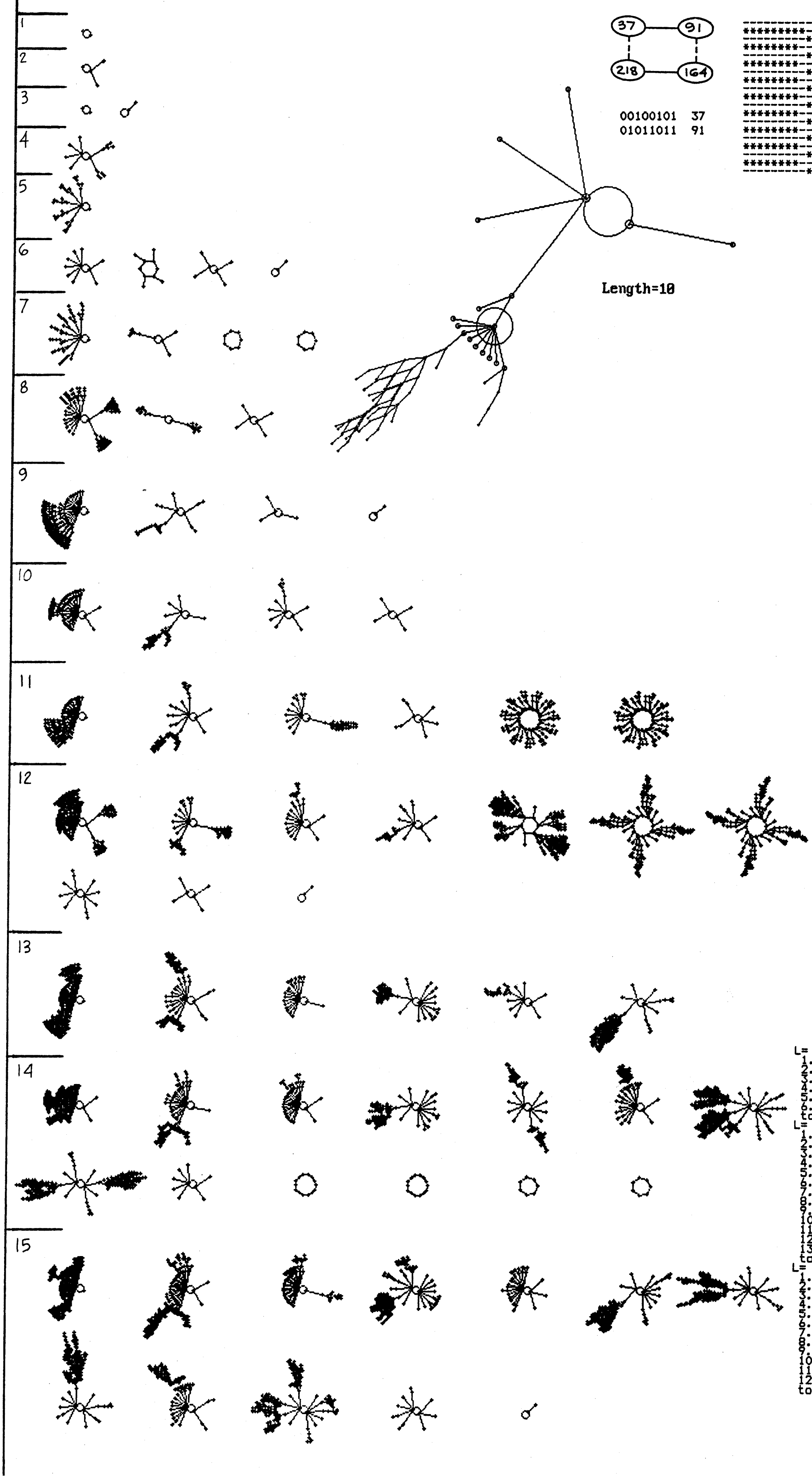
3-rule 36 =00100100

ty. at no(p)s g ml mp

```
L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.000 1(1)2 1 1 2
 2.001 3(1)2 1 1 2
 total NATs = 4
L= 4
 1.0000 1(1)8 6 2 6
 2.0001 4(1)2 1 1 2
 total NATs = 5
L= 5
 1.00000 1(1)22 16 2 12
 2.00001 5(1)2 1 1 2
 total NATs = 6
L= 6
 1.000000 1(1)46 32 2 20
 2.000001 6(1)2 1 1 2
 3.001001 3(1)2 1 1 2
 total NATs = 10
L= 7
 1.0000000 1(1)86 64 2 30
 2.0000001 7(1)4 3 1 4
 3.0001001 7(1)2 1 1 2
 total NATs = 15
L= 8
 1.00000000 1(1)152 114 2 46
 2.00000001 8(1)10 8 2 8
 3.00001001 8(1)2 1 1 2
 4.00010001 4(1)2 1 1 2
 total NATs = 21
L= 9
 1.000000000 1(1)272 217 2 74
 2.000000001 9(1)22 17 2 14
 3.000001001 9(1)2 1 1 2
 4.000010001 9(1)2 1 1 2
 5.001001001 3(1)2 1 1 2
 total NATs = 31
L= 10
 1.0000000000 1(1)504 412 2 122
 2.0000000001 10(1)42 33 2 22
 3.0000001001 10(1)4 3 1 4
 4.0000010001 10(1)2 1 1 2
 5.0000100001 5(1)4 3 1 4
 6.0001001001 10(1)2 1 1 2
 total NATs = 46
L= 11
 1.00000000000 1(1)948 793 2 200
 2.00000000001 11(1)76 61 2 34
 3.00000001001 11(1)10 8 2 8
 4.00000010001 11(1)4 3 1 4
 5.00000100001 11(1)6 5 1 6
 6.00001001001 11(1)2 1 1 2
 7.00010001001 11(1)2 1 1 2
 total NATs = 67
L= 12
 1.000000000000 1(1)1790 1530 2 324
 2.000000000001 12(1)136 113 2 54
 3.000000001001 12(1)22 17 2 14
 4.000000010001 12(1)10 8 2 8
 5.000000100001 12(1)12 10 2 10
 6.000001000001 6(1)10 9 1 10
 7.000001001001 12(1)2 1 1 2
 8.000010001001 12(1)2 1 1 2
 9.000010010001 12(1)2 1 1 2
 10.000100010001 4(1)2 1 1 2
 11.001001001001 3(1)2 1 1 2
 total NATs = 98
L= 13
 1.0000000000000 1(1)3356 2926 2 522
 2.0000000000001 13(1)250 214 2 88
 3.0000000001001 13(1)42 33 2 22
 4.0000000010001 13(1)22 17 2 14
 5.0000000100001 13(1)22 18 2 16
 6.0000001000001 13(1)20 18 2 16
 7.0000001001001 13(1)4 3 1 4
 8.0000010001001 13(1)2 1 1 2
 9.0000010010001 13(1)2 1 1 2
 10.0000100001001 13(1)4 3 1 4
 11.0000100010001 13(1)2 1 1 2
 12.0001001001001 13(1)2 1 1 2
 total NATs = 144
L= 14
 1.00000000000000 1(1)6262 5560 2 842
 2.00000000000001 14(1)468 408 2 144
 3.00000000001001 14(1)76 61 2 34
 4.00000000010001 14(1)42 33 2 22
 5.00000000100001 14(1)42 35 2 26
 6.00000001000001 14(1)34 30 2 24
 7.00000001001001 14(1)10 8 2 8
 8.00000010000001 7(1)40 36 2 26
 9.00000010001001 14(1)4 3 1 4
 10.00000010010001 14(1)4 3 1 4
 11.00000100001001 14(1)6 5 1 6
 12.00000100010001 14(1)2 1 1 2
 13.00000100100001 14(1)6 5 1 6
 14.00001000010001 14(1)4 3 1 4
 15.00001001001001 14(1)2 1 1 2
 16.00010001001001 14(1)2 1 1 2
 17.00010010001001 7(1)2 1 1 2
 total NATs = 211
L= 15
 1.000000000000000 1(1)11662 10516 2 1362
 2.000000000000001 15(1)882 782 2 234
 3.000000000001001 15(1)136 113 2 54
 4.000000000010001 15(1)76 61 2 34
 5.000000000100001 15(1)78 67 2 42
 6.000000001000001 15(1)62 55 2 38
 7.000000001001001 15(1)22 17 2 14
 8.000000010000001 15(1)68 61 2 40
 9.000000010001001 15(1)10 8 2 8
 10.000000010010001 15(1)10 8 2 8
 11.000000100001001 15(1)12 10 2 10
 12.000000100010001 15(1)4 3 1 4
 13.000000100100001 15(1)12 10 2 10
 14.000001000001001 15(1)10 9 1 10
 15.000001000010001 15(1)6 5 1 6
 16.000001000100001 15(1)6 5 1 6
 17.000001001001001 15(1)2 1 1 2
 18.000010000100001 5(1)8 7 1 8
 19.000010001001001 15(1)2 1 1 2
 20.000010010001001 15(1)2 1 1 2
 21.000010010010001 15(1)2 1 1 2
 22.000100010001001 15(1)2 1 1 2
 23.001001001001001 3(1)2 1 1 2
 total NATs = 309
```

λ ratio = .75 Z = .75 0000110000110011-0000110000110011-rule 204672051
=3-rule 37 -00100101 Length=1 -15



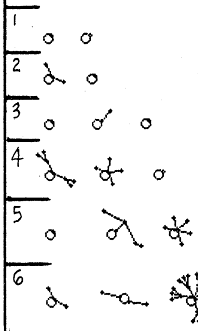


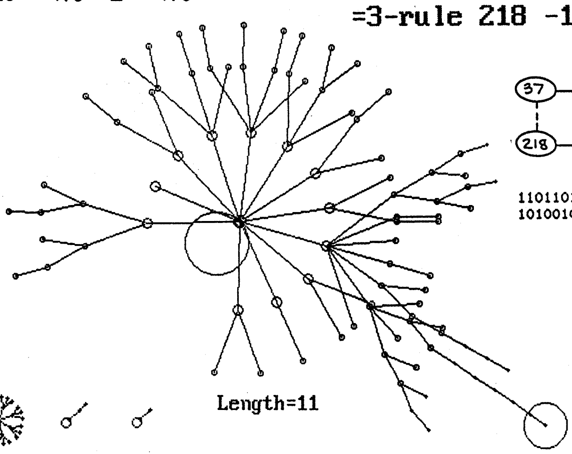
3-rule 37 =00100101

ty. at no(p)s g ml mp

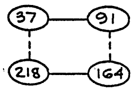
```
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)4 2 1 3
 total NATs = 1
L= 3
 1.000 1(2)2 0 0 1
 2.001 3(1)2 1 1 2
 total NATs = 4
L= 4
 1.0000 1(2)16 8 3 5
 total NATs = 1
L= 5
 1.00000 1(2)32 15 4 6
 total NATs = 1
L= 6
 1.000000 1(2)10 8 1 7
 2.000001 3(6)10 4 1·2
 3.000011 3(2)6 4 1 3
 4.001001 3(1)2 1 1 2
 total NATs = 10
L= 7
 1.0000000 1(2)44 14 5 8
 2.1000011 7(2)10 6 3 4
 3.0001011 1(7)7 0 0 1
 4.0001101 1(7)7 0 0 1
 total NATs = 10
L= 8
 1.00000000 1(2)112 48 5 13
 2.10000011 4(2)24 14 4 5
 3.00000011 8(2)6 4 1 3
 total NATs = 13
L= 9
 1.000000000 1(2)227 90 7 19
 2.100000011 9(2)26 12 10 7
 3.000000011 9(2)5 3 1 3
 4.001001001 3(1)2 1 1 2
 total NATs = 22
L= 10
 1.0000000000 1(2)94 67 4 26
 2.1000000011 10(2)75 35 13 9
 3.0000000011 10(2)15 9 5 6
 4.0000001111 5(2)6 4 1 3
 total NATs = 26
L= 11
 1.00000000000 1(2)112 55 6 34
 2.10000000011 11(2)89 48 12 13
 3.00000000011 11(2)54 21 11 9
 4.00000001111 11(2)9 5 2 4
 5.01111011000 1(33)132 44 3 3
 6.01111000011 1(33)132 44 3 3
 total NATs = 36
L= 12
 1.000000000000 1(2)226 116 5 47
 2.100000000011 12(2)86 53 6 18
 3.000000000011 12(2)30 20 7 12
 4.001111000000 12(2)26 13 8 6
 5.000001000001 3(6)362 150 8 8
 6.100001100000 3(12)164 72 11 5
 7.110000100000 3(12)164 72 11 5
 8.000000011111 6(2)12 8 2 5
 9.000011000011 3(2)6 4 1 3
 10.001001001001 3(1)2 1 1 2
 total NATs = 58
L= 13
 1.0000000000000 1(2)366 208 8 66
 2.1000000000011 13(2)141 70 12 25
 3.0000000000011 13(2)25 19 2 15
 4.1100000011111 13(2)87 42 8 10
 5.0011111000000 13(2)31 18 8 7
 6.0011000011100 13(2)318 142 18 9
 total NATs = 66
L= 14
 1.00000000000000 1(2)340 226 5 92
 2.10000000000011 14(2)127 77 11 34
 3.00000000000011 14(2)39 27 4 21
 4.11000000011111 14(2)168 95 11 13
 5.00111111000000 7(2)72 38 10 9
 6.00000000001111 14(2)86 43 9 16
 7.00111000011100 7(2)600 301 16 12
 8.11000001100001 14(2)378 174 18 9
 9.00000011000011 14(2)9 7 1 6
 10.00001001101101 1(14)14 0 0 1
 11.00001011011001 1(14)14 0 0 1
 12.00010110001011 1(7)7 0 0 1
 13.00011010001101 1(7)7 0 0 1
 total NATs = 103
L= 15
 1.000000000000000 1(2)497 270 6 126
 2.100000000000011 15(2)275 151 19 47
 3.000000000000011 15(2)75 53 6 30
 4.110000000011111 15(2)202 100 10 17
 5.000000000001111 15(2)29 21 2 15
 6.001111000011100 15(2)461 221 17 18
 7.110000001100001 15(2)315 148 24 13
 8.110000001110000 15(2)315 148 24 13
 9.000000000011111 15(2)233 125 15 22
 10.110000001110001 15(2)233 127 11 16
 11.000000011000011 15(2)13 8 2 6
 12.001001001001001 3(1)2 1 1 2
 total NATs = 154
```

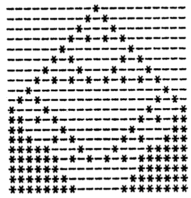
λ ratio = .75   Z = .75   1111001111001100-1111001111001100-rule   4090295244
=3-rule 218 -11011010   Length=1 -15

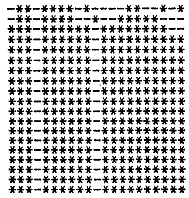


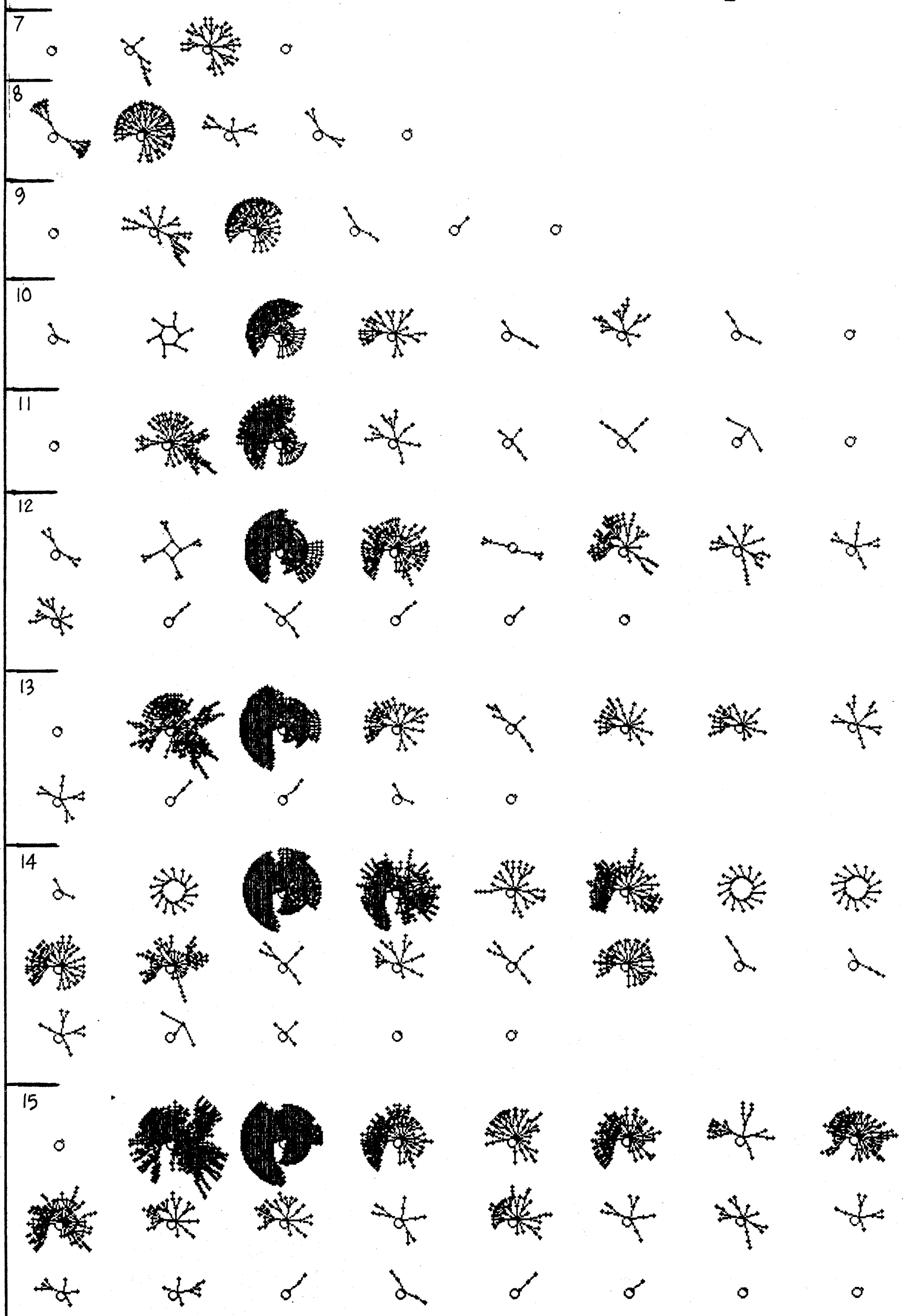


Length=11



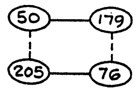




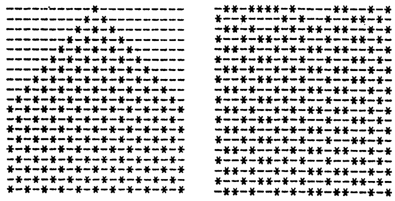


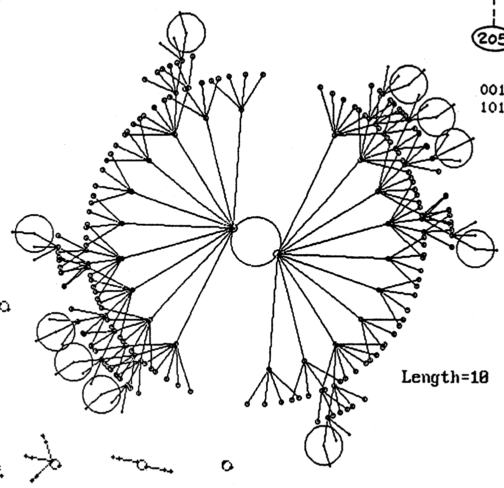

```
3-rule 218 =11011010

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)3 2 1 3
 2.11 1(1)1 0 0 1
 total NATs = 2
L= 3
 1.000 1(1)1 0 0 1
 2.110 3(1)2 1 1 2
 3.111 1(1)1 0 0 1
 total NATs = 5
L= 4
 1.0000 1(1)7 4 2 3
 2.1111 1(1)5 4 1 5
 3.0111 4(1)1 0 0 1
 total NATs = 6
L= 5
 1.00000 1(1)1 0 0 1
 2.11110 5(1)5 2 3 2
 3.11111 1(1)6 5 1 6
 total NATs = 7
L= 6
 1.000000 1(1)3 2 1 3
 2.100010 3(2)4 2 1 2
 3.111111 1(1)25 18 2 7
 4.101111 6(1)3 1 2 2
 5.110110 3(1)2 1 1 2
 total NATs = 14
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111110 7(1)11 5 6 4
 3.1111111 1(1)43 21 3 8
 4.0110111 7(1)1 0 0 1
 total NATs = 16
L= 8
 1.00000000 1(1)31 16 4 3
 2.11111111 1(1)93 48 3 13
 3.11011111 8(1)12 7 3 5
 4.10111011 4(1)7 4 2 3
 5.01101111 8(1)1 0 0 1
 total NATs = 22
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111110 9(1)37 15 7 7
 3.111111111 1(1)118 72 3 19
 4.101110111 9(1)5 2 2 3
 5.110110110 3(1)2 1 1 2
 6.011011111 9(1)1 0 0 1
 total NATs = 32
L= 10
 1.0000000000 1(1)3 2 1 3
 2.1000000010 5(6)12 6 1 2
 3.1111111111 1(1)336 205 4 26
 4.1110111111 10(1)41 19 4 9
 5.1011011111 10(1)5 2 3 3
 6.1101111011 5(1)21 12 4 6
 7.1011101111 10(1)5 2 2 3
 8.0110110111 10(1)1 0 0 1
 total NATs = 52
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111110 11(1)94 45 10 13
 3.11111111111 1(1)606 275 7 34
 4.11011110111 11(1)19 9 3 6
 5.01101111111 11(1)5 3 2 4
 6.11011101111 11(1)8 3 3 4
 7.10111011011 11(1)4 2 2 2
 8.01101101111 11(1)1 0 0 1
 total NATs = 68
L= 12
 1.000000000000 1(1)7 4 2 3
 2.010000000100 6(4)16 8 2 2
 3.111111111111 1(1)1133 568 5 47
 4.111101111111 12(1)142 76 5 18
 5.010001010001 3(2)8 4 2 2
 6.111011111011 6(1)67 31 5 8
 7.011110111111 12(1)25 11 5 7
 8.011011111111 12(1)11 6 2 5
 9.110111011111 12(1)15 10 3 7
 10.101110110111 12(1)3 1 2 2
 11.101110111011 4(1)7 3 2 4
 12.101111011011 12(1)3 1 2 2
 13.110110110110 3(1)2 1 1 2
 14.011011011111 12(1)1 0 0 1
 total NATs = 108
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.1111111111110 13(1)303 149 12 25
 3.1111111111111 1(1)1951 910 7 66
 4.1110111110111 13(1)64 33 4 9
 5.1011011110111 13(1)11 5 4 4
 6.1101111011111 13(1)34 15 4 10
 7.1111011101111 13(1)29 15 4 10
 8.0110111111111 13(1)17 9 3 6
 9.1011110111011 13(1)12 7 2 5
 10.1011101101111 13(1)3 1 2 2
 11.1011111011011 13(1)3 1 2 2
 12.1101101101111 13(1)3 2 1 3
 13.0110110110111 13(1)1 0 0 1
 total NATs = 145
L= 14
 1.00000000000000 1(1)3 2 1 3
 2.10000000000010 7(14)28 14 1 2
 3.11111111111111 1(1)3683 1897 6 92
 4.11111011111111 14(1)489 255 6 34
 5.11101101111111 14(1)37 18 5 9
 6.11110111111011 7(1)183 96 6 12
 7.10000010101000 1(14)28 14 1 2
 8.10000010001010 1(14)28 14 1 2
 9.11101111101111 14(1)77 41 4 12
 10.10111101111111 14(1)69 36 5 16
 11.11011110110111 14(1)10 4 3 4
 12.11011110111011 14(1)20 11 3 7
 13.11011111011011 14(1)10 4 3 4
 14.11110111011111 14(1)55 32 4 13
 15.11011101101111 14(1)5 2 3 3
 16.01111110110111 14(1)5 2 3 3
 17.10111011101111 14(1)11 6 2 5
 18.10111011011011 14(1)4 2 2 2
 19.11011011011111 14(1)4 3 1 4
 20.01101101101111 14(1)1 0 0 1
 21.01101110110111 7(1)1 0 0 1
 total NATs = 221
```

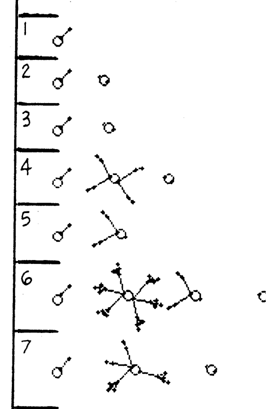
λ ratio = .75 Z = .625 **0000111100001100-0000111100001100-rule 252448524**
=3-rule 50 -00110010 Length=1 -14

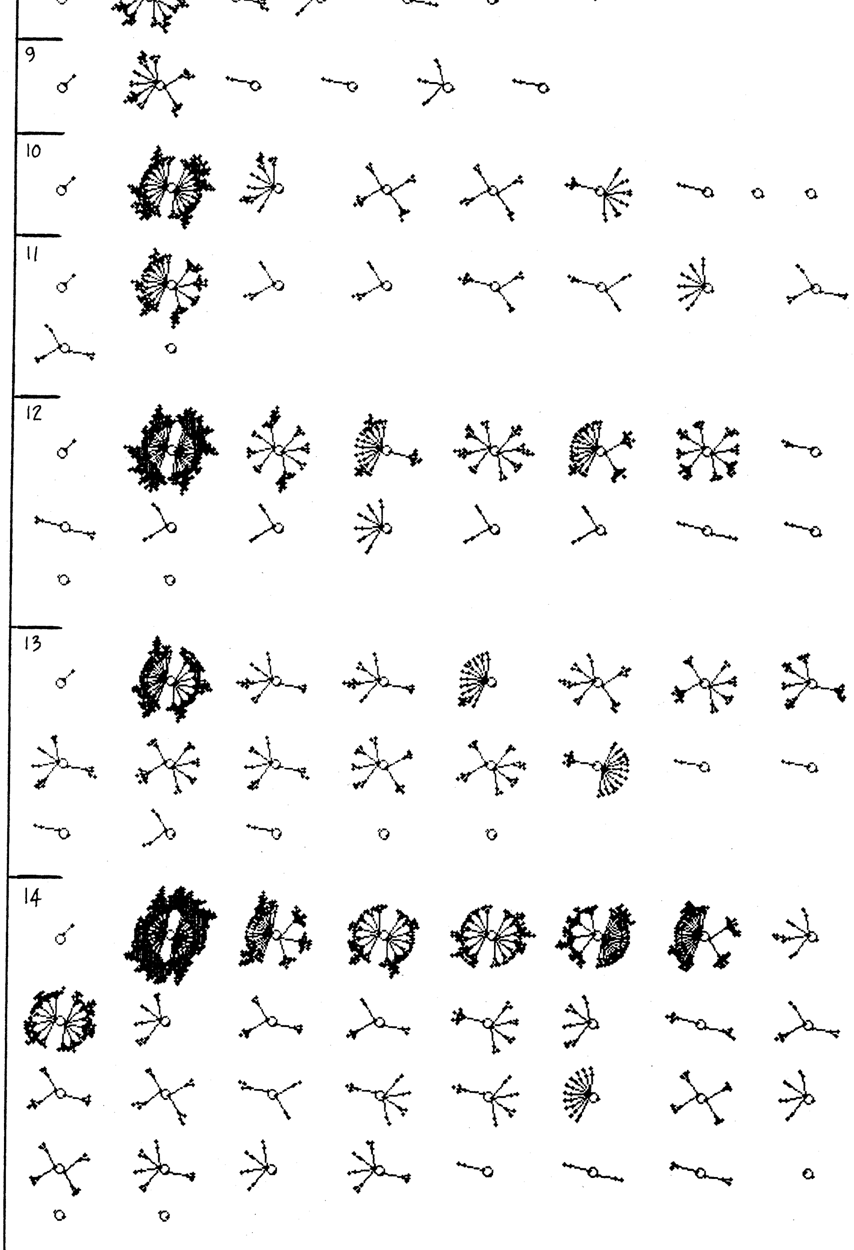


00110010 50
10110011 179







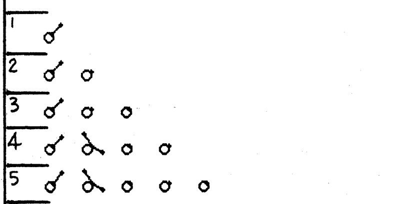


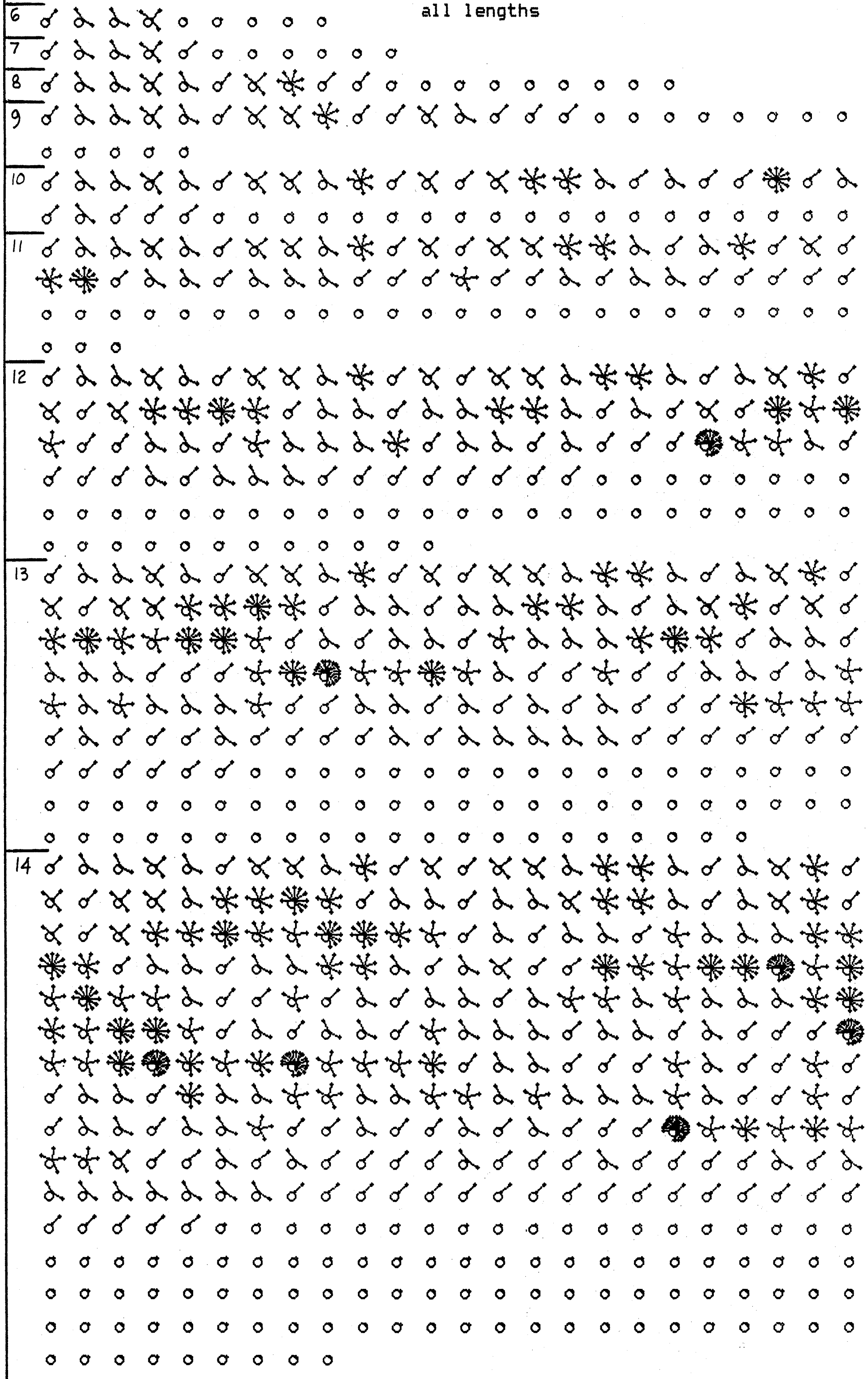

```
3-rule 50 =00110010

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.000 1(1)2 1 1 2
 2.001 3(2)2 0 0 1
 total NATs = 4
L= 4
 1.0000 1(1)2 1 1 2
 2.1010 1(2)10 4 2 3
 3.0011 2(2)2 0 0 1
 total NATs = 4
L= 5
 1.00000 1(1)2 1 1 2
 2.10010 5(2)6 2 2 3
 total NATs = 6
L= 6
 1.000000 1(1)2 1 1 2
 2.010101 1(2)32 18 3 4
 3.100100 3(2)6 2 2 3
 4.001011 6(2)2 0 0 1
 total NATs = 11
L= 7
 1.0000000 1(1)2 1 1 2
 2.0100101 7(2)16 8 3 4
 3.0010011 7(2)2 0 0 1
 total NATs = 15
L= 8
 1.00000000 1(1)2 1 1 2
 2.10101010 1(2)90 60 4 7
 3.01001011 4(2)12 6 3 3
 4.10010100 8(2)8 3 2 4
 5.10101100 8(2)6 2 2 2
 6.00110011 2(2)2 0 0 1
 total NATs = 24
L= 9
 1.000000000 1(1)2 1 1 2
 2.101001010 9(2)42 26 4 7
 3.100101100 9(2)4 1 2 2
 4.100100110 9(2)4 1 2 2
 5.100100100 3(2)8 3 2 4
 6.101001100 9(2)4 1 2 2
 total NATs = 40
L= 10
 1.0000000000 1(1)2 1 1 2
 2.0101010101 1(2)242 180 5 11
 3.1010010100 5(2)26 14 4 7
 4.0100101011 10(2)20 12 3 4
 5.0101010011 10(2)16 8 3 3
 6.0101011011 10(2)22 12 3 7
 7.1001001100 10(2)4 1 2 2
 8.0010011011 5(2)2 0 0 1
 9.0010110011 10(2)2 0 0 1
 total NATs = 62
L= 11
 1.00000000000 1(1)2 1 1 2
 2.01010010101 11(2)110 78 5 11
 3.01001010011 11(2)8 3 3 3
 4.01001011001 11(2)8 3 3 3
 5.01001011011 11(2)14 8 3 4
 6.01010110011 11(2)10 4 3 3
 7.10010100100 11(2)14 6 2 7
 8.10010101100 11(2)10 5 2 3
 9.10010011010 11(2)10 5 2 3
 10.00100110011 11(2)2 0 0 1
 total NATs = 100
L= 12
 1.000000000000 1(1)2 1 1 2
 2.101010101010 1(2)640 514 6 18
 3.010100101011 6(2)58 40 5 7
 4.101001010100 12(2)52 33 4 11
 5.101010101100 12(2)42 26 4 5
 6.101010100100 12(2)58 38 4 11
 7.010010101011 12(2)54 38 3 7
 8.010010110011 12(2)6 2 3 2
 9.100101101100 12(2)8 4 2 2
 10.100101100100 12(2)6 2 2 3
 11.100101001100 12(2)6 2 2 3
 12.100100100100 3(2)14 6 2 7
 13.100100110100 12(2)6 2 2 3
 14.100100110010 12(2)6 2 2 3
 15.101011001100 12(2)6 2 2 2
 16.101001101100 12(2)4 1 2 2
 17.001011001011 6(2)2 0 0 1
 18.001100110011 2(2)2 0 0 1
 total NATs = 163
L= 13
 1.0000000000000 1(1)2 1 1 2
 2.1010100101010 13(2)288 224 6 18
 3.1010010101100 13(2)20 11 4 5
 4.1010010100110 13(2)20 11 4 5
 5.1010010100100 13(2)30 16 4 11
 6.1010101001100 13(2)26 14 4 5
 7.0100101011011 13(2)30 20 3 7
 8.0100101001011 13(2)32 21 3 7
 9.0100101010011 13(2)20 11 3 5
 10.0100101101101 13(2)30 20 3 7
 11.0100101100101 13(2)20 11 3 5
 12.0101010010011 13(2)26 16 3 5
 13.0101010011011 13(2)26 16 3 5
 14.0101011011011 13(2)36 21 3 11
 15.1001011001100 13(2)4 1 2 2
 16.1001001101100 13(2)4 1 2 2
 17.1001001100110 13(2)4 1 2 2
 18.1001001100100 13(2)6 2 2 3
 19.1010011001100 13(2)4 1 2 2
 20.0010011001011 13(2)2 0 0 1
 21.0010011010011 13(2)2 0 0 1
 total NATs = 261
```

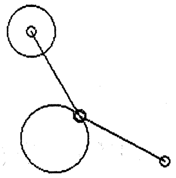
λ ratio = .75 $Z = .625$

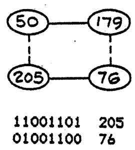
1111000011110011-1111000011110011-rule =3-rule 205 -11001101

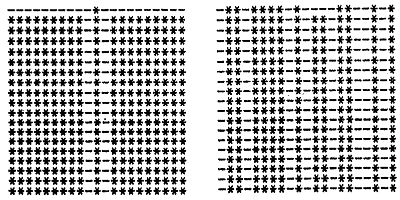
4042518771 Length=1 -14







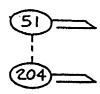




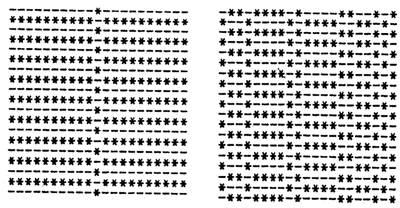
```
3-rule 205 =11001101
ty. at no(p)s g ml mp
L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.111 1(1)2 1 1 2
 2.001 3(1)1 0 0 1
 3.011 3(1)1 0 0 1
 total NATs = 7
L= 4
 1.1111 1(1)2 1 1 2
 2.0101 2(1)3 2 1 3
 3.0011 4(1)1 0 0 1
 4.0111 4(1)1 0 0 1
 total NATs = 11
L= 5
 1.11111 1(1)2 1 1 2
 2.01101 5(1)3 2 1 3
 3.00101 5(1)1 0 0 1
 4.00111 5(1)1 0 0 1
 5.01111 5(1)1 0 0 1
 total NATs = 21
L= 6
 1.111111 1(1)2 1 1 2
 2.011101 6(1)3 2 1 3
 3.011011 3(1)3 2 1 3
 4.010101 2(1)4 3 1 4
 5.001001 3(1)1 0 0 1
 6.001011 6(1)1 0 0 1
 7.001101 6(1)1 0 0 1
 8.001111 6(1)1 0 0 1
 9.011111 6(1)1 0 0 1
 total NATs = 39
L= 7
 1.1111111 1(1)2 1 1 2
 2.0111101 7(1)3 2 1 3
 3.0111011 7(1)3 2 1 3
 4.0110101 7(1)4 3 1 4
 5.0101001 7(1)2 1 1 2
 6.0010011 7(1)1 0 0 1
 7.0010111 7(1)1 0 0 1
 8.0011011 7(1)1 0 0 1
 9.0011101 7(1)1 0 0 1
 10.0011111 7(1)1 0 0 1
 11.0111111 7(1)1 0 0 1
 total NATs = 71
L= 8
 1.11111111 1(1)2 1 1 2
 2.01111101 8(1)3 2 1 3
 3.01111011 8(1)3 2 1 3
 4.01110101 8(1)4 3 1 4
 5.01110111 4(1)3 2 1 3
 6.01101001 8(1)2 1 1 2
 7.01101011 8(1)4 3 1 4
 8.01010101 2(1)7 6 1 7
 9.01010011 8(1)2 1 1 2
 10.01011001 8(1)2 1 1 2
 11.00100101 8(1)1 0 0 1
 12.00100111 8(1)1 0 0 1
 13.00101111 8(1)1 0 0 1
 14.00110011 4(1)1 0 0 1
 15.00110111 8(1)1 0 0 1
 16.00111011 8(1)1 0 0 1
 17.00111101 8(1)1 0 0 1
 18.00111111 8(1)1 0 0 1
 19.01111111 8(1)1 0 0 1
 total NATs = 131
L= 9
 1.111111111 1(1)2 1 1 2
 2.011111101 9(1)3 2 1 3
 3.011111011 9(1)3 2 1 3
 4.011110101 9(1)4 3 1 4
 5.011110111 9(1)3 2 1 3
 6.011101001 9(1)2 1 1 2
 7.011101011 9(1)4 3 1 4
 8.011101101 9(1)4 3 1 4
 9.011010101 9(1)7 6 1 7
 10.011010011 9(1)2 1 1 2
 11.011011001 9(1)2 1 1 2
 12.011011011 3(1)4 3 1 4
 13.010100101 9(1)3 2 1 3
 14.010100111 9(1)2 1 1 2
 15.010110011 9(1)2 1 1 2
 16.010111001 9(1)2 1 1 2
 17.001001001 3(1)1 0 0 1
 18.001001011 9(1)1 0 0 1
 19.001001101 9(1)1 0 0 1
 20.001001111 9(1)1 0 0 1
 21.001010011 9(1)1 0 0 1
 22.001011111 9(1)1 0 0 1
 23.001100111 9(1)1 0 0 1
 24.001101111 9(1)1 0 0 1
 25.001110111 9(1)1 0 0 1
 26.001111011 9(1)1 0 0 1
 27.001111101 9(1)1 0 0 1
 28.001111111 9(1)1 0 0 1
 29.011111111 9(1)1 0 0 1
 total NATs = 241
L= 10
 1.1111111111 1(1)2 1 1 2
 2.0111111101 10(1)3 2 1 3
 3.0111111011 10(1)3 2 1 3
 4.0111110101 10(1)4 3 1 4
 5.0111110111 10(1)3 2 1 3
 6.0111101001 10(1)2 1 1 2
 7.0111101011 10(1)4 3 1 4
 8.0111101101 10(1)4 3 1 4
 9.0111101111 5(1)3 2 1 3
 10.0111010101 10(1)7 6 1 7
 11.0111010011 10(1)2 1 1 2
 12.0111010111 10(1)4 3 1 4
 13.0111011001 10(1)2 1 1 2
 14.0111011011 10(1)4 3 1 4
 15.0110101101 5(1)7 6 1 7
 16.0110101011 10(1)7 6 1 7
 17.0110100101 10(1)3 2 1 3
 18.0110100111 10(1)2 1 1 2
 19.0110101001 10(1)3 2 1 3
 20.0110110011 10(1)2 1 1 2
 21.0110111001 10(1)2 1 1 2
 22.0101010101 2(1)11 10 1 11
 23.0101001001 10(1)2 1 1 2

 45.0011111011 10(1)1 0 0 1
 46.0011111101 10(1)1 0 0 1
 47.0011111111 10(1)1 0 0 1
 48.0111111111 10(1)1 0 0 1
 total NATs = 443
```

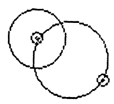
λ ratio = 1 $Z = 1$

0000111100001111-0000111100001111-rule 252645135
=3-rule 51 -00110011
Length=1 -14

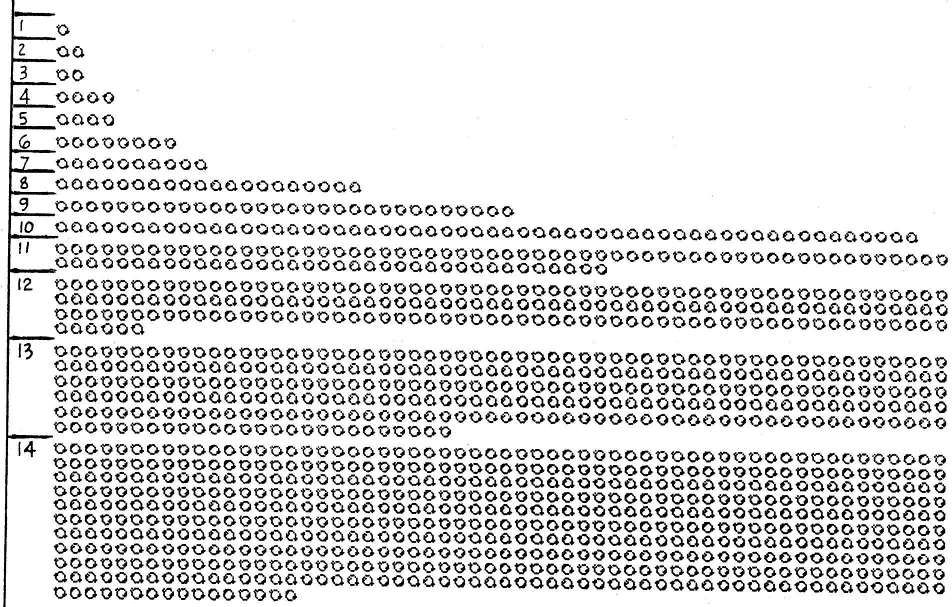


00110011 51





all lengths

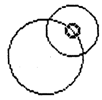


3-rule 51 =00110011

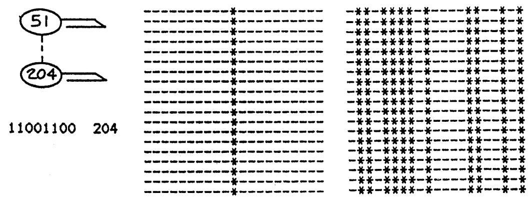
ty. at no(p)s g ml mp

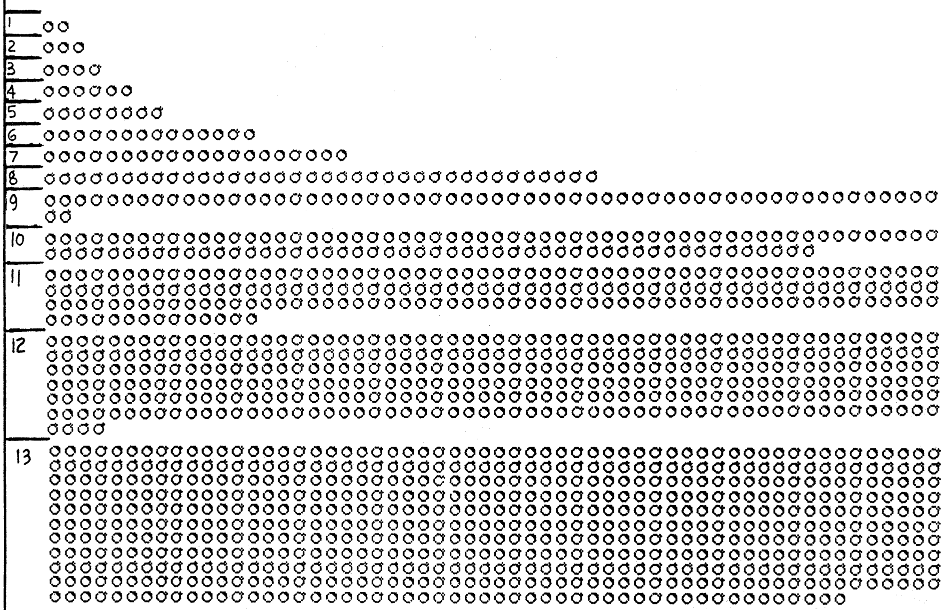
L= 1
1.0 1(2)2 0 0 1
total NATs = 1
L= 2
1.00 1(2)2 0 0 1
2.01 1(2)2 0 0 1
total NATs = 2
L= 3
1.000 1(2)2 0 0 1
2.001 3(2)2 0 0 1
total NATs = 4
L= 4
1.0000 1(2)2 0 0 1
2.0001 4(2)2 0 0 1
3.0011 2(2)2 0 0 1
4.0101 1(2)2 0 0 1
total NATs = 8
L= 5
1.00000 1(2)2 0 0 1
2.00001 5(2)2 0 0 1
3.00011 5(2)2 0 0 1
4.00101 5(2)2 0 0 1
total NATs = 16
L= 6
1.000000 1(2)2 0 0 1
2.000001 6(2)2 0 0 1
3.000011 6(2)2 0 0 1
4.000101 6(2)2 0 0 1
5.000111 3(2)2 0 0 1
6.001001 3(2)2 0 0 1
7.001011 6(2)2 0 0 1
8.010101 1(2)2 0 0 1
total NATs = 32
L= 7
1.0000000 1(2)2 0 0 1
2.0000001 7(2)2 0 0 1
3.0000011 7(2)2 0 0 1
4.0000101 7(2)2 0 0 1
5.0000111 7(2)2 0 0 1
6.0001001 7(2)2 0 0 1
7.0001011 7(2)2 0 0 1
8.0001101 7(2)2 0 0 1
9.0010011 7(2)2 0 0 1
10.0010101 7(2)2 0 0 1
total NATs = 64
L= 8
1.00000000 1(2)2 0 0 1
2.00000001 8(2)2 0 0 1
3.00000011 8(2)2 0 0 1
4.00000101 8(2)2 0 0 1
5.00000111 8(2)2 0 0 1
6.00001001 8(2)2 0 0 1
7.00001011 8(2)2 0 0 1
8.00001101 8(2)2 0 0 1
9.00001111 4(2)2 0 0 1
10.00010001 4(2)2 0 0 1
11.00010011 8(2)2 0 0 1
12.00010101 8(2)2 0 0 1
13.00010111 8(2)2 0 0 1
14.00011001 8(2)2 0 0 1
15.00011011 8(2)2 0 0 1
16.00100101 8(2)2 0 0 1
17.00101011 8(2)2 0 0 1
18.00101101 4(2)2 0 0 1
19.00110011 2(2)2 0 0 1
20.01010101 1(2)2 0 0 1
total NATs = 128
L= 9
1.000000000 1(2)2 0 0 1
2.000000001 9(2)2 0 0 1
3.000000011 9(2)2 0 0 1
4.000000101 9(2)2 0 0 1
5.000000111 9(2)2 0 0 1
6.000001001 9(2)2 0 0 1
7.000001011 9(2)2 0 0 1
8.000001101 9(2)2 0 0 1
9.000001111 9(2)2 0 0 1
10.000010001 9(2)2 0 0 1
11.000010011 9(2)2 0 0 1
12.000010101 9(2)2 0 0 1
13.000010111 9(2)2 0 0 1
14.000011001 9(2)2 0 0 1
15.000011011 9(2)2 0 0 1
16.000011101 9(2)2 0 0 1
17.000100011 9(2)2 0 0 1
18.000100101 9(2)2 0 0 1
19.000100111 9(2)2 0 0 1
20.000101001 9(2)2 0 0 1
21.000101011 9(2)2 0 0 1
22.000101101 9(2)2 0 0 1
23.000110011 9(2)2 0 0 1
24.000110101 9(2)2 0 0 1
25.000110111 9(2)2 0 0 1
26.001001001 3(2)2 0 0 1
27.001001011 9(2)2 0 0 1
28.001001101 9(2)2 0 0 1
29.001010011 9(2)2 0 0 1
30.001010101 9(2)2 0 0 1
total NATs = 256

λ ratio = 1 $Z = 1$ 1111000011110000-1111000011110000-rule 4042322160
=3-rule 204 -11001100   Length=1 -13



all lengths

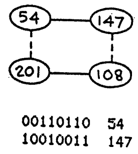


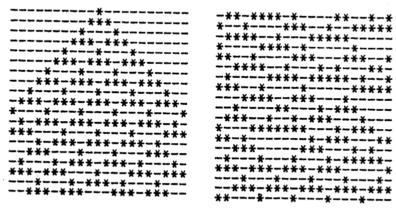


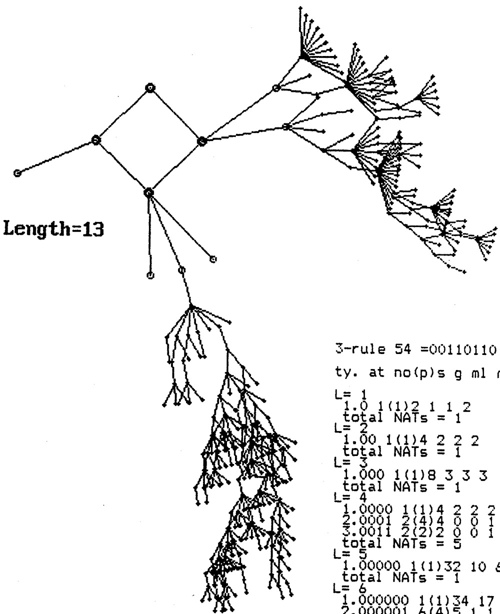

```
3-rule 204 =11001100
ty. at no(p)s g ml mp
L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 2(1)1 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 4
L= 3
 1.000 1(1)1 0 0 1
 2.001 3(1)1 0 0 1
 3.011 3(1)1 0 0 1
 4.111 1(1)1 0 0 1
 total NATs = 8
L= 4
 1.0000 1(1)1 0 0 1
 2.0001 4(1)1 0 0 1
 3.0011 4(1)1 0 0 1
 4.0101 2(1)1 0 0 1
 5.0111 4(1)1 0 0 1
 6.1111 1(1)1 0 0 1
 total NATs = 16
L= 5
 1.00000 1(1)1 0 0 1
 2.00001 5(1)1 0 0 1
 3.00011 5(1)1 0 0 1
 4.00101 5(1)1 0 0 1
 5.00111 5(1)1 0 0 1
 6.01011 5(1)1 0 0 1
 7.01111 5(1)1 0 0 1
 8.11111 1(1)1 0 0 1
 total NATs = 32
L= 6
 1.000000 1(1)1 0 0 1
 2.000001 6(1)1 0 0 1
 3.000011 6(1)1 0 0 1
 4.000101 6(1)1 0 0 1
 5.000111 6(1)1 0 0 1
 6.001001 3(1)1 0 0 1
 7.001011 6(1)1 0 0 1
 8.001101 6(1)1 0 0 1
 9.001111 6(1)1 0 0 1
 10.010101 2(1)1 0 0 1
 11.010111 6(1)1 0 0 1
 12.011011 3(1)1 0 0 1
 13.011111 6(1)1 0 0 1
 14.111111 1(1)1 0 0 1
 total NATs = 64
L= 7
 1.0000000 1(1)1 0 0 1
 2.0000001 7(1)1 0 0 1
 3.0000011 7(1)1 0 0 1
 4.0000101 7(1)1 0 0 1
 5.0000111 7(1)1 0 0 1
 6.0001001 7(1)1 0 0 1
 7.0001011 7(1)1 0 0 1
 8.0001101 7(1)1 0 0 1
 9.0001111 7(1)1 0 0 1
 10.0010011 7(1)1 0 0 1
 11.0010101 7(1)1 0 0 1
 12.0010111 7(1)1 0 0 1
 13.0011011 7(1)1 0 0 1
 14.0011101 7(1)1 0 0 1
 15.0011111 7(1)1 0 0 1
 16.0101011 7(1)1 0 0 1
 17.0101111 7(1)1 0 0 1
 18.0110111 7(1)1 0 0 1
 19.0111111 7(1)1 0 0 1
 20.1111111 1(1)1 0 0 1
 total NATs = 128
L= 8
 1.00000000 1(1)1 0 0 1
 2.00000001 8(1)1 0 0 1
 3.00000011 8(1)1 0 0 1
 4.00000101 8(1)1 0 0 1
 5.00000111 8(1)1 0 0 1
 6.00001001 8(1)1 0 0 1
 7.00001011 8(1)1 0 0 1
 8.00001101 8(1)1 0 0 1
 9.00001111 8(1)1 0 0 1
 10.00010001 4(1)1 0 0 1
 11.00010011 8(1)1 0 0 1
 12.00010101 8(1)1 0 0 1
 13.00010111 8(1)1 0 0 1
 14.00011001 8(1)1 0 0 1
 15.00011011 8(1)1 0 0 1
 16.00011101 8(1)1 0 0 1
 17.00011111 8(1)1 0 0 1
 18.00100101 8(1)1 0 0 1
 19.00100111 8(1)1 0 0 1
 20.00101011 8(1)1 0 0 1
 21.00101101 8(1)1 0 0 1
 22.00101111 8(1)1 0 0 1
 23.00110011 4(1)1 0 0 1
 24.00110101 8(1)1 0 0 1
 25.00110111 8(1)1 0 0 1
 26.00111011 8(1)1 0 0 1
 27.00111101 8(1)1 0 0 1
 28.00111111 8(1)1 0 0 1
 29.01010101 2(1)1 0 0 1
 30.01010111 8(1)1 0 0 1
 31.01011011 8(1)1 0 0 1
 32.01011111 8(1)1 0 0 1
 33.01101111 8(1)1 0 0 1
 34.01110111 4(1)1 0 0 1
 35.01111111 8(1)1 0 0 1
 36.11111111 1(1)1 0 0 1
 total NATs = 256
```

λ ratio = 1 $Z = .75$

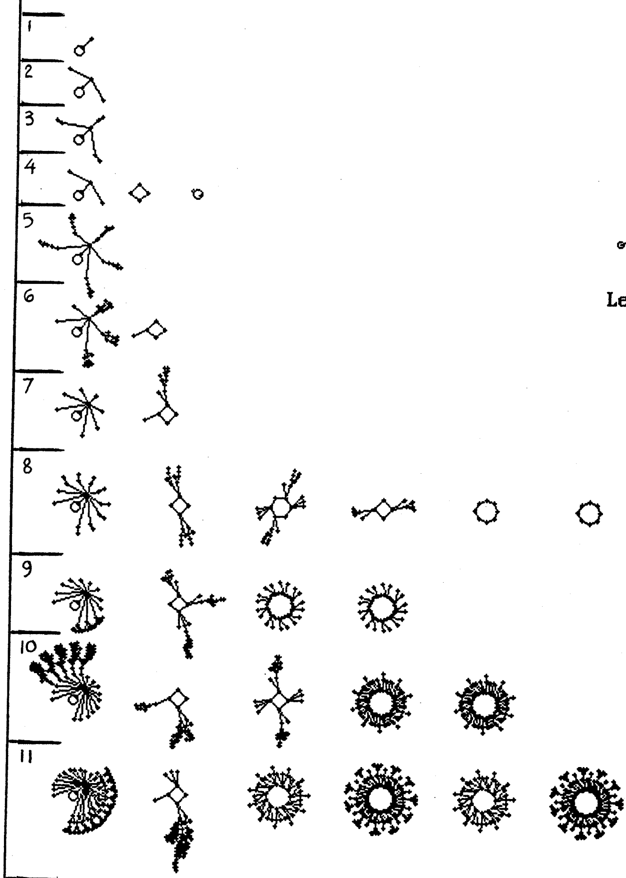
00001111100111100-00001111100111100-rule 255594300
=3-rule 54 -00110110
Length=1 -15

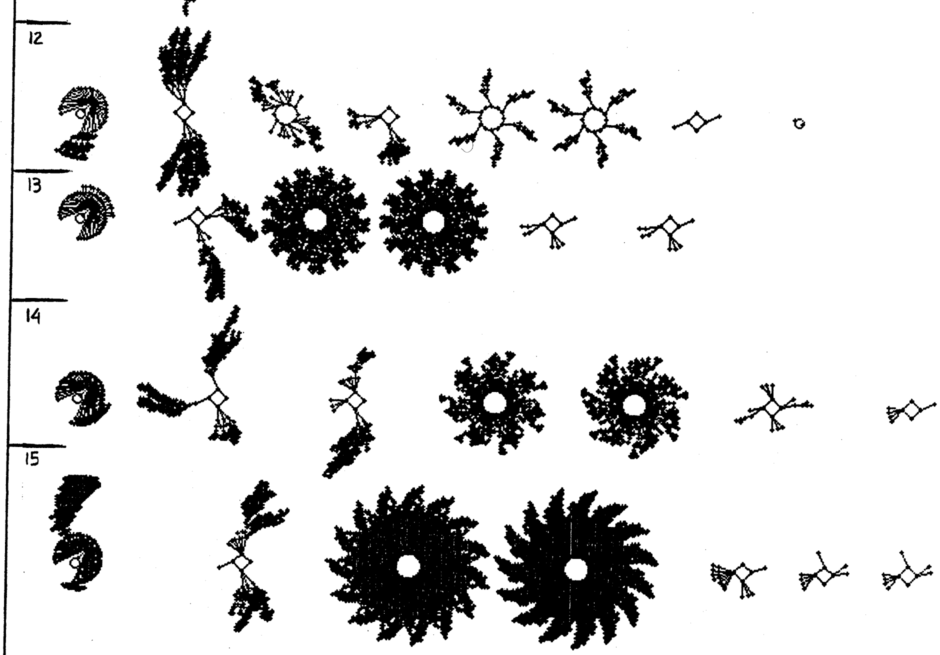






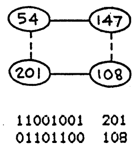

```
3-rule 54 =00110110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.000 1(1)8 3 3 3
 total NATs = 1
L= 4
 1.0000 1(1)4 2 2 2
 2.0001 2(4)4 0 0 1
 3.0011 2(2)2 0 0 1
 total NATs = 5
L= 5
 1.00000 1(1)32 10 6 5
 total NATs = 1
L= 6
 1.000000 1(1)34 17 5 5
 2.000001 6(4)5 1 1 2
 total NATs = 7
L= 7
 1.0000000 1(1)9 7 2 7
 2.0000001 7(4)17 6 6 3
 total NATs = 8
L= 8
 1.00000000 1(1)20 10 3 10
 2.01000100 2(4)24 8 4 3
 3.00000011 4(6)28 12 4 4
 4.00000101 4(4)14 8 2 3
 5.00010111 1(8)8 0 0 1
 6.00011101 1(8)8 0 0 1
 7.00110011 2(2)2 0 0 1
 total NATs = 15
L= 9
 1.000000000 1(1)26 21 3 12
 2.011111100 9(4)44 20 9 5
 3.100011100 1(27)45 18 1 3
 4.100010011 1(27)45 18 1 3
 total NATs = 12
L= 10
 1.0000000000 1(1)164 72 10 17
 2.0100000100 10(4)46 21 7 5
 3.1000000100 5(4)44 26 6 7
 4.1000011100 1(30)90 50 2 4
 5.1000010011 1(30)90 50 2 4
 total NATs = 18
L= 11
 1.00000000000 1(1)101 66 5 22
 2.01000000100 11(4)125 69 16 9
 3.10000111100 1(11)55 33 2 4
 4.00000010111 1(99)231 99 3 4
 5.10000100111 1(11)55 33 2 4
 6.00000011101 1(99)231 99 3 4
 total NATs = 16
L= 12
 1.000000000000 1(1)166 113 8 29
 2.000100010001 2(4)1036 552 21 7
 3.110000001111 6(10)124 82 7 12
 4.010000010000 6(4)105 63 7 9
 5.110001000101 2(12)102 48 7 5
 6.010001110100 2(12)102 48 7 5
 7.000001000101 12(4)6 2 1 2
 8.001100110011 2(2)2 0 0 1
 total NATs = 33
L= 13
 1.0000000000000 1(1)54 39 3 39
 2.0111000011100 13(4)406 250 20 16
 3.0010001110100 1(169)1274 702 8 8
 4.1110001000010 1(169)1274 702 8 8
 5.1000001001000 13(4)12 6 2 4
 6.1000001000100 13(4)12 6 2 4
 total NATs = 42
L= 14
 1.00000000000000 1(1)95 79 3 51
 2.00010000010001 14(4)695 417 25 16
 3.00100000010000 7(4)415 286 20 21
 4.10000000011100 2(112)805 497 8 11
 5.10000000010011 2(112)805 497 8 11
 6.10000001001000 14(4)22 12 4 4
 7.00000100000101 14(4)9 5 1 5
 total NATs = 54
L= 15
 1.000000000000000 1(1)1583 1093 21 68
 2.000100000010001 15(4)1014 664 20 28
 3.001000111101110 1(330)7680 4845 19 15
 4.111000100011110 1(330)7680 4845 19 15
 5.100000100100000 15(4)19 11 2 8
 6.000000100000101 15(4)11 7 1 5
 7.000000101000001 15(4)11 7 1 5
 total NATs = 63
```

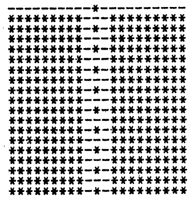


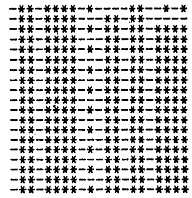


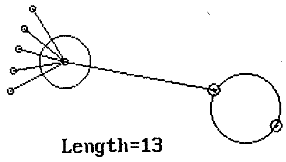
λ ratio = 1 $Z = .75$

1111000011000011-1111000011000011-rule 4039372995
=3-rule 201 -11001001
Length=1 -13

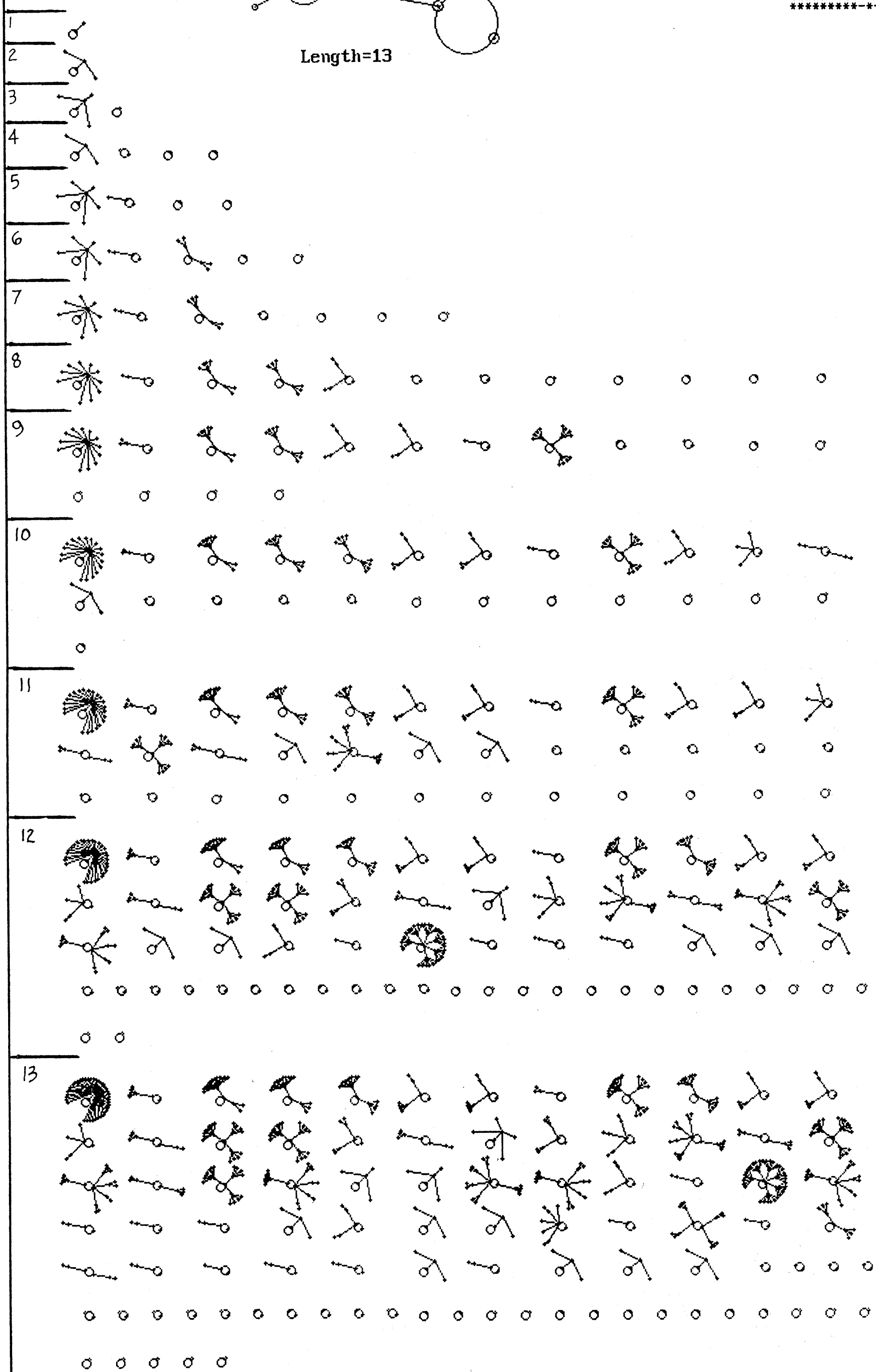








Length=13

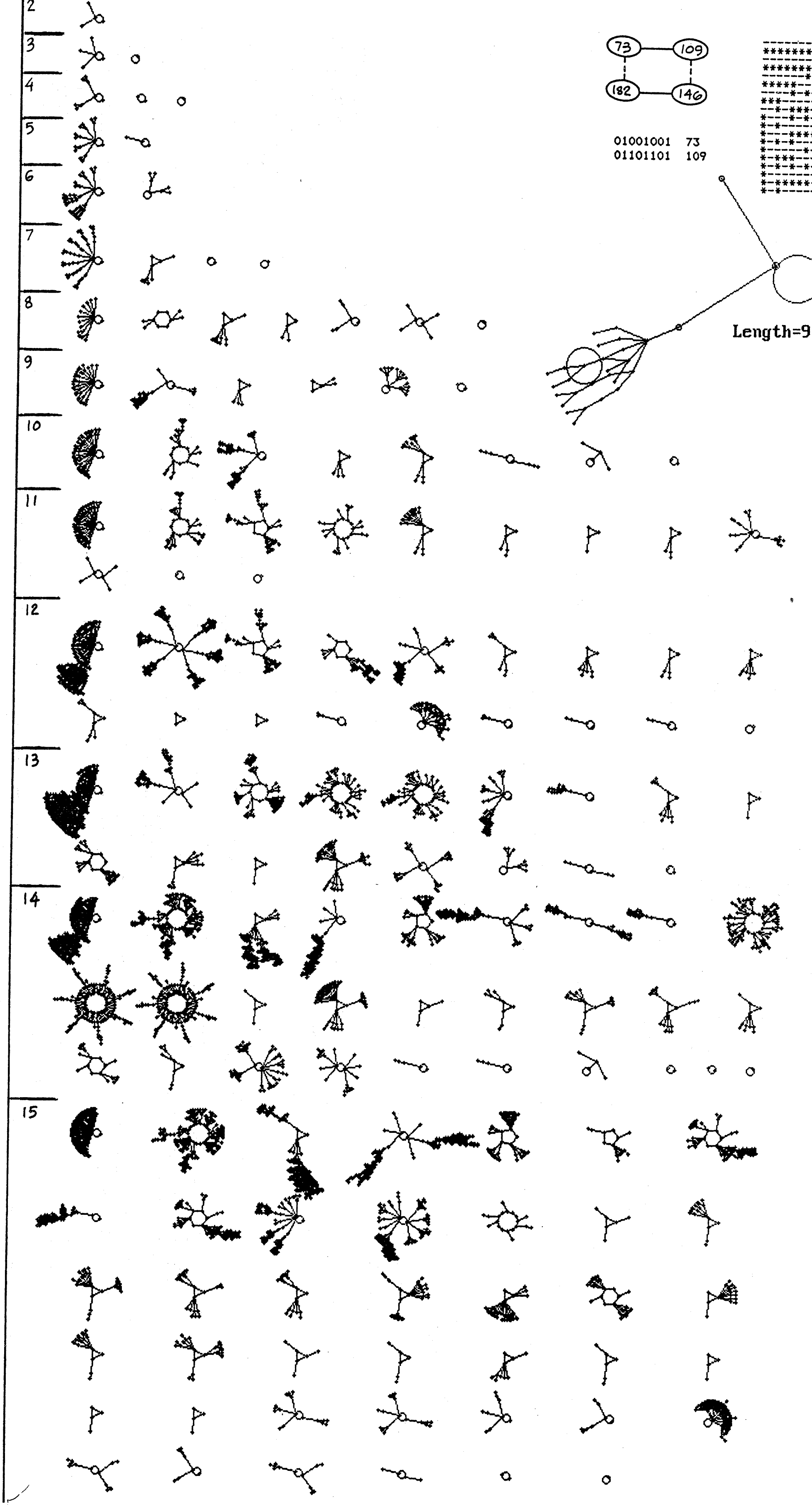



```
3-rule 201 =11001001

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.111 1(1)5 3 2 3
 2.011 3(1)1 0 0 1
 total NATs = 4
L= 4
 1.1111 1(1)4 2 2 2
 2.0001 2(2)2 0 0 1
 3.0011 4(1)1 0 0 1
 4.0111 4(1)1 0 0 1
 total NATs = 11
L= 5
 1.11111 1(1)7 5 2 5
 2.01100 5(2)3 1 1 2
 3.00111 5(1)1 0 0 1
 4.01111 5(1)1 0 0 1
 total NATs = 16
L= 6
 1.111111 1(1)7 5 2 5
 2.011100 6(2)4 1 2 2
 3.011011 3(1)7 4 2 3
 4.001111 6(1)1 0 0 1
 5.011111 6(1)1 0 0 1
 total NATs = 22
L= 7
 1.1111111 1(1)9 7 2 7
 2.0111100 7(2)4 1 2 2
 3.0111011 7(1)8 5 2 3
 4.0001011 7(2)2 0 0 1
 5.0011011 7(1)1 0 0 1
 6.0011111 7(1)1 0 0 1
 7.0111111 7(1)1 0 0 1
 total NATs = 43
L= 8
 1.11111111 1(1)12 10 2 10
 2.01111100 8(2)4 1 2 2
 3.01111011 8(1)9 6 2 4
 4.01110111 4(1)9 6 2 3
 5.01100011 8(2)6 2 2 3
 6.00010001 2(2)2 0 0 1
 7.00010111 8(2)2 0 0 1
 8.00110011 4(1)1 0 0 1
 9.00110111 8(1)1 0 0 1
 10.00111011 8(1)1 0 0 1
 11.00111111 8(1)1 0 0 1
 12.01111111 8(1)1 0 0 1
 total NATs = 75
L= 9
 1.111111111 1(1)14 12 2 12
 2.011111100 9(2)5 2 2 2
 3.011111011 9(1)10 7 2 5
 4.011110111 9(1)10 7 2 4
 5.011100011 9(2)6 2 2 3
 6.011101100 9(2)6 2 2 3
 7.011000100 9(2)3 1 1 2
 8.011011011 3(1)16 12 2 4
 9.000101111 9(2)2 0 0 1
 10.000110011 9(2)2 0 0 1
 11.001100111 9(1)1 0 0 1
 12.001101111 9(1)1 0 0 1
 13.001110111 9(1)1 0 0 1
 14.001111011 9(1)1 0 0 1
 15.001111111 9(1)1 0 0 1
 16.011111111 9(1)1 0 0 1
 total NATs = 130
L= 10
 1.1111111111 1(1)19 17 2 17
 2.0111111100 10(2)5 2 2 2
 3.0111111011 10(1)12 9 2 7
 4.0111110111 10(1)11 8 2 5
 5.0111101111 5(1)11 8 2 4
 6.0111100011 10(2)7 3 2 3
 7.0111101100 10(2)7 3 2 3
 8.0111000100 10(2)4 1 2 2
 9.0111011011 10(1)16 12 2 4
 10.0111000111 10(2)6 2 2 3
 11.0110001100 5(2)5 3 1 4
 12.0110001011 10(2)6 2 2 2
 13.0110110011 10(1)4 2 2 2
 14.0001011111 10(2)2 0 0 1
 15.0001100111 10(2)2 0 0 1
 16.0001101011 5(2)2 0 0 1
 17.0001110011 10(2)2 0 0 1
 18.0011001111 10(1)1 0 0 1
 19.0011011111 10(1)1 0 0 1
 20.0011100111 5(1)1 0 0 1
 21.0011101111 10(1)1 0 0 1
 22.0011110111 10(1)1 0 0 1
 23.0011111011 10(1)1 0 0 1
 24.0011111111 10(1)1 0 0 1
 25.0111111111 10(1)1 0 0 1
 total NATs = 221
```

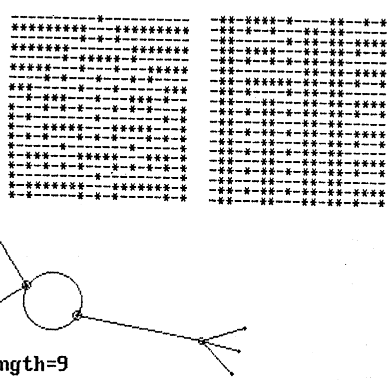
λ ratio = .75   Z = .75

00110000011000011-00110000011000011-rule   818098371

=3-rule 73 -01001001   Length=1 -15



01001001 73
01101101 109



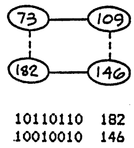
3-rule 73 =01001001

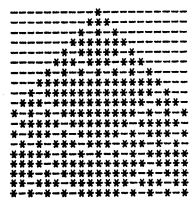
ty. at no(p)s g ml mp

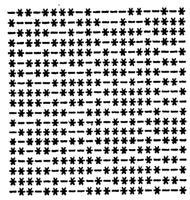
```
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)4 2 1 3
 total NATs = 1
L= 3
 1.000 1(2)5 3 1 4
 2.011 3(1)1 0 0 1
 total NATs = 4
L= 4
 1.0000 1(2)8 4 2 3
 2.0001 2(2)2 0 0 1
 3.0011 4(1)1 0 0 1
 total NATs = 7
L= 5
 1.00000 1(2)17 10 2 6
 2.01100 5(2)3 1 1 2
 total NATs = 6
L= 6
 1.000000 1(2)43 18 5 6
 2.011011 3(1)7 4 2 3
 total NATs = 4
L= 7
 1.0000000 1(2)44 14 5 8
 2.0000011 7(3)9 5 2 3
 3.0001011 7(2)2 0 0 1
 4.0011011 7(1)1 0 0 1
 total NATs = 22
L= 8
 1.00000000 1(2)16 12 2 11
 2.00000001 4(6)10 4 1 3
 3.00000011 8(3)12 6 2 4
 4.00000101 8(3)5 2 1 3
 5.01100011 8(2)6 2 2 3
 6.00010001 2(2)6 4 1 3
 7.00110011 4(1)1 0 0 1
 total NATs = 35
L= 9
 1.000000000 1(2)32 21 2 13
 2.010110100 9(2)35 15 7 5
 3.000000101 9(3)6 3 1 4
 4.000000111 9(3)5 2 1 3
 5.011011011 3(1)16 12 2 4
 6.000110011 9(2)2 0 0 1
 total NATs = 40
L= 10
 1.0000000000 1(2)59 40 2 18
 2.0000000001 10(8)28 14 5 5
 3.0001100011 5(2)67 28 8 7
 4.0000001001 10(3)6 3 1 4
 5.0101011011 10(3)16 9 2 4
 6.0110001011 10(2)8 2 3 2
 7.0110110011 10(1)4 2 2 2
 8.0001101011 5(2)2 0 0 1
 total NATs = 61
L= 11
 1.00000000000 1(2)79 55 2 23
 2.00000000001 11(8)34 18 5 5
 3.00000000011 11(5)44 25 6 9
 4.00000001011 11(12)30 14 2 3
 5.01001011011 11(3)27 16 2 7
 6.01010110000 11(3)6 2 2 3
 7.00000110011 11(3)4 1 1 2
 8.00000110101 11(3)6 2 2 3
 9.01100011011 11(2)19 9 3 5
 10.01100010001 11(2)6 4 1 3
 11.00010110011 11(2)2 0 0 1
 12.00110011011 11(1)1 0 0 1
 total NATs = 122
L= 12
 1.000000000000 1(2)527 322 12 30
 2.010001000100 2(2)218 114 8 5
 3.000000000011 12(5)60 37 6 12
 4.000110000011 12(6)51 22 8 5
 5.010001011011 12(2)75 41 7 8
 6.010010110000 12(3)9 4 2 3
 7.000000101011 12(3)9 4 2 5
 8.000000110011 12(3)7 2 2 3
 9.000000110101 12(3)9 4 2 5
 10.010010000011 12(3)9 4 2 3
 11.000001010111 4(3)3 0 0 1
 12.000001110101 4(3)3 0 0 1
 13.011000101100 12(2)5 2 2 2
 14.011011011011 3(1)47 36 3 7
 15.011000110100 12(2)5 2 2 2
 16.011000110011 12(2)4 1 2 2
 17.011011001100 12(2)4 1 2 2
 18.001100110011 4(1)1 0 0 1
 total NATs = 162
L= 13
 1.0000000000000 1(2)860 533 12 40
 2.0100011000100 13(2)68 37 8 7
 3.0000000000011 13(8)93 60 6 16
 4.0000000001011 13(18)75 39 7 6
 5.0000000001101 13(18)75 39 7 6
 6.0001100011011 13(2)81 43 8 11
 7.0100010110011 13(2)19 5 7 4
 8.0000001001011 13(3)13 7 2 5
 9.0000001010111 13(3)4 1 1 2
 10.0000001100011 13(6)29 14 2 8
 11.0111101100000 13(3)13 7 2 5
 12.0000001110101 13(3)4 1 1 2
 13.0101011011011 13(3)47 32 3 7
 14.0110100011011 13(2)26 14 3 3
 15.0110110110011 13(1)9 6 2 3
 16.0110001101011 13(2)6 2 2 2
 17.0001100110011 13(2)2 0 0 1
 total NATs = 209
```

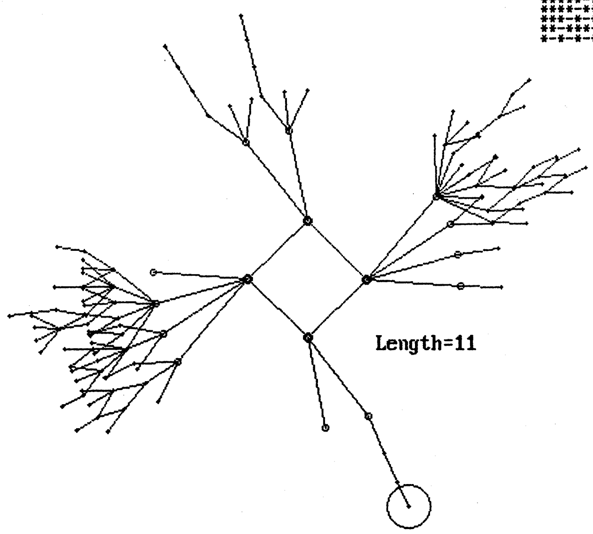
λ ratio = .75 $Z = .75$ **1100111100111100-1100111100111100-rule**   **3476868924**

=3-rule 182 -10110110   **Length=1 -15**

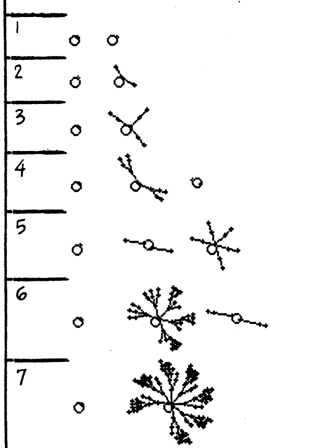


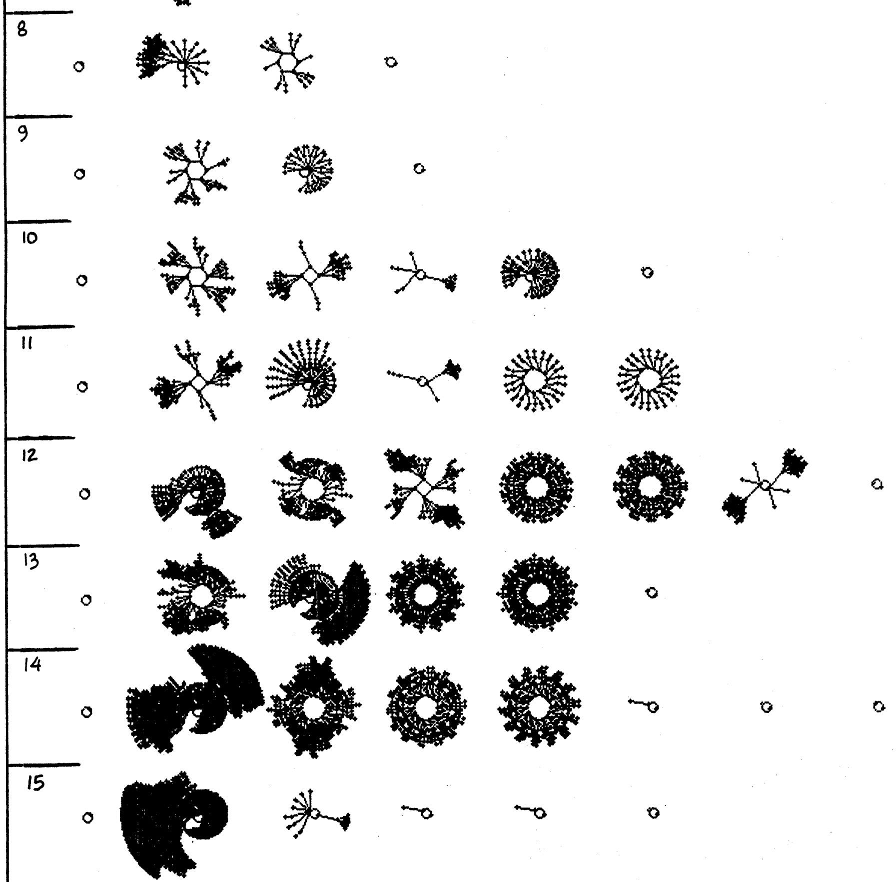






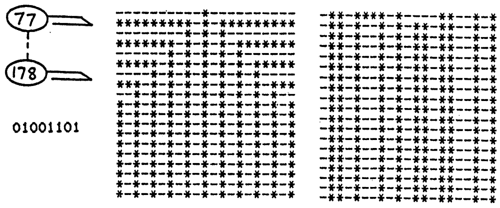
Length=11

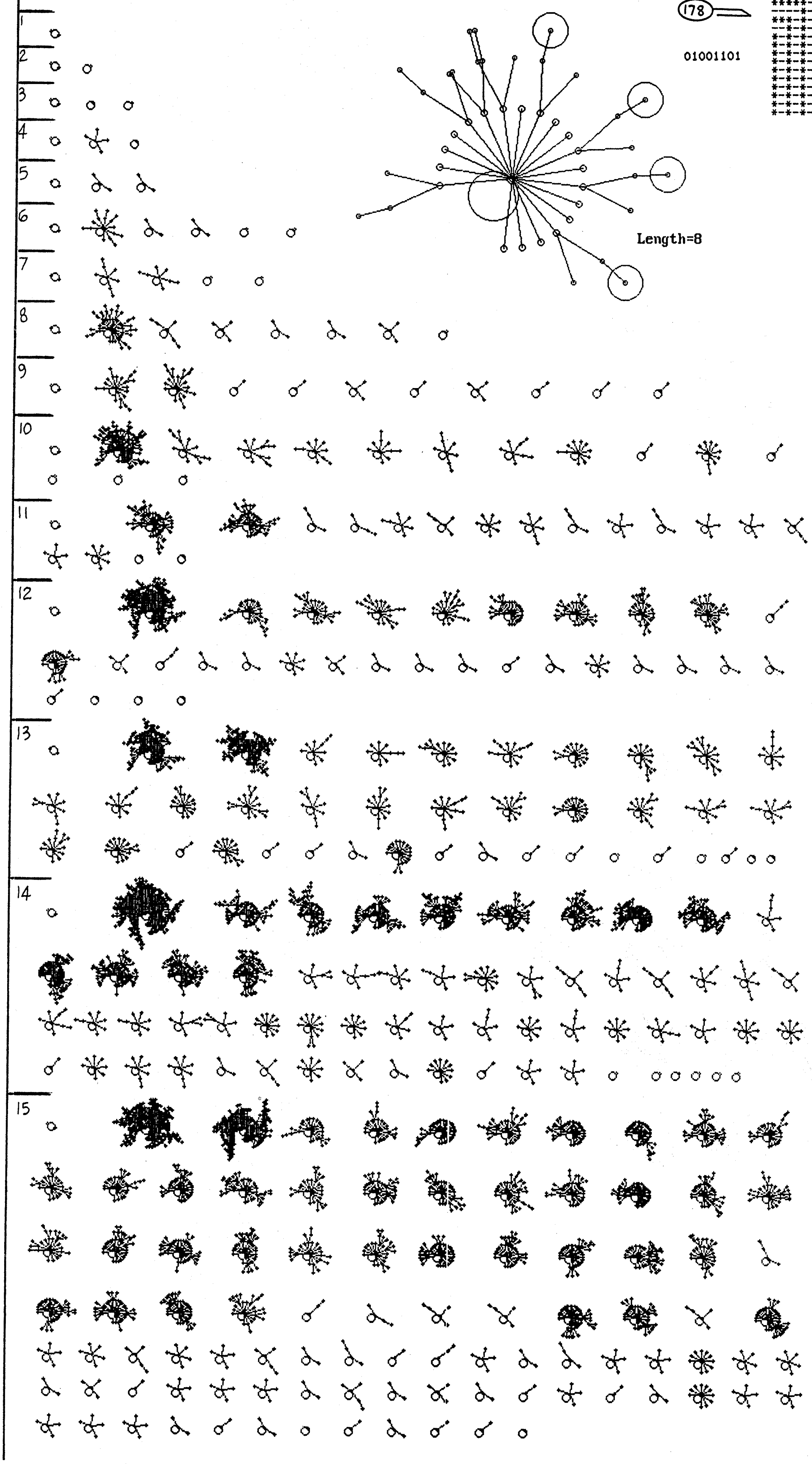





```
3-rule 182 =10110110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.11 1(1)3 2 1 3
 total NATs = 2
L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)7 3 2 4
 total NATs = 2
L= 4
 1.0000 1(1)1 0 0 1
 2.1111 1(1)11 4 3 3
 3.0011 2(2)2 0 0 1
 total NATs = 4
L= 5
 1.00000 1(1)1 0 0 1
 2.10011 5(2)4 2 1 2
 3.11111 1(1)11 5 2 6
 total NATs = 7
L= 6
 1.000000 1(1)1 0 0 1
 2.111111 1(1)45 24 4 6
 3.010111 3(2)6 2 2 2
 total NATs = 5
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111111 1(1)127 49 6 8
 total NATs = 2
L= 8
 1.00000000 1(1)1 0 0 1
 2.11111111 1(1)139 68 7 11
 3.11001111 4(6)28 12 2 4
 4.00110011 2(2)2 0 0 1
 total NATs = 8
L= 9
 1.000000000 1(1)1 0 0 1
 2.111001111 9(6)50 23 4 5
 3.111111111 1(1)43 30 2 13
 4.001100111 9(2)2 0 0 1
 total NATs = 20
L= 10
 1.0000000000 1(1)1 0 0 1
 2.1011111011 5(6)82 40 5 5
 3.1010010111 5(4)84 36 5 7
 4.1100111001 5(2)20 11 3 4
 5.1111111111 1(1)73 45 3 18
 6.0011010111 10(2)2 0 0 1
 total NATs = 27
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11010010111 11(4)131 68 6 9
 3.11111111111 1(1)133 55 6 23
 4.11100111010 11(2)33 19 4 7
 5.11001011011 1(11)55 22 2 3
 6.01001111011 1(11)55 22 2 3
 total NATs = 26
L= 12
 1.000000000000 1(1)1 0 0 1
 2.111111111111 1(1)329 160 8 30
 3.011100111011 6(14)220 120 5 12
 4.011111110111 6(4)238 122 8 8
 5.101001111101 1(36)264 132 3 4
 6.111001011011 1(36)264 132 3 4
 7.101110101110 3(2)162 90 6 8
 8.001100110011 2(2)2 0 0 1
 total NATs = 21
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.1011100111011 13(14)444 252 7 16
 3.1111111111111 1(1)1535 780 10 40
 4.1010011111011 1(39)429 234 4 5
 5.1110010111011 1(39)429 234 4 5
 6.0011001100111 13(2)2 0 0 1
 total NATs = 30
L= 14
 1.00000000000000 1(1)1 0 0 1
 2.11111111111111 1(1)10783 6167 17 52
 3.01110101110111 7(14)708 416 7 15
 4.01010111110111 1(14)280 154 4 5
 5.11010101110111 1(14)280 154 4 5
 6.10011100110011 14(2)3 1 1 2
 7.00110011010111 14(2)2 0 0 1
 8.00110101100111 7(2)2 0 0 1
 total NATs = 46
L= 15
 1.000000000000000 1(1)1 0 0 1
 2.111111111111111 1(1)32522 19313 20 69
 3.110011100111001 5(2)25 16 3 8
 4.100111001101011 15(2)3 1 1 2
 5.100111010110011 15(2)3 1 1 2
 6.001100111010111 15(2)2 0 0 1
 total NATs = 52
```

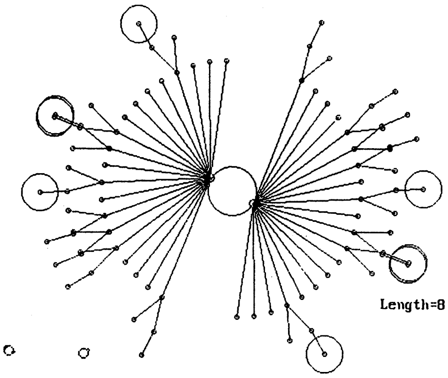
λ ratio = 1 $Z = .5$ **0011000011110011-0011000011110011-rule 821244147**
=3-rule 77 -01001101 **Length=1 -15**



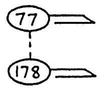



```
3-rule 77 =01001101

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(2)2 0 0 1
 2.001 3(1)1 0 0 1
 3.011 3(1)1 0 0 1
 total NATs = 7
L= 4
 1.0000 1(2)2 0 0 1
 2.0101 2(1)5 4 1 5
 3.0011 4(1)1 0 0 1
 total NATs = 7
L= 5
 1.00000 1(2)2 0 0 1
 2.01101 5(1)3 2 1 3
 3.00101 5(1)3 2 1 3
 total NATs = 11
L= 6
 1.000000 1(2)2 0 0 1
 2.010101 2(1)16 9 2 10
 3.011011 3(1)3 2 1 3
 4.001001 3(1)3 2 1 3
 5.001011 6(1)1 0 0 1
 6.001101 6(1)1 0 0 1
 total NATs = 21
L= 7
 1.0000000 1(2)2 0 0 1
 2.0100101 7(1)8 5 2 6
 3.0101011 7(1)8 5 2 6
 4.0010011 7(1)1 0 0 1
 5.0011011 7(1)1 0 0 1
 total NATs = 29
L= 8
 1.00000000 1(2)2 0 0 1
 2.01010101 2(1)45 28 3 21
 3.01001011 8(1)6 3 2 4
 4.01101011 8(1)4 3 1 4
 5.01010011 8(1)3 2 1 3
 6.01011001 8(1)3 2 1 3
 7.00100101 8(1)4 3 1 4
 8.00110011 4(1)1 0 0 1
 total NATs = 47
L= 9
 1.000000000 1(2)2 0 0 1
 2.010110101 9(1)21 14 3 13
 3.010010101 9(1)21 14 3 13
 4.011010011 9(1)2 1 1 2
 5.011011001 9(1)2 1 1 2
 6.011011011 3(1)4 3 1 4
 7.010110011 9(1)2 1 1 2
 8.001001001 3(1)4 3 1 4
 9.001001011 9(1)2 1 1 2
 10.001001101 9(1)2 1 1 2
 11.001010011 9(1)2 1 1 2
 total NATs = 79
L= 10
 1.0000000000 1(2)2 0 0 1
 2.0101010101 2(1)121 85 4 46
 3.0101101011 5(1)13 8 3 7
 4.0100101001 5(1)13 8 3 7
 5.0100101011 10(1)10 7 2 8
 6.0100101101 10(1)10 7 2 8
 7.0101010011 10(1)8 5 2 6
 8.0101011001 10(1)8 5 2 6
 9.0101011011 10(1)11 9 2 10
 10.0110110011 10(1)2 1 1 2
 11.0101001001 10(1)11 9 2 10
 12.0010010011 10(1)2 1 1 2
 13.0010011011 10(1)1 0 0 1
 14.0010110011 10(1)1 0 0 1
 15.0011001101 10(1)1 0 0 1
 total NATs = 123
L= 11
 1.00000000000 1(2)2 0 0 1
 2.01010010101 11(1)55 41 4 28
 3.01010101011 11(1)55 41 4 28
 4.01001010011 11(1)4 2 2 3
 5.01001011001 11(1)4 2 2 3
 6.01001011011 11(1)7 5 2 6
 7.01010110011 11(1)5 3 2 4
 8.01101011011 11(1)7 6 1 7
 9.01101001001 11(1)7 5 2 6
 10.01101001101 11(1)4 2 2 3
 11.01101010011 11(1)5 4 1 5
 12.01101011001 11(1)4 2 2 3
 13.01101100101 11(1)5 4 1 5
 14.01010010011 11(1)5 4 1 5
 15.01010011001 11(1)5 3 2 4
 16.01011001001 11(1)5 4 1 5
 17.00100100101 11(1)7 6 1 7
 18.00100110011 11(1)1 0 0 1
 19.00110011011 11(1)1 0 0 1
 total NATs = 199
L= 12
 1.000000000000 1(2)2 0 0 1
 2.010101010101 2(1)320 241 5 98
 3.010100101011 12(1)29 21 4 18
 4.010110101011 12(1)26 20 3 16
 5.010101010011 12(1)21 14 3 13
 6.010101011001 12(1)21 14 3 13
 7.010101011011 12(1)29 24 3 21
 8.010010101011 12(1)27 22 2 17
 9.010010100101 12(1)26 20 3 16
 10.010010110101 12(1)27 22 2 17
 11.010010110011 12(1)3 1 2 2
 12.010101001001 12(1)29 24 3 21
 13.011010010011 12(1)4 3 1 4
 14.011010011001 12(1)3 1 2 2
 15.011010011011 12(1)3 2 1 3
 16.011010110011 12(1)3 2 1 3
 17.011011011011 3(1)7 6 1 7
 18.011011001001 12(1)4 3 1 4
 19.011011001011 12(1)3 2 1 3
 20.011011001101 12(1)3 2 1 3
 21.010100110011 12(1)3 2 1 3
 22.010110010011 12(1)2 1 1 2
 23.010110011001 12(1)3 2 1 3
 24.001001001001 3(1)7 6 1 7
 25.001001001011 12(1)3 2 1 3
 26.001001001101 12(1)3 2 1 3
 27.001001010011 12(1)3 2 1 3
 28.001001100101 12(1)3 2 1 3
 29.001010011011 12(1)2 1 1 2
 30.001011001011 6(1)1 0 0 1
 31.001100110011 4(1)1 0 0 1
 32.001101001101 6(1)1 0 0 1
 total NATs = 325
```

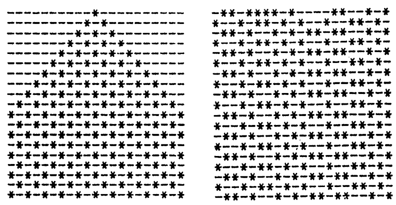
λ ratio = 1 $Z = .5$ 1100111100001100-1100111100001100-rule 3473723148
=3-rule 178 -10110010 Length=1 -15

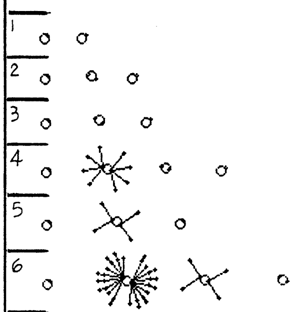


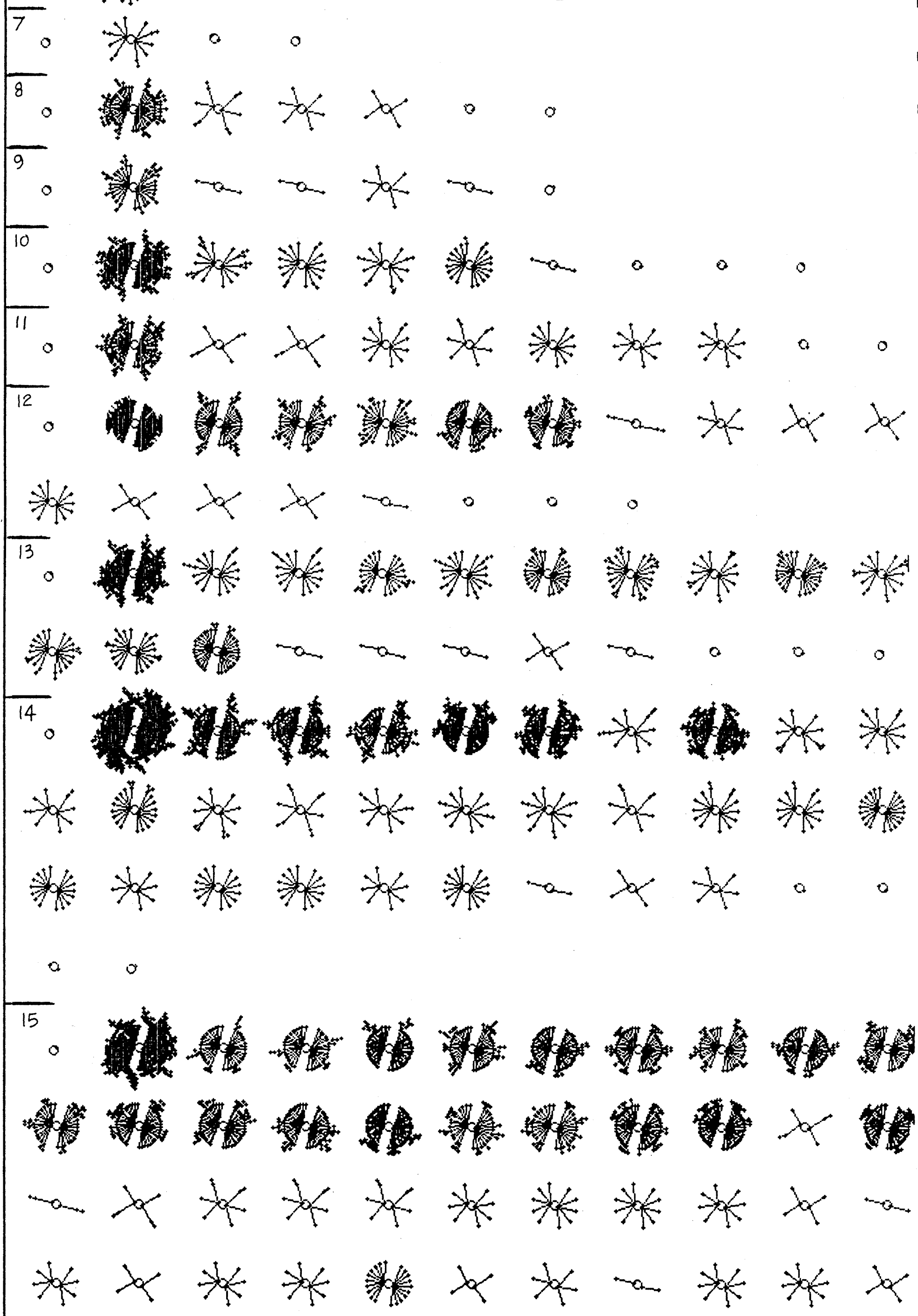
Length=8



10110010 178







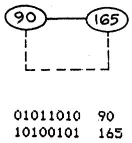

```
3-rule 178 =10110010

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 1(2)2 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)1 0 0 1
 2.001 3(2)2 0 0 1
 3.111 1(1)1 0 0 1
 total NATs = 5
L= 4
 1.0000 1(1)1 0 0 1
 2.1010 1(2)10 8 1 5
 3.0011 2(2)2 0 0 1
 4.1111 1(1)1 0 0 1
 total NATs = 5
L= 5
 1.00000 1(1)1 0 0 1
 2.10010 5(2)6 4 1 3
 3.11111 1(1)1 0 0 1
 total NATs = 7
L= 6
 1.000000 1(1)1 0 0 1
 2.010101 1(2)32 18 2 10
 3.100100 3(2)6 4 1 3
 4.001011 6(2)2 0 0 1
 5.111111 1(1)1 0 0 1
 total NATs = 12
L= 7
 1.0000000 1(1)1 0 0 1
 2.0100101 7(2)16 10 2 6
 3.0010011 7(2)2 0 0 1
 4.1111111 1(1)1 0 0 1
 total NATs = 16
L= 8
 1.00000000 1(1)1 0 0 1
 2.10101010 1(2)90 56 3 21
 3.01001011 4(2)12 6 2 4
 4.10010100 8(2)8 6 1 4
 5.10101100 8(2)6 4 1 3
 6.00110011 2(2)2 0 0 1
 7.11111111 1(1)1 0 0 1
 total NATs = 25
L= 9
 1.000000000 1(1)1 0 0 1
 2.101001010 9(2)42 28 3 13
 3.100101100 9(2)4 2 1 2
 4.100100110 9(2)4 2 1 2
 5.100100100 3(2)8 6 1 4
 6.101001100 9(2)4 2 1 2
 7.111111111 1(1)1 0 0 1
 total NATs = 41
L= 10
 1.0000000000 1(1)1 0 0 1
 2.0101010101 1(2)242 170 4 46
 3.1010010100 5(2)26 16 3 7
 4.0100101011 10(2)20 14 2 8
 5.0101010011 10(2)16 10 2 6
 6.0101011011 10(2)22 18 2 10
 7.1001001100 10(2)4 2 1 2
 8.0010011011 5(2)2 0 0 1
 9.0010110011 10(2)2 0 0 1
 10.1111111111 1(1)1 0 0 1
 total NATs = 63
L= 11
 1.00000000000 1(1)1 0 0 1
 2.01010010101 11(2)110 82 4 28
 3.01001010011 11(2)8 4 2 3
 4.01001011001 11(2)8 4 2 3
 5.01001011011 11(2)14 10 2 6
 6.01010110011 11(2)10 6 2 4
 7.10010100100 11(2)14 12 1 7
 8.10010101100 11(2)10 8 1 5
 9.10010011010 11(2)10 8 1 5
 10.00100110011 11(2)2 0 0 1
 11.11111111111 1(1)1 0 0 1
 total NATs = 101
L= 12
 1.000000000000 1(1)1 0 0 1
 2.101010101010 1(2)640 482 5 98
 3.010100101011 6(2)58 42 4 18
 4.101001010100 12(2)52 40 3 16
 5.101010101100 12(2)42 28 3 13
 6.101010100100 12(2)58 48 3 21
 7.010010101011 12(2)54 44 2 17
 8.010010110011 12(2)6 2 2 2
 9.100101101100 12(2)8 6 1 4
 10.100101100100 12(2)6 4 1 3
 11.100101001100 12(2)6 4 1 3
 12.100100100100 3(2)14 12 1 7
 13.100100110100 12(2)6 4 1 3
 14.100100110010 12(2)6 4 1 3
 15.101011001100 12(2)6 4 1 3
 16.101001101100 12(2)4 2 1 2
 17.001011001011 6(2)2 0 0 1
 18.001100110011 2(2)2 0 0 1
 19.111111111111 1(1)1 0 0 1
 total NATs = 164
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.1010100101010 13(2)288 230 5 60
 3.1010010101100 13(2)20 14 3 8
 4.1010010100110 13(2)20 14 3 8
 5.1010010100100 13(2)30 24 3 11
 6.1010101001100 13(2)26 16 3 9
 7.0100101011011 13(2)30 26 2 13
 8.0100101001011 13(2)32 24 2 10
 9.0100101010011 13(2)20 14 2 7
 10.0100101101101 13(2)30 26 2 13
 11.0100101100101 13(2)20 14 2 7
 12.0101010010011 13(2)26 20 2 10
 13.0101010011011 13(2)26 20 2 10
 14.0101011011011 13(2)36 32 2 16
 15.1001011001100 13(2)4 2 1 2
 16.1001001101100 13(2)4 2 1 2
 17.1001001100110 13(2)4 2 1 2
 18.1001001100100 13(2)6 4 1 3
 19.1010011001100 13(2)4 2 1 2
 20.0010011001011 13(2)2 0 0 1
 21.0010011010011 13(2)2 0 0 1
 22.1111111111111 1(1)1 0 0 1
 total NATs = 262
```

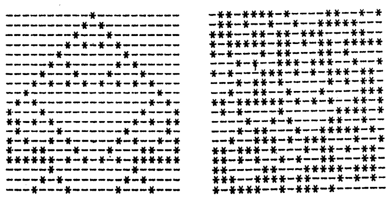
116

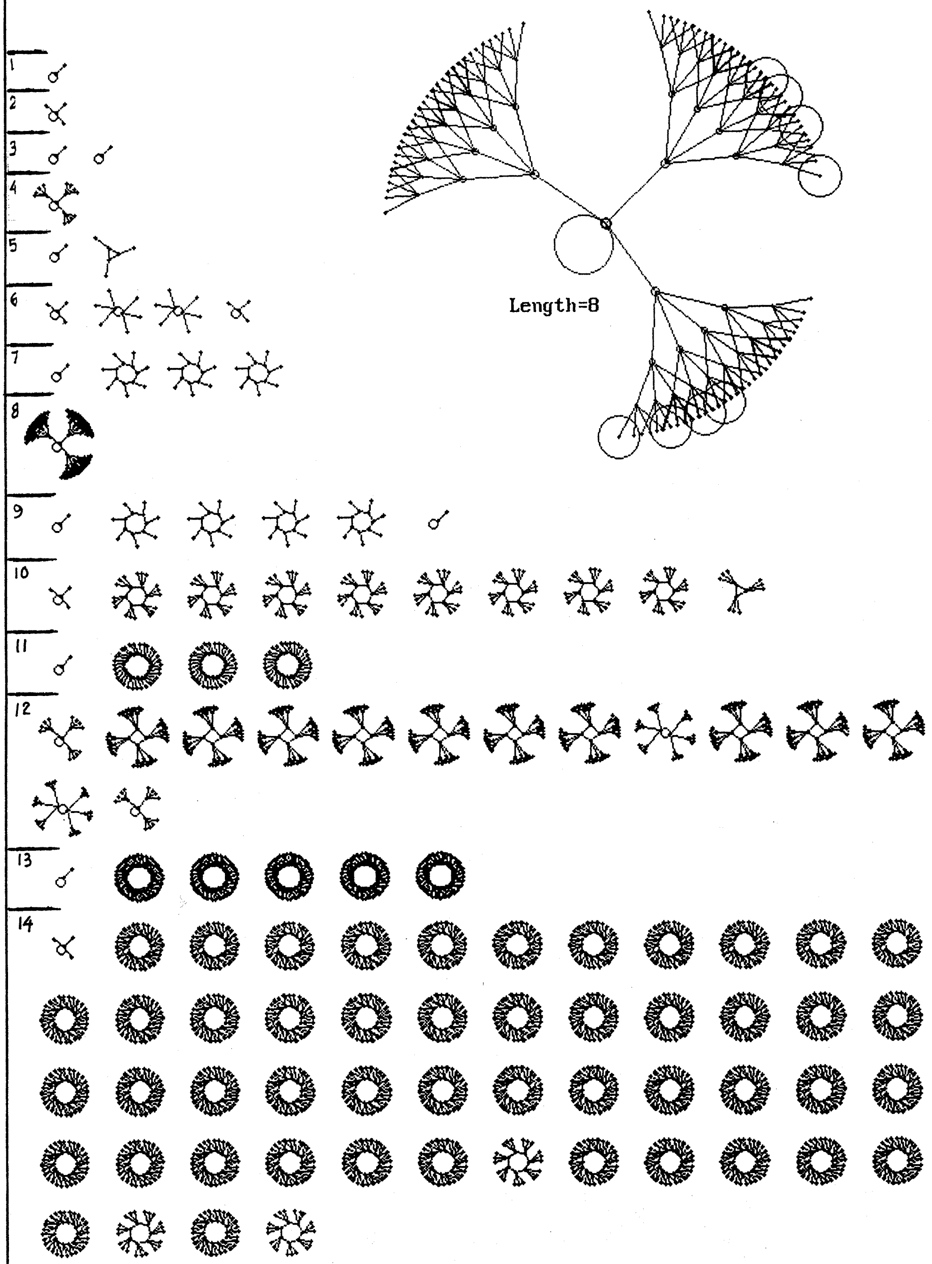
intentionally blank

λ ratio = 1 $Z = 1$

0011001111001100-0011001111001100-rule 869020620
=3-rule 90 -01011010
Length=1 -14



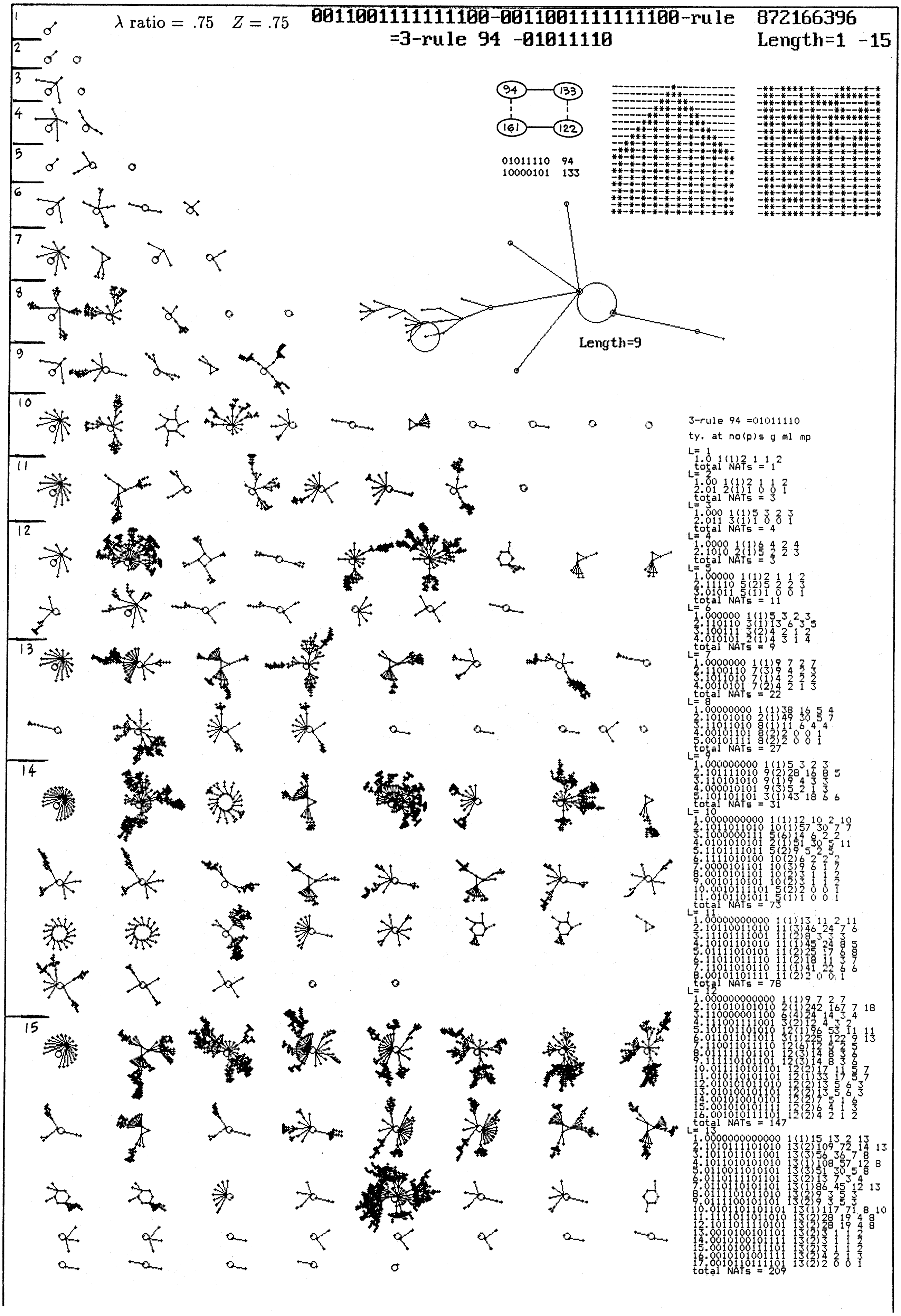




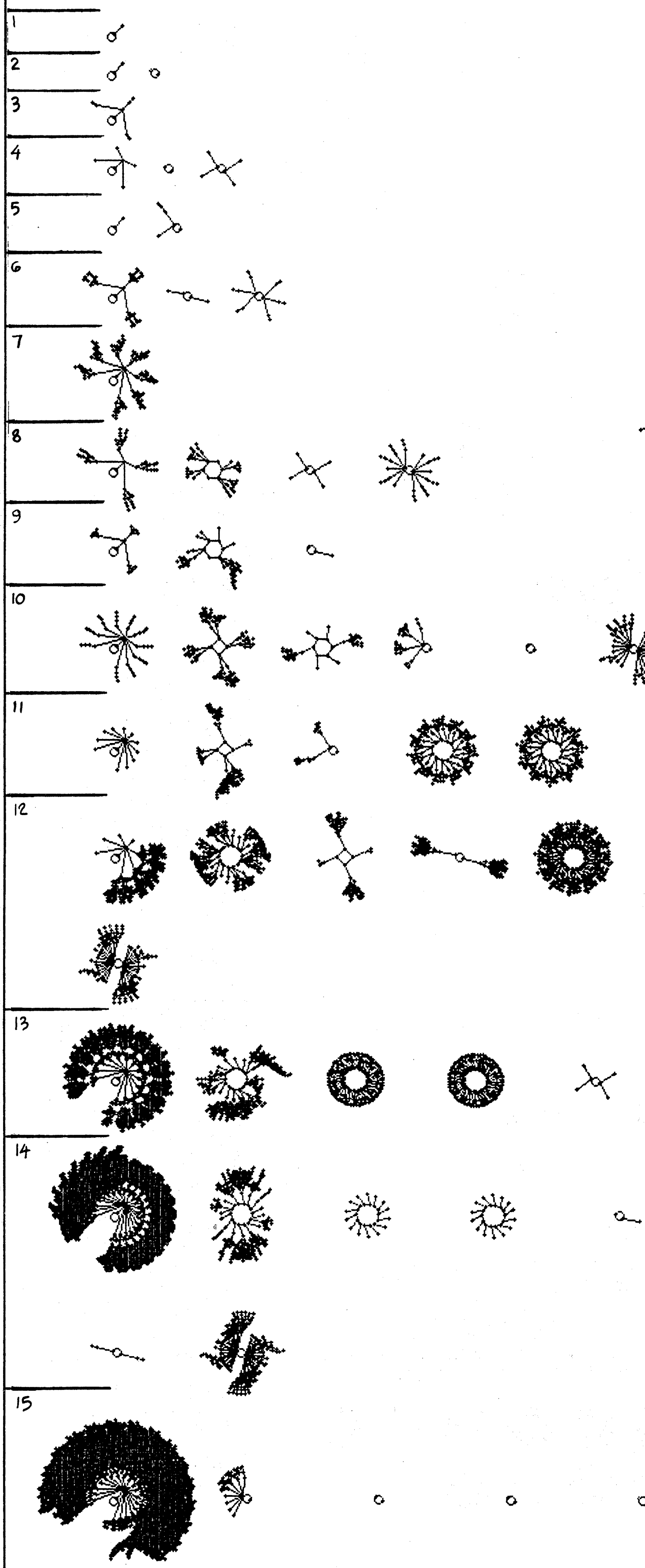
3-rule 90 =01011010

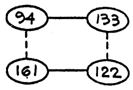
ty. at no(p)s g ml mp

L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)4 3 1 4
total NATs = 1
L= 3
1.000 1(1)2 1 1 2
2.110 3(1)2 1 1 2
total NATs = 4
L= 4
1.0000 1(1)16 12 2 4
total NATs = 1
L= 5
1.00000 1(1)2 1 1 2
2.10010 5(3)6 3 1 2
total NATs = 6
L= 6
1.000000 1(1)4 3 1 4
2.100010 3(2)8 6 1 4
3.100111 3(2)8 6 1 4
4.101101 3(1)4 3 1 4
total NATs = 10
L= 7
1.0000000 1(1)2 1 1 2
2.1000010 7(7)14 7 1 2
3.1010011 1(7)14 7 1 2
4.1011100 1(7)14 7 1 2
total NATs = 10
L= 8
1.00000000 1(1)256 192 4 4
total NATs = 1
L= 9
1.000000000 1(1)2 1 1 2
2.100000010 9(7)14 7 1 2
3.100001101 9(7)14 7 1 2
4.100010011 9(7)14 7 1 2
5.100011100 9(7)14 7 1 2
6.110110110 3(1)2 1 1 2
total NATs = 40
L= 10
1.0000000000 1(1)4 3 1 4
2.1000000010 5(6)24 18 1 4
3.1000000111 5(6)24 18 1 4
4.1000001101 5(6)24 18 1 4
5.1000010110 5(6)24 18 1 4
6.1000010011 5(6)24 18 1 4
7.1000011100 5(6)24 18 1 4
8.1000101111 5(6)24 18 1 4
9.1000100101 5(6)24 18 1 4
10.1000110001 5(3)12 9 1 4
total NATs = 46
L= 11
1.00000000000 1(1)2 1 1 2
2.10000000010 11(31)62 31 1 2
3.10000010011 11(31)62 31 1 2
4.10000011100 11(31)62 31 1 2
total NATs = 34
L= 12
1.000000000000 1(1)16 12 2 4
2.010000000100 6(4)64 48 2 4
3.110000001100 6(4)64 48 2 4
4.110000011101 6(4)64 48 2 4
5.010000100110 6(4)64 48 2 4
6.110000101110 6(4)64 48 2 4
7.010000110111 6(4)64 48 2 4
8.110001001000 6(4)64 48 2 4
9.010001010001 3(2)32 24 2 4
10.110001011001 6(4)64 48 2 4
11.010001110011 6(4)64 48 2 4
12.110001111011 6(4)64 48 2 4
13.110011110011 3(2)32 24 2 4
14.110110110110 3(1)16 12 2 4
total NATs = 70
L= 13
1.0000000000000 1(1)2 1 1 2
2.1000000000010 13(63)126 63 1 2
3.1000000010011 13(63)126 63 1 2
4.1000000011100 13(63)126 63 1 2
5.1000000101111 13(63)126 63 1 2
6.1000000111110 13(63)126 63 1 2
total NATs = 66
L= 14
1.00000000000000 1(1)4 3 1 4
2.10000000000010 7(14)56 42 1 4
3.10000000000111 7(14)56 42 1 4
4.10000000001101 7(14)56 42 1 4
5.10000000010110 7(14)56 42 1 4
6.10000000010011 7(14)56 42 1 4
7.10000000011100 7(14)56 42 1 4
8.10000000101111 7(14)56 42 1 4
9.10000000100101 7(14)56 42 1 4
10.10000000111110 7(14)56 42 1 4
11.10000000110100 7(14)56 42 1 4
12.10000000110001 7(14)56 42 1 4
13.10000001010010 7(14)56 42 1 4
14.10000001010111 7(14)56 42 1 4
15.10000001011000 7(14)56 42 1 4
16.10000001000110 7(14)56 42 1 4
17.10000001000011 7(14)56 42 1 4
18.10000001001001 7(14)56 42 1 4
19.10000001111010 7(14)56 42 1 4
20.10000001110000 7(14)56 42 1 4
21.10000001110101 7(14)56 42 1 4
22.10000001101011 7(14)56 42 1 4
23.10000001100100 7(14)56 42 1 4
24.10000010100111 7(14)56 42 1 4
25.10000010101000 1(14)56 42 1 4
26.10000010101101 7(14)56 42 1 4
27.10000010110110 7(14)56 42 1 4
28.10000010111100 7(14)56 42 1 4
29.10000010111001 7(14)56 42 1 4
30.10000010001010 1(14)56 42 1 4
31.10000010001111 7(14)56 42 1 4
32.10000010000101 7(14)56 42 1 4
33.10000010011110 7(14)56 42 1 4
34.10000010011011 7(14)56 42 1 4
35.10000011111000 7(14)56 42 1 4
36.10000011111101 7(14)56 42 1 4
37.10000011101100 7(14)56 42 1 4
38.10000011101001 7(14)56 42 1 4
39.10000011011010 7(14)56 42 1 4
40.10000011011111 7(14)56 42 1 4
41.10000011001110 7(14)56 42 1 4
42.10000011001011 7(14)56 42 1 4
43.10000011000001 7(7)28 21 1 4
44.10000111111001 1(14)56 42 1 4
45.10000111011011 7(14)56 42 1 4
46.10000110011111 1(14)56 42 1 4
47.10001110110101 1(14)56 42 1 4
48.10001101011011 1(14)56 42 1 4
49.10010111101100 1(14)56 42 1 4
50.10010111001011 1(7)28 21 1 4
51.10010011011110 1(14)56 42 1 4
52.10011101001110 1(7)28 21 1 4
total NATs = 298

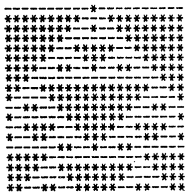

λ ratio = .75 Z = .75
0011001111111100-0011001111111100-rule 872166396
=3-rule 94 -01011110 Length=1 -15
94
133
161
122
01011110 94
10000101 133
Length=9
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
3-rule 94 =01011110
ty. at no(p)s g ml mp
L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)2 1 1 2
2.01 2(1)1 0 0 1
total NATs = 3
L= 3
1.000 1(1)5 3 2 3
2.011 3(1)1 0 0 1
total NATs = 4
L= 4
1.0000 1(1)6 4 2 4
2.1010 2(1)5 2 2 3
total NATs = 3
L= 5
1.00000 1(1)2 1 1 2
2.11110 5(2)5 2 2 3
3.01011 5(1)1 0 0 1
total NATs = 11
L= 6
1.000000 1(1)5 3 2 3
2.110110 3(1)13 6 3 5
3.100111 3(2)4 2 1 2
4.010101 2(1)4 3 1 4
total NATs = 9
L= 7
1.0000000 1(1)9 7 2 7
2.1100110 7(3)9 4 2 3
3.1011010 7(1)4 2 2 2
4.0010101 7(2)4 2 1 3
total NATs = 22
L= 8
1.00000000 1(1)38 16 5 4
2.10101010 2(1)49 30 5 7
3.11011010 8(1)11 6 4 4
4.00101101 8(2)2 0 0 1
5.00101111 8(2)2 0 0 1
total NATs = 27
L= 9
1.000000000 1(1)5 3 2 3
2.101111010 9(2)28 16 8 5
3.110101010 9(1)9 4 3 3
4.000010101 9(3)5 2 1 3
5.101101101 3(1)43 18 6 6
total NATs = 31
L= 10
1.0000000000 1(1)12 10 2 10
2.1011011010 10(1)57 30 7 7
3.1000000111 5(6)14 6 2 2
4.0101010101 2(1)51 30 5 11
5.1101111011 5(2)9 5 2 5
6.1111010100 10(2)6 2 2 2
7.0000101101 10(3)9 6 1 7
8.0010101101 10(2)3 1 1 2
9.0010110101 10(2)3 1 1 2
10.0010111101 5(2)2 0 0 1
11.0101101011 5(1)1 0 0 1
total NATs = 73
L= 11
1.00000000000 1(1)13 11 2 11
2.10110011010 11(3)46 24 7 6
3.11101111001 11(2)8 3 3 3
4.10101101010 11(1)45 24 8 5
5.01111010101 11(2)25 17 6 8
6.11011011110 11(2)18 11 3 7
7.11011010110 11(1)41 22 6 6
8.00101101111 11(2)2 0 0 1
total NATs = 78
L= 12
1.000000000000 1(1)9 7 2 7
2.101010101010 2(1)242 167 7 18
3.110000001100 6(4)24 14 3 4
4.111001111001 3(2)12 4 3 2
5.101101101010 12(1)96 53 11 11
6.011011011011 3(1)225 122 9 13
7.110011011110 12(6)12 5 2 5
8.011111101101 12(3)14 8 3 6
9.111110101101 12(3)14 8 3 6
10.011110101101 12(2)17 11 5 7
11.010110101101 12(1)33 17 5 7
12.010101011010 12(2)13 5 6 3
13.010100101101 12(2)13 5 6 3
14.001010010101 12(2)7 5 1 6
15.001010101111 12(2)6 4 1 3
16.001010111101 12(2)4 2 1 2
total NATs = 147
L= 13
1.0000000000000 1(1)15 13 2 13
2.1010111101010 13(2)109 72 14 13
3.1011011011001 13(3)56 36 7 8
4.1011010101010 13(1)108 57 12 8
5.0110011010101 13(3)51 30 5 8
6.0110111101101 13(2)13 7 3 4
7.0110110101101 13(1)86 45 12 13
8.0111101011010 13(2)9 3 5 3
9.0111100101101 13(2)9 3 5 3
10.0101101101101 13(1)117 71 8 10
11.1111011011010 13(2)28 19 4 8
12.1011011110101 13(2)28 19 4 8
13.0010100101101 13(2)3 1 1 2
14.0010100101111 13(2)3 1 1 2
15.0010100111101 13(2)3 1 1 2
16.0010101001111 13(2)4 2 1 3
17.0010110111101 13(2)2 0 0 1
total NATs = 209

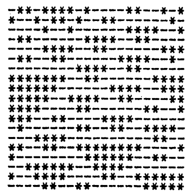
λ ratio = .75 Z = .75 **11001100000000011-11001100000000011-rule 3422800899**
=3-rule 161 -10100001 Length=1 -15

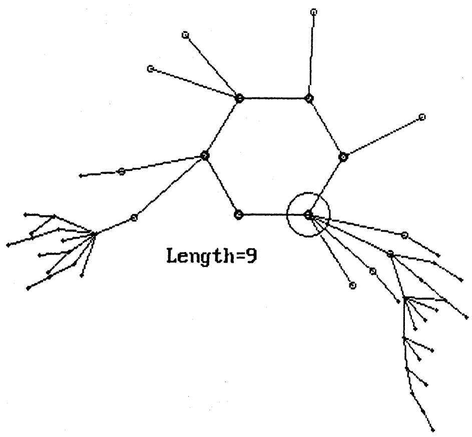




10100001 161
01111010 122



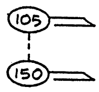




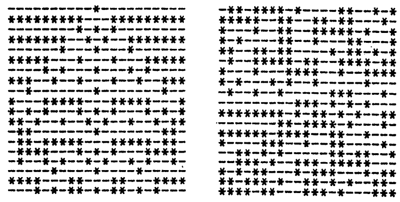

```
3-rule 161 =10100001

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.111 1(1)8 3 3 3
 total NATs = 1
L= 4
 1.1111 1(1)6 4 2 4
 2.0001 2(2)2 0 0 1
 3.0101 1(2)6 4 1 3
 total NATs = 4
L= 5
 1.11111 1(1)2 1 1 2
 2.00001 5(2)6 2 3 3
 total NATs = 6
L= 6
 1.111111 1(1)38 21 5 5
 2.000011 3(2)4 2 1 2
 3.010101 1(2)14 6 2 4
 total NATs = 5
L= 7
 1.1111111 1(1)128 63 7 7
 total NATs = 1
L= 8
 1.11111111 1(1)54 16 7 4
 2.00000001 4(6)40 22 3 4
 3.00010001 2(2)6 4 1 3
 4.01010101 1(2)30 12 3 7
 total NATs = 8
L= 9
 1.111111111 1(1)17 12 3 4
 2.000000001 9(6)52 26 7 6
 3.000010001 9(2)3 1 1 2
 total NATs = 19
L= 10
 1.1111111111 1(1)42 10 5 10
 2.0011111001 5(4)100 52 6 7
 3.0000000011 5(6)48 26 5 7
 4.0010000100 5(2)32 19 3 6
 5.0000100011 10(2)2 0 0 1
 6.0101010101 1(2)62 30 4 11
 total NATs = 27
L= 11
 1.11111111111 1(1)13 11 2 11
 2.00111111001 11(4)134 74 9 8
 3.00010000110 11(2)19 9 5 3
 4.00111000001 1(11)176 99 4 6
 5.00111001000 1(11)176 99 4 6
 total NATs = 25
L= 12
 1.111111111111 1(1)342 229 9 13
 2.000000000001 6(14)282 164 5 11
 3.001111110011 6(4)98 50 7 5
 4.000110000110 3(2)132 60 8 7
 5.001111000001 1(36)468 252 5 6
 6.001111001000 1(36)468 252 5 6
 7.000100010001 2(2)8 6 1 4
 8.010101010101 1(2)126 70 5 18
 total NATs = 21
L= 13
 1.1111111111111 1(1)2576 1495 12 13
 2.0000000000001 13(14)396 229 11 13
 3.0111100000010 1(39)195 130 2 5
 4.0111100100000 1(39)195 130 2 5
 5.0000100010001 13(2)6 4 1 3
 total NATs = 29
L= 14
 1.11111111111111 1(1)13246 8127 18 21
 2.00000000000011 7(14)384 210 9 10
 3.00000011001111 1(14)28 14 1 2
 4.00000011110011 1(14)28 14 1 2
 5.00001000010001 14(2)3 1 1 2
 6.00001000100011 14(2)4 2 1 2
 7.00001000110001 7(2)6 2 2 2
 8.01010101010101 1(2)254 154 6 29
 total NATs = 46
L= 15
 1.111111111111111 1(1)32483 20208 23 21
 2.001000010000100 5(2)36 26 3 9
 3.000010000100011 15(2)2 0 0 1
 4.000010000110001 15(2)2 0 0 1
 5.000010001000011 15(2)3 1 1 2
 total NATs = 51
```

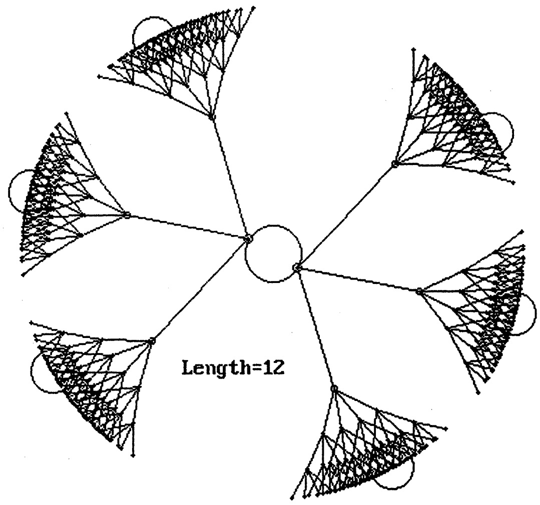
λ ratio = 1 Z = 1

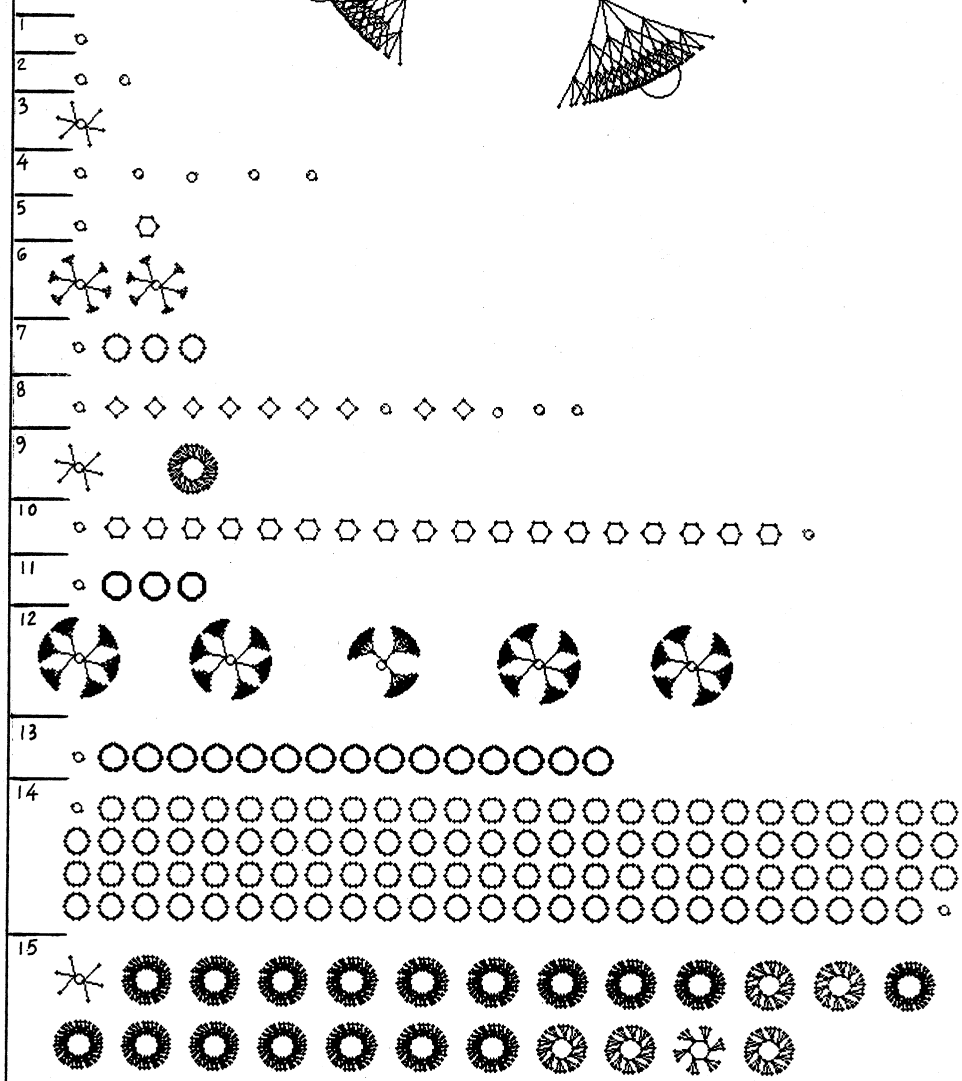
0011110011000011-0011110011000011-rule 1019428035 =3-rule 105 -01101001 **Length=1 -15**



01101001 105

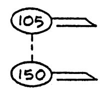




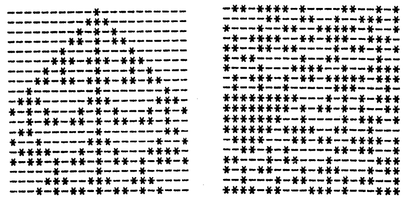


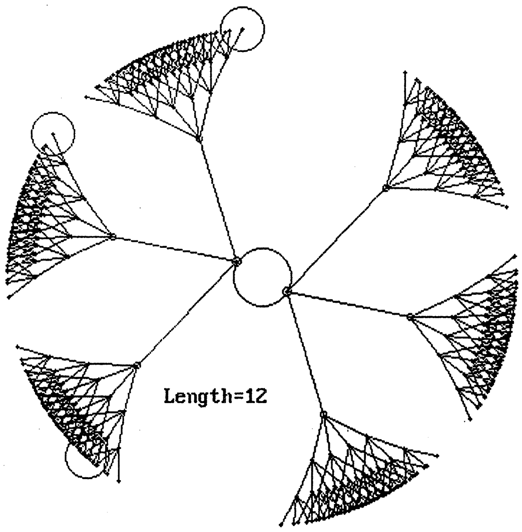

```
3-rule 105 =01101001
ty. at no(p)s g ml mp
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.000 1(2)8 6 1 4
 total NATs = 1
L= 4
 1.0000 1(2)2 0 0 1
 2.0001 2(2)2 0 0 1
 3.0011 4(1)1 0 0 1
 4.0101 1(2)2 0 0 1
 5.0111 2(2)2 0 0 1
 total NATs = 10
L= 5
 1.00000 1(2)2 0 0 1
 2.00001 5(6)6 0 0 1
 total NATs = 6
L= 6
 1.000000 1(2)32 24 2 4
 2.010101 1(2)32 24 2 4
 total NATs = 2
L= 7
 1.0000000 1(2)2 0 0 1
 2.0000001 7(14)14 0 0 1
 3.0001011 1(14)14 0 0 1
 4.0001101 1(14)14 0 0 1
 total NATs = 10
L= 8
 1.00000000 1(2)2 0 0 1
 2.00000001 8(4)4 0 0 1
 3.00000011 8(4)4 0 0 1
 4.00000101 4(4)4 0 0 1
 5.00000111 8(4)4 0 0 1
 6.00001011 8(4)4 0 0 1
 7.00001101 8(4)4 0 0 1
 8.00001111 4(4)4 0 0 1
 9.00010001 2(2)2 0 0 1
 10.00010111 8(4)4 0 0 1
 11.00011011 4(4)4 0 0 1
 12.00110011 4(1)1 0 0 1
 13.01010101 1(2)2 0 0 1
 14.01110111 2(2)2 0 0 1
 total NATs = 70
L= 9
 1.000000000 1(2)8 6 1 4
 2.011111100 9(14)56 42 1 4
 total NATs = 10
L= 10
 1.0000000000 1(2)2 0 0 1
 2.0000000001 10(6)6 0 0 1
 3.0000000011 10(6)6 0 0 1
 4.0000000101 10(6)6 0 0 1
 5.0000000111 10(6)6 0 0 1
 6.0000001001 10(6)6 0 0 1
 7.0000001011 10(6)6 0 0 1
 8.0000010011 10(6)6 0 0 1
 9.0000010111 10(6)6 0 0 1
 10.0000011001 10(6)6 0 0 1
 11.0000011011 10(6)6 0 0 1
 12.0000011101 10(6)6 0 0 1
 13.0000011111 5(6)6 0 0 1
 14.0000100001 5(6)6 0 0 1
 15.0000100011 10(6)6 0 0 1
 16.0000100111 10(6)6 0 0 1
 17.0000101111 10(6)6 0 0 1
 18.0000110101 10(6)6 0 0 1
 19.0000111101 10(6)6 0 0 1
 20.0101010101 1(2)2 0 0 1
 total NATs = 172
L= 11
 1.00000000000 1(2)2 0 0 1
 2.00000000001 11(62)62 0 0 1
 3.00000001011 11(62)62 0 0 1
 4.00000001101 11(62)62 0 0 1
 total NATs = 34
L= 12
 1.000000000000 1(2)512 384 4 4
 2.000100010001 2(2)512 384 4 4
 3.001100110011 4(1)256 192 4 4
 4.010101010101 1(2)512 384 4 4
 5.110111011101 2(2)512 384 4 4
 total NATs = 10
L= 13
 1.0000000000000 1(2)2 0 0 1
 2.0000000000001 13(42)42 0 0 1
 3.0000000000011 13(42)42 0 0 1
 4.0000000000101 13(42)42 0 0 1
 5.0000000001011 13(42)42 0 0 1
 6.0000000001101 13(42)42 0 0 1
 7.0000000010011 13(42)42 0 0 1
 8.0000000010111 13(42)42 0 0 1
 9.0000000011001 13(42)42 0 0 1
 10.0000000011101 13(42)42 0 0 1
 11.0000000100101 13(42)42 0 0 1
 12.0000000100111 13(42)42 0 0 1
 13.0000000101001 13(42)42 0 0 1
 14.0000000101111 13(42)42 0 0 1
 15.0000001011111 13(42)42 0 0 1
 16.0000001101111 13(42)42 0 0 1
 total NATs = 196
```

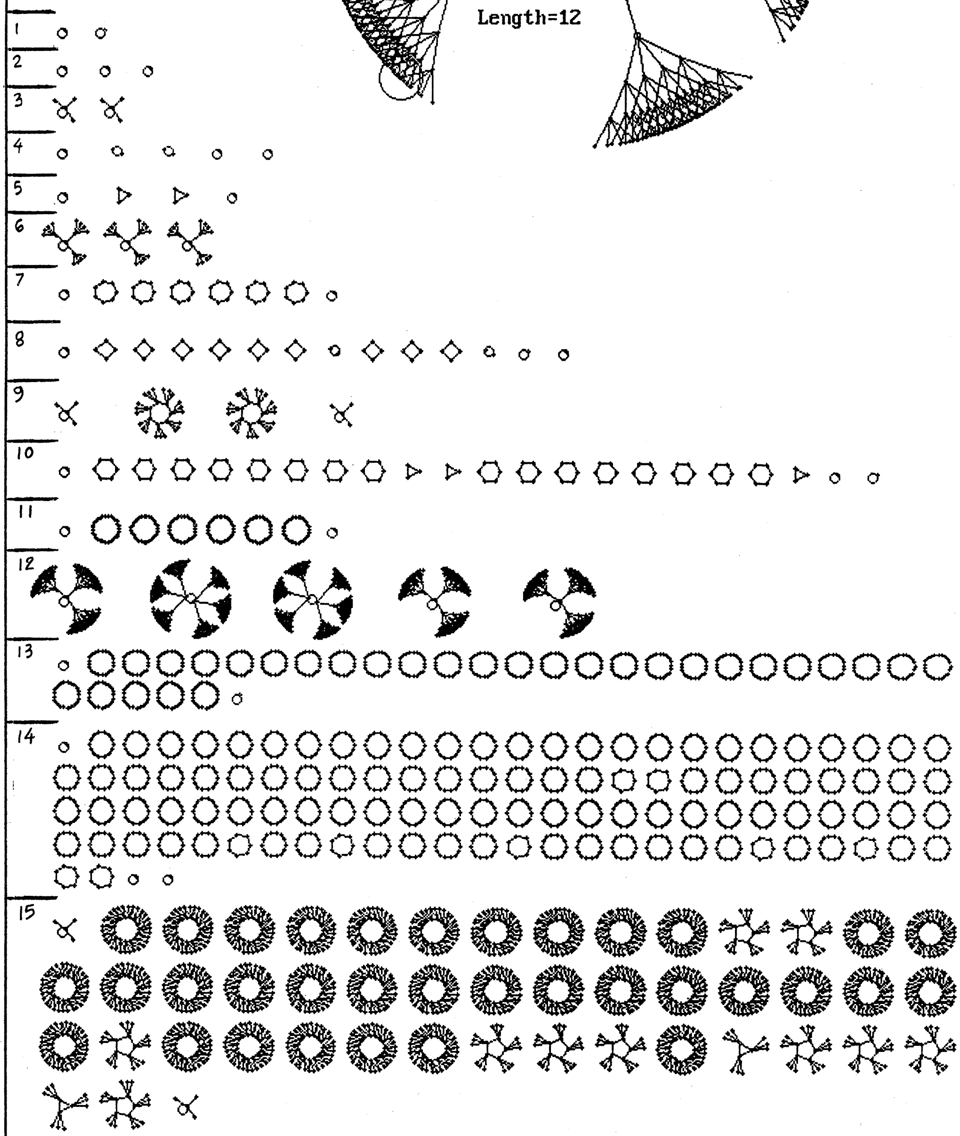
λ ratio = 1 $Z = 1$ 1100001100111100-1100001100111100-rule 3275539260
=3-rule 150 -10010110   Length=1 -15



10010110 150








```
3-rule 150 =10010110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 2(1)1 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 4
L= 3
 1.000 1(1)4 3 1 4
 2.111 1(1)4 3 1 4
 total NATs = 2
L= 4
 1.0000 1(1)1 0 0 1
 2.0001 4(2)2 0 0 1
 3.0011 2(2)2 0 0 1
 4.0101 2(1)1 0 0 1
 5.1111 1(1)1 0 0 1
 total NATs = 10
L= 5
 1.00000 1(1)1 0 0 1
 2.00001 5(3)3 0 0 1
 3.00011 5(3)3 0 0 1
 4.11111 1(1)1 0 0 1
 total NATs = 12
L= 6
 1.000000 1(1)16 12 2 4
 2.010101 2(1)16 12 2 4
 3.111111 1(1)16 12 2 4
 total NATs = 4
L= 7
 1.0000000 1(1)1 0 0 1
 2.0000001 7(7)7 0 0 1
 3.0000011 7(7)7 0 0 1
 4.0001011 1(7)7 0 0 1
 5.0001101 1(7)7 0 0 1
 6.0010111 1(7)7 0 0 1
 7.0011101 1(7)7 0 0 1
 8.1111111 1(1)1 0 0 1
 total NATs = 20
L= 8
 1.00000000 1(1)1 0 0 1
 2.00000001 8(4)4 0 0 1
 3.00000011 8(4)4 0 0 1
 4.00000101 4(4)4 0 0 1
 5.00001011 8(4)4 0 0 1
 6.00001101 8(4)4 0 0 1
 7.00001111 4(4)4 0 0 1
 8.00010001 4(2)2 0 0 1
 9.00010111 8(4)4 0 0 1
 10.00011111 8(4)4 0 0 1
 11.00100111 4(4)4 0 0 1
 12.00110011 2(2)2 0 0 1
 13.01010101 2(1)1 0 0 1
 14.11111111 1(1)1 0 0 1
 total NATs = 70
L= 9
 1.000000000 1(1)4 3 1 4
 2.100000011 9(7)28 21 1 4
 3.100000100 9(7)28 21 1 4
 4.111111111 1(1)4 3 1 4
 total NATs = 20
L= 10
 1.0000000000 1(1)1 0 0 1
 2.0000000001 10(6)6 0 0 1
 3.0000000011 10(6)6 0 0 1
 4.0000000101 10(6)6 0 0 1
 5.0000001011 10(6)6 0 0 1
 6.0000001101 10(6)6 0 0 1
 7.0000001111 10(6)6 0 0 1
 8.0000010011 10(6)6 0 0 1
 9.0000010111 10(6)6 0 0 1
 10.0000011111 10(3)3 0 0 1
 11.0000100001 5(3)3 0 0 1
 12.0000100101 10(6)6 0 0 1
 13.0000100111 10(6)6 0 0 1
 14.0000101011 10(6)6 0 0 1
 15.0000101111 10(6)6 0 0 1
 16.0000111101 10(6)6 0 0 1
 17.0001000111 10(6)6 0 0 1
 18.0001001001 10(6)6 0 0 1
 19.0001001011 10(6)6 0 0 1
 20.0001100011 5(3)3 0 0 1
 21.0101010101 2(1)1 0 0 1
 22.1111111111 1(1)1 0 0 1
 total NATs = 184
L= 11
 1.00000000000 1(1)1 0 0 1
 2.00000000001 11(31)31 0 0 1
 3.00000000011 11(31)31 0 0 1
 4.00000001011 11(31)31 0 0 1
 5.00000001101 11(31)31 0 0 1
 6.00000010111 11(31)31 0 0 1
 7.00000011101 11(31)31 0 0 1
 8.11111111111 1(1)1 0 0 1
 total NATs = 68
L= 12
 1.000000000000 1(1)256 192 4 4
 2.000100010001 4(2)512 384 4 4
 3.001100110011 2(2)512 384 4 4
 4.010101010101 2(1)256 192 4 4
 5.111111111111 1(1)256 192 4 4
 total NATs = 10
```

122

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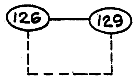
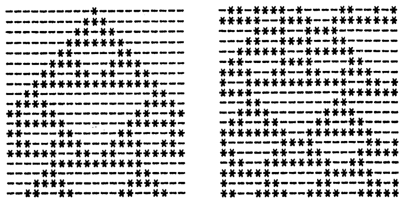
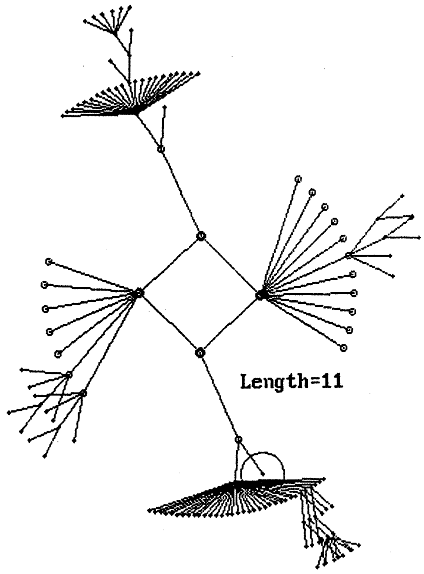
λ ratio = .5 $Z = .5$

0011111111111100-0011111111111100-rule 1073496060
=3-rule 126 -01111110 Length=1 -15

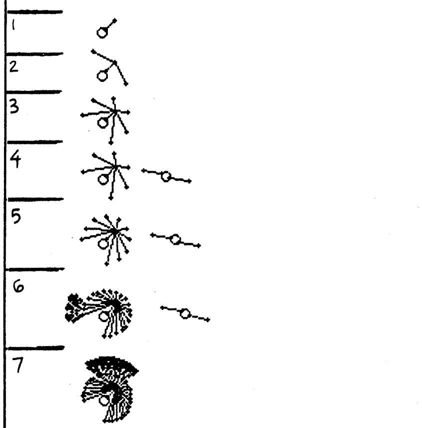
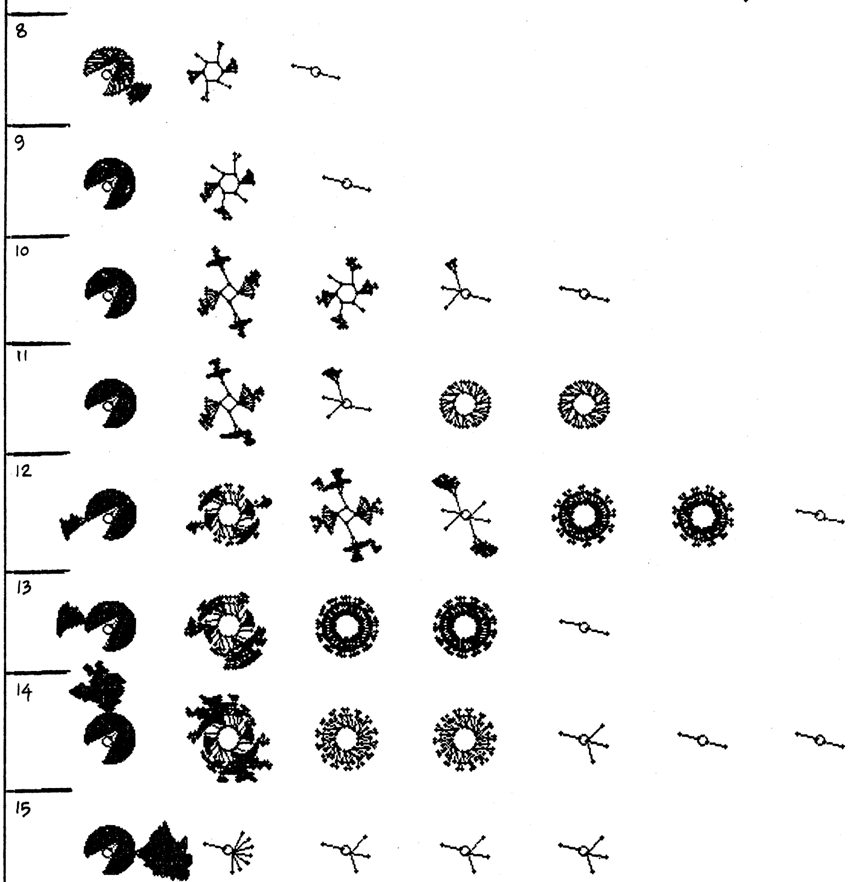


01111110 126
10000001 129





Length=11


```
3-rule 126 =01111110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.000 1(1)8 6 2 6
 total NATs = 1
L= 4
 1.0000 1(1)8 6 2 6
 2.1011 2(2)4 2 1 2
 total NATs = 3
L= 5
 1.00000 1(1)12 10 2 10
 2.10011 5(2)4 2 1 2
 total NATs = 6
L= 6
 1.000000 1(1)52 35 5 20
 2.100111 3(2)4 2 1 2
 total NATs = 4
L= 7
 1.0000000 1(1)128 91 5 28
 total NATs = 1
L= 8
 1.00000000 1(1)136 106 6 46
 2.10000011 4(6)28 20 2 8
 3.10111011 2(2)4 2 1 2
 total NATs = 7
L= 9
 1.000000000 1(1)80 78 2 78
 2.100000011 9(6)44 33 3 12
 3.100111011 9(2)4 2 1 2
 total NATs = 19
L= 10
 1.0000000000 1(1)124 122 2 122
 2.1100000110 5(4)104 82 6 22
 3.1000000111 5(6)56 44 3 12
 4.1110111101 5(2)12 7 3 4
 5.1001110111 10(2)4 2 1 2
 total NATs = 26
L= 11
 1.00000000000 1(1)200 198 2 198
 2.11000000110 11(4)136 109 6 34
 3.11100111101 11(2)24 18 4 8
 4.10001111101 1(11)44 33 1 4
 5.10001101111 1(11)44 33 1 4
 total NATs = 25
L= 12
 1.000000000000 1(1)548 495 8 324
 2.100000000011 6(14)284 242 5 54
 3.110000001100 6(4)188 154 6 34
 4.111001111001 3(2)124 96 6 14
 5.110001100111 1(36)168 120 2 4
 6.110001111100 1(36)168 120 2 4
 7.101110111011 2(2)4 2 1 2
 total NATs = 20
L= 13
 1.0000000000000 1(1)2160 1820 9 520
 2.1000000000011 13(14)428 371 5 90
 3.1100001100111 1(39)208 143 2 4
 4.1100001111100 1(39)208 143 2 4
 5.1001110111011 13(2)4 2 1 2
 total NATs = 29
L= 14
 1.00000000000000 1(1)11932 10327 18 842
 2.10000000000111 7(14)588 520 7 90
 3.11000011001111 1(14)84 56 2 4
 4.11000011111100 1(14)84 56 2 4
 5.10011100111011 14(2)6 4 1 4
 6.10011101110111 14(2)4 2 1 2
 7.10011101111011 7(2)4 2 1 2
 total NATs = 45
L= 15
 1.000000000000000 1(1)32448 28621 20 1366
 2.100111001110011 5(2)10 8 1 8
 3.100111001110111 15(2)6 4 1 4
 4.100111001111011 15(2)6 4 1 4
 5.100111011100111 15(2)6 4 1 4
 total NATs = 51
```

124

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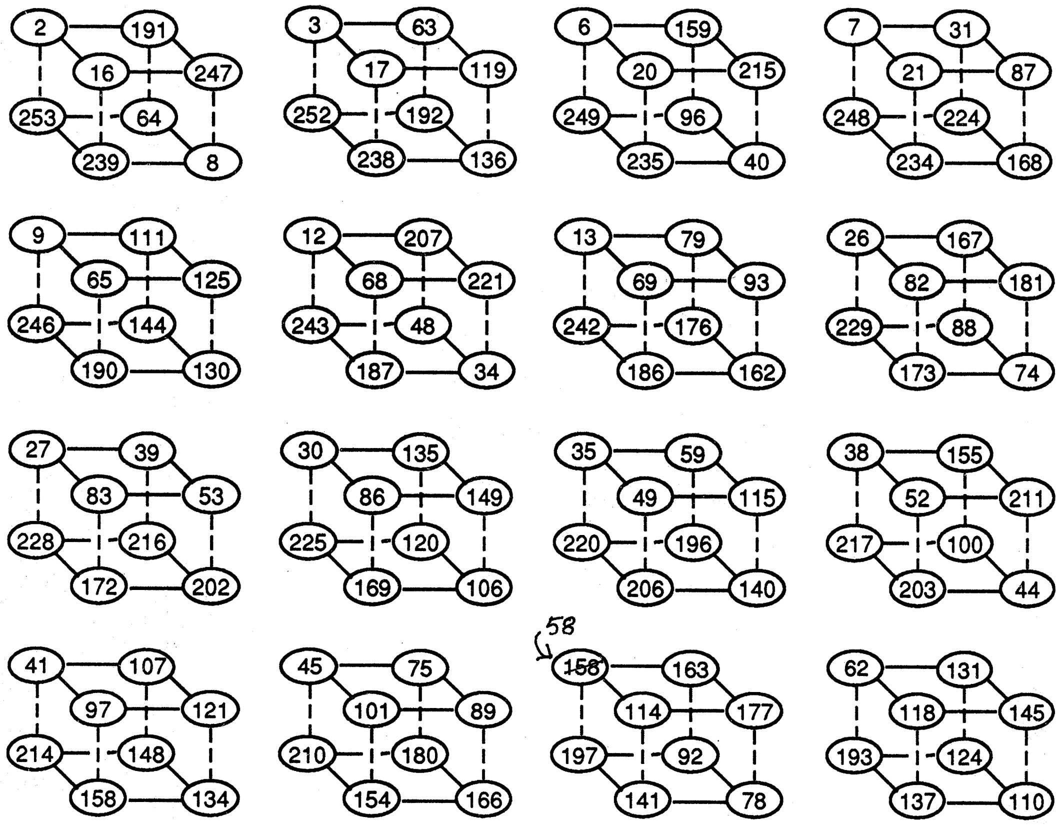
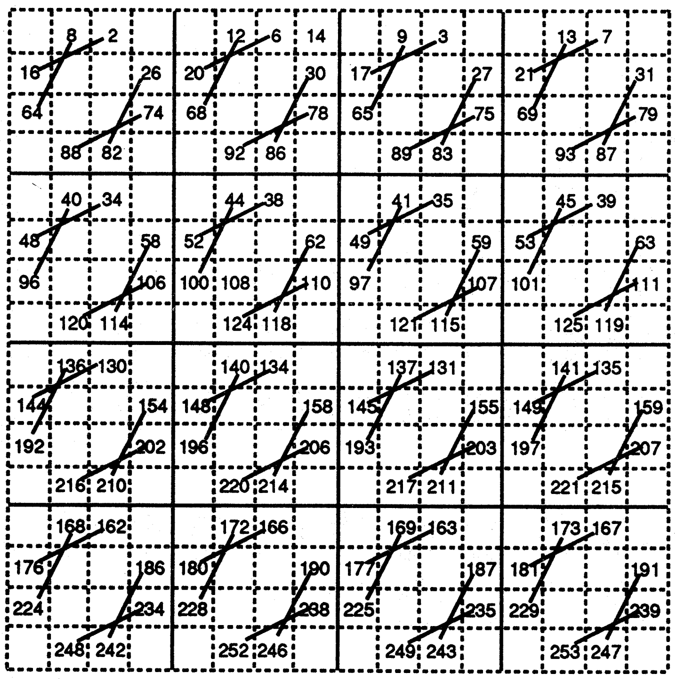
A2.3.3 Semi-Asymmetric Rule Clusters (see section 3.3.8)

There are no collapsed clusters among the semi-asymmetric rules.

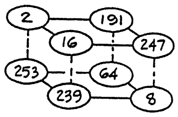




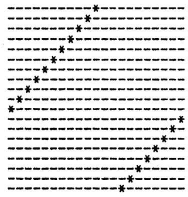
The rule-space matrix (see appendix 4).

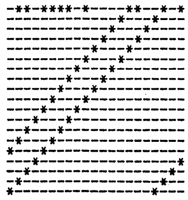
λ ratio = .25 Z = .25

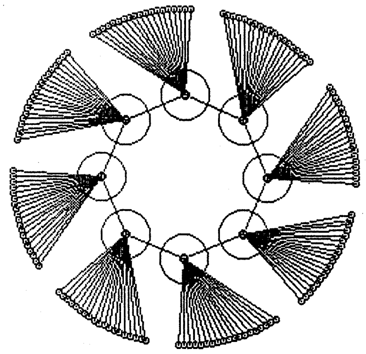
0000000000001100-0000000000001100-rule 786444
=3-rule 2 -00000010   Length=1 -15



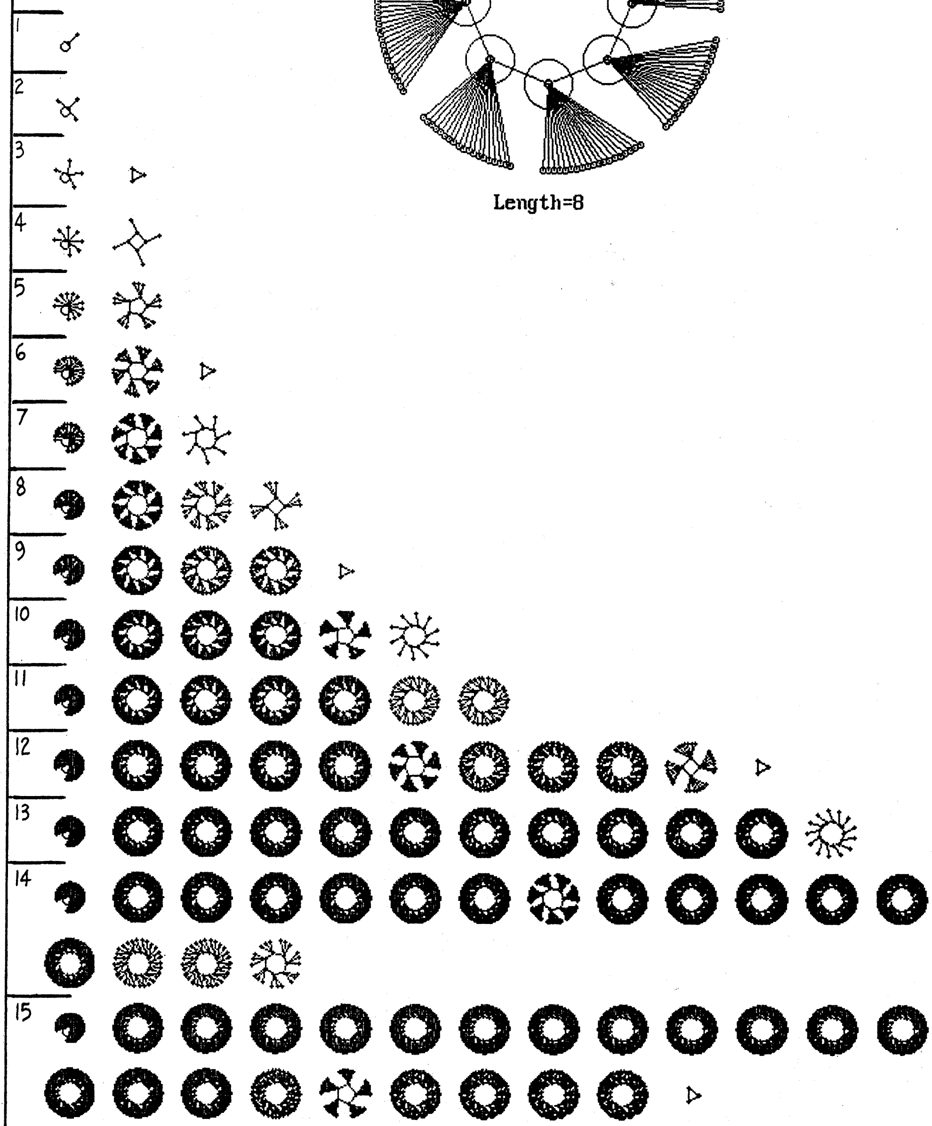
00000010 2
10111111 191
1111 0111 247
0001000 16





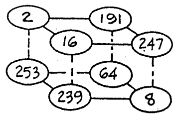


Length=8

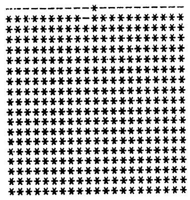


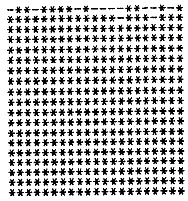

```
3-rule 2 =00000010

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 3 1 4
 total NATs = 1
L= 3
 1.000 1(1)5 4 1 5
 2.001 1(3)3 0 0 1
 total NATs = 2
L= 4
 1.0000 1(1)8 7 1 8
 2.0001 1(4)8 4 1 2
 total NATs = 2
L= 5
 1.00000 1(1)12 11 1 12
 2.00001 1(5)20 15 1 4
 total NATs = 2
L= 6
 1.000000 1(1)19 18 1 19
 2.000001 1(6)42 36 1 7
 3.001001 1(3)3 0 0 1
 total NATs = 3
L= 7
 1.0000000 1(1)30 29 1 30
 2.0000001 1(7)84 77 1 12
 3.0001001 1(7)14 7 1 2
 total NATs = 3
L= 8
 1.00000000 1(1)48 47 1 48
 2.00000001 1(8)160 152 1 20
 3.00001001 1(8)32 24 1 4
 4.00010001 1(4)16 12 1 4
 total NATs = 4
L= 9
 1.000000000 1(1)77 76 1 77
 2.000000001 1(9)297 288 1 33
 3.000001001 1(9)63 54 1 7
 4.000010001 1(9)72 63 1 8
 5.001001001 1(3)3 0 0 1
 total NATs = 5
L= 10
 1.0000000000 1(1)124 123 1 124
 2.0000000001 1(10)540 530 1 54
 3.0000001001 1(10)120 110 1 12
 4.0000010001 1(10)140 130 1 14
 5.0000100001 1(5)80 75 1 16
 6.0001001001 1(10)20 10 1 2
 total NATs = 6
L= 11
 1.00000000000 1(1)200 199 1 200
 2.00000000001 1(11)968 957 1 88
 3.00000001001 1(11)220 209 1 20
 4.00000010001 1(11)264 253 1 24
 5.00000100001 1(11)308 297 1 28
 6.00001001001 1(11)44 33 1 4
 7.00010001001 1(11)44 33 1 4
 total NATs = 7
L= 12
 1.000000000000 1(1)323 322 1 323
 2.000000000001 1(12)1716 1704 1 143
 3.000000001001 1(12)396 384 1 33
 4.000000010001 1(12)480 468 1 40
 5.000000100001 1(12)576 564 1 48
 6.000001000001 1(6)294 288 1 49
 7.000001001001 1(12)84 72 1 7
 8.000010001001 1(12)96 84 1 8
 9.000010010001 1(12)96 84 1 8
 10.000100010001 1(4)32 28 1 8
 11.001001001001 1(3)3 0 0 1
 total NATs = 11
L= 13
 1.0000000000000 1(1)522 521 1 522
 2.0000000000001 1(13)3016 3003 1 232
 3.0000000001001 1(13)702 689 1 54
 4.0000000010001 1(13)858 845 1 66
 5.0000000100001 1(13)1040 1027 1 80
 6.0000001000001 1(13)1092 1079 1 84
 7.0000001001001 1(13)156 143 1 12
 8.0000010001001 1(13)182 169 1 14
 9.0000010010001 1(13)182 169 1 14
 10.0000100001001 1(13)208 195 1 16
 11.0000100010001 1(13)208 195 1 16
 12.0001001001001 1(13)26 13 1 2
 total NATs = 12
L= 14
 1.00000000000000 1(1)844 843 1 844
 2.00000000000001 1(14)5264 5250 1 376
 3.00000000001001 1(14)1232 1218 1 88
 4.00000000010001 1(14)1512 1498 1 108
 5.00000000100001 1(14)1848 1834 1 132
 6.00000001000001 1(14)1960 1946 1 140
 7.00000001001001 1(14)280 266 1 20
 8.00000010000001 1(7)1008 1001 1 144
 9.00000010001001 1(14)336 322 1 24
 10.00000010010001 1(14)336 322 1 24
 11.00000100001001 1(14)392 378 1 28
 12.00000100010001 1(14)392 378 1 28
 13.00000100100001 1(14)392 378 1 28
 14.00001000010001 1(14)448 434 1 32
 15.00001001001001 1(14)56 42 1 4
 16.00010001001001 1(14)56 42 1 4
 17.00010010001001 1(7)28 21 1 4
 total NATs = 17
L= 15
 1.000000000000000 1(1)1365 1364 1 1365
 2.000000000000001 1(15)9135 9120 1 609
 3.000000000001001 1(15)2145 2130 1 143
 4.000000000010001 1(15)2640 2625 1 176
 5.000000000100001 1(15)3240 3225 1 216
 6.000000001000001 1(15)3465 3450 1 231
 7.000000001001001 1(15)495 480 1 33
 8.000000010000001 1(15)3600 3585 1 240
 9.000000010001001 1(15)600 585 1 40
 10.000000010010001 1(15)600 585 1 40
 11.000000100001001 1(15)720 705 1 48
 12.000000100010001 1(15)720 705 1 48
 13.000000100100001 1(15)720 705 1 48
 14.000001000001001 1(15)735 720 1 49
 15.000001000010001 1(15)840 825 1 56
 16.000001000100001 1(15)840 825 1 56
 17.000001001001001 1(15)105 90 1 7
 18.000010000100001 1(5)320 315 1 64
 19.000010001001001 1(15)120 105 1 8
 20.000010010001001 1(15)120 105 1 8
 21.000010010010001 1(15)120 105 1 8
 22.000100010001001 1(15)120 105 1 8
 23.001001001001001 1(3)3 0 0 1
 total NATs = 23
```

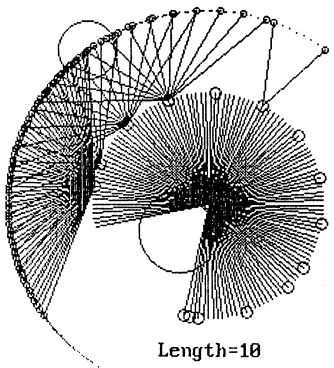
λ ratio = .25 Z = .25   1111111111110011-1111111111110011-rule   4294180851
=3-rule 253 -11111101   Length=1 -15



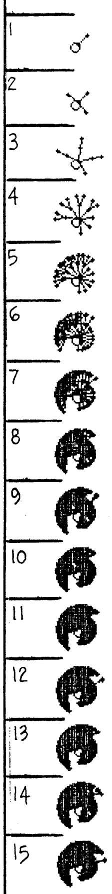

```
11111101  253
01000000  64
00001000  8
11101111  239
```







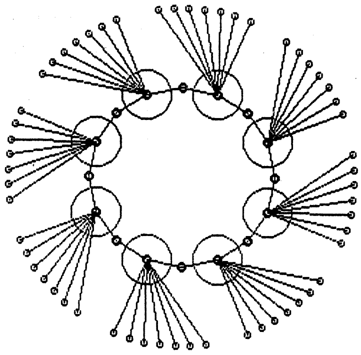
Length=10



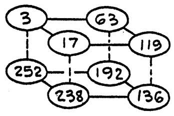

```
3-rule 253 =11111101

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)4 3 1 4
 total NATs = 1
L= 3
 1.111 1(1)8 4 2 5
 total NATs = 1
L= 4
 1.1111 1(1)16 11 2 8
 total NATs = 1
L= 5
 1.11111 1(1)32 26 2 12
 total NATs = 1
L= 6
 1.111111 1(1)64 54 2 19
 total NATs = 1
L= 7
 1.1111111 1(1)128 113 2 30
 total NATs = 1
L= 8
 1.11111111 1(1)256 235 2 48
 total NATs = 1
L= 9
 1.111111111 1(1)512 481 2 77
 total NATs = 1
L= 10
 1.1111111111 1(1)1024 978 2 124
 total NATs = 1
L= 11
 1.11111111111 1(1)2048 1981 2 200
 total NATs = 1
L= 12
 1.111111111111 1(1)4096 3998 2 323
 total NATs = 1
L= 13
 1.1111111111111 1(1)8192 8048 2 522
 total NATs = 1
L= 14
 1.11111111111111 1(1)16384 16173 2 844
 total NATs = 1
L= 15
 1.111111111111111 1(1)32768 32459 2 1365
 total NATs = 1
```

λ ratio = .5 Z = .5 0000000000001111-0000000000001111-rule 983055

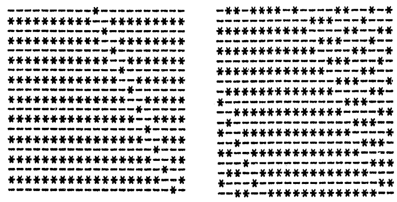
=3-rule 3 -00000011 Length=1 -14

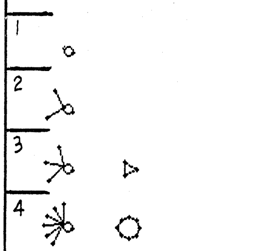


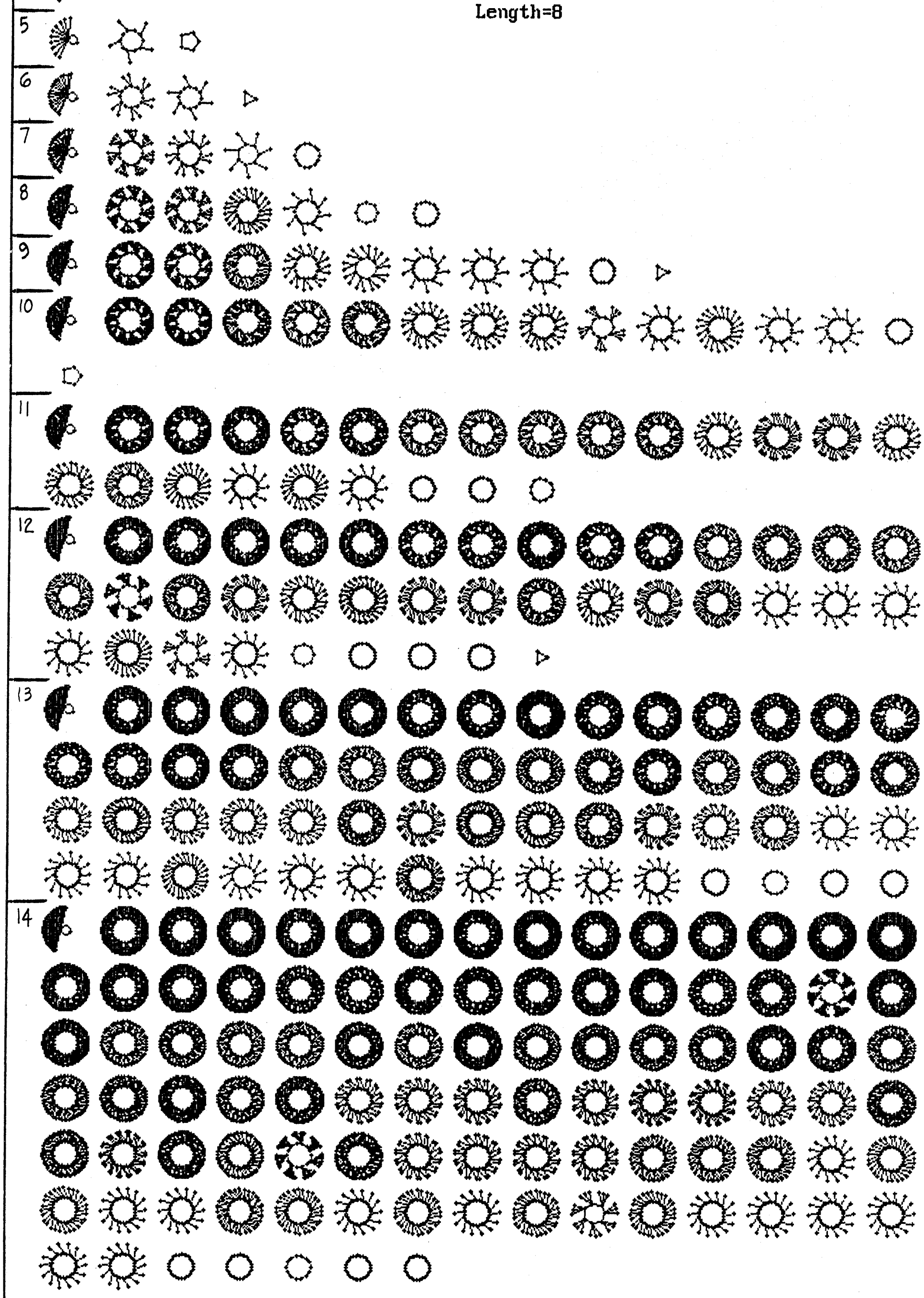
Length=8



00000011 3
00111111 63
01110111 119
00010001 17







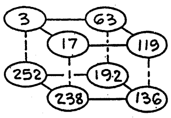
3-rule 3 =00000011

ty. at no(p)s g ml mp

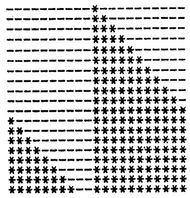
```
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)4 2 1 3
 total NATs = 1
L= 3
 1.000 1(2)5 3 1 4
 2.001 1(3)3 0 0 1
 total NATs = 2
L= 4
 1.0000 1(2)8 6 1 7
 2.0001 1(8)8 0 0 1
 total NATs = 2
L= 5
 1.00000 1(2)12 10 1 11
 2.00001 1(10)15 5 1 2
 3.00011 1(5)5 0 0 1
 total NATs = 3
L= 6
 1.000000 1(2)19 17 1 18
 2.000001 1(12)24 12 1 3
 3.000011 1(12)18 6 1 2
 4.001001 1(3)3 0 0 1
 total NATs = 4
L= 7
 1.0000000 1(2)30 28 1 29
 2.0000001 1(14)42 28 1 5
 3.0000011 1(14)28 14 1 3
 4.0111000 1(7)14 7 1 2
 5.0001001 1(14)14 0 0 1
 total NATs = 5
L= 8
 1.00000000 1(2)48 46 1 47
 2.00000001 1(16)72 56 1 8
 3.00000011 1(16)48 32 1 5
 4.01111000 1(16)40 24 1 3
 5.00001001 1(16)24 8 1 2
 6.00010001 1(8)8 0 0 1
 7.00010011 1(16)16 0 0 1
 total NATs = 7
L= 9
 1.000000000 1(2)77 75 1 76
 2.000000001 1(18)126 108 1 13
 3.000000011 1(18)81 63 1 8
 4.011111000 1(18)63 45 1 5
 5.000001001 1(18)36 18 1 3
 6.011110000 1(9)27 18 1 3
 7.000010001 1(18)27 9 1 2
 8.000010011 1(18)27 9 1 2
 9.000011001 1(18)27 9 1 2
 10.000100011 1(18)18 0 0 1
 11.001001001 1(3)3 0 0 1
 total NATs = 11
L= 10
 1.0000000000 1(2)124 122 1 123
 2.0000000001 1(20)220 200 1 21
 3.0000000011 1(20)140 120 1 13
 4.0111111000 1(20)100 80 1 8
 5.0000001001 1(20)60 40 1 5
 6.0111110000 1(20)80 60 1 5
 7.0000010001 1(20)40 20 1 3
 8.0000010011 1(20)40 20 1 3
 9.0000011001 1(20)40 20 1 3
 10.0000100001 1(10)25 15 1 4
 11.0000100011 1(20)30 10 1 2
 12.0111001000 1(20)40 20 1 2
 13.0000110001 1(20)30 10 1 2
 14.0000110011 1(20)30 10 1 2
 15.0001001001 1(20)20 0 0 1
 16.0001100011 1(5)5 0 0 1
 total NATs = 16
L= 11
 1.00000000000 1(2)200 198 1 199
 2.00000000001 1(22)385 363 1 34
 3.00000000011 1(22)242 220 1 21
 4.01111111000 1(22)165 143 1 13
 5.00000001001 1(22)99 77 1 8
 6.01111110000 1(22)121 99 1 8
 7.00000010001 1(22)66 44 1 5
 8.00000010011 1(22)66 44 1 5
 9.01111100000 1(11)55 44 1 5
 10.00000011001 1(22)66 44 1 5
 11.00000100001 1(22)77 55 1 6
 12.00000100011 1(22)44 22 1 3
 13.01111001000 1(22)55 33 1 3
 14.01111000010 1(22)55 33 1 3
 15.00000110001 1(22)44 22 1 3
 16.00000110011 1(22)44 22 1 3
 17.00001000011 1(22)55 33 1 4
 18.01110011000 1(22)44 22 1 2
 19.00001001001 1(22)33 11 1 2
 20.01110000110 1(22)44 22 1 2
 21.00001100011 1(22)33 11 1 2
 22.00010001001 1(22)22 0 0 1
 23.00010010011 1(22)22 0 0 1
 24.00010011001 1(11)11 0 0 1
 total NATs = 24
L= 12
 1.000000000000 1(2)323 321 1 322
 2.000000000001 1(24)672 648 1 55
 3.000000000011 1(24)420 396 1 34
 4.011111111000 1(24)276 252 1 21
 5.000000001001 1(24)168 144 1 13
 6.011111110000 1(24)192 168 1 13
 7.000000010001 1(24)108 84 1 8
 8.000000010011 1(24)108 84 1 8
 9.011111100000 1(24)156 132 1 8
 10.000000011001 1(24)108 84 1 8
 11.000000100001 1(24)132 108 1 10
 12.000000100011 1(24)72 48 1 5
 13.011111001000 1(24)84 60 1 5
 14.011111000010 1(24)84 60 1 5
 15.000000110001 1(24)72 48 1 5

 30.000010010011 1(24)36 12 1 2
 31.000010011001 1(24)36 12 1 2
 32.011100001100 1(24)48 24 1 2
 33.000011000011 1(12)30 18 1 4
 34.000011001001 1(24)36 12 1 2
 35.000100010001 1(8)8 0 0 1
 36.000100010011 1(24)24 0 0 1
 37.000100011001 1(24)24 0 0 1
 38.000100100011 1(24)24 0 0 1
 39.001001001001 1(3)3 0 0 1
 total NATs = 39
```

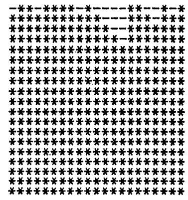
λ ratio = .5   Z = .5

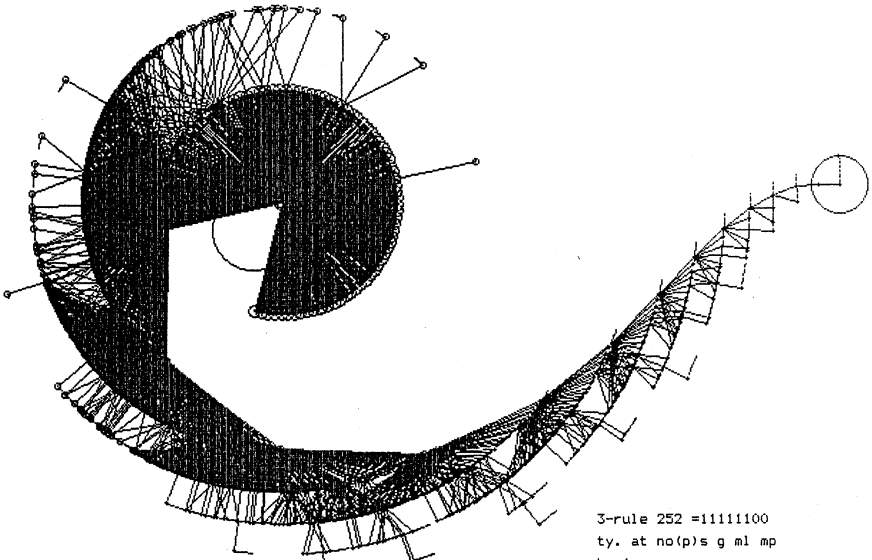
11111111111110000-11111111111110000-rule 4293984240
=3-rule 252 -11111100
Length=1 -15



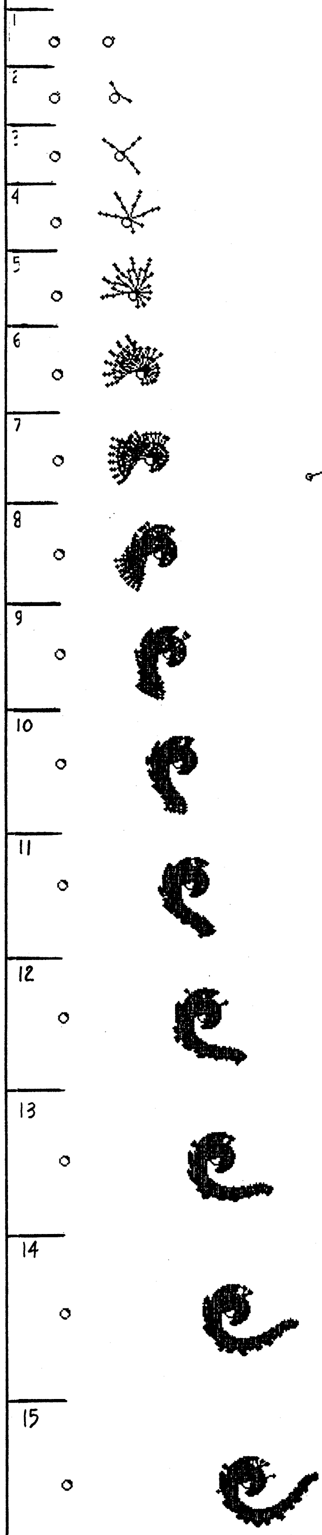
11111100	252
11000000	192
10001000	136
11101110	238





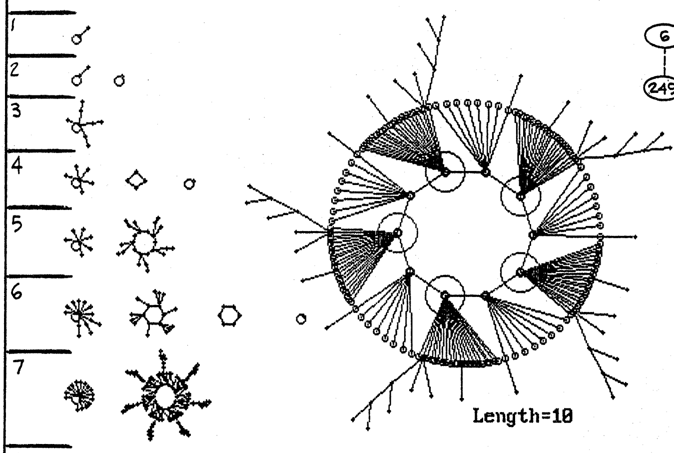


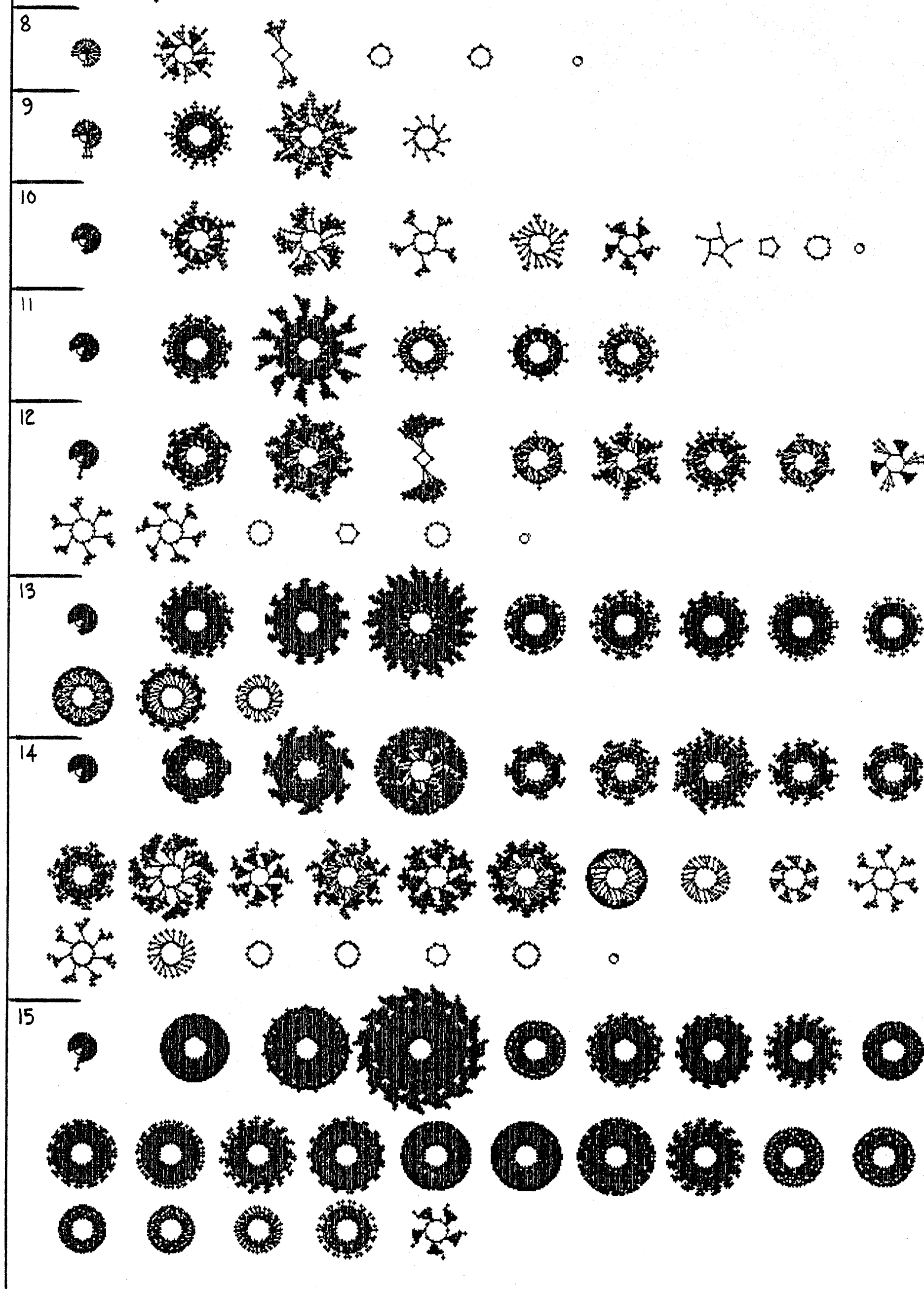
Length=15

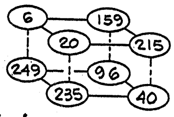



```
3-rule 252 =11111100

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.11 1(1)3 2 1 3
 total NATs = 2
L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)7 3 2 4
 total NATs = 2
L= 4
 1.0000 1(1)1 0 0 1
 2.1111 1(1)15 6 3 7
 total NATs = 2
L= 5
 1.00000 1(1)1 0 0 1
 2.11111 1(1)31 15 4 11
 total NATs = 2
L= 6
 1.000000 1(1)1 0 0 1
 2.111111 1(1)63 35 5 18
 total NATs = 2
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111111 1(1)127 77 6 29
 total NATs = 2
L= 8
 1.00000000 1(1)1 0 0 1
 2.11111111 1(1)255 166 7 47
 total NATs = 2
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111111 1(1)511 354 8 76
 total NATs = 2
L= 10
 1.0000000000 1(1)1 0 0 1
 2.1111111111 1(1)1023 747 9 123
 total NATs = 2
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111111 1(1)2047 1562 10 199
 total NATs = 2
L= 12
 1.000000000000 1(1)1 0 0 1
 2.111111111111 1(1)4095 3243 11 322
 total NATs = 2
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.1111111111111 1(1)8191 6695 12 521
 total NATs = 2
L= 14
 1.00000000000000 1(1)1 0 0 1
 2.11111111111111 1(1)16383 13757 13 843
 total NATs = 2
L= 15
 1.000000000000000 1(1)1 0 0 1
 2.111111111111111 1(1)32767 28158 14 1364
 total NATs = 2
```

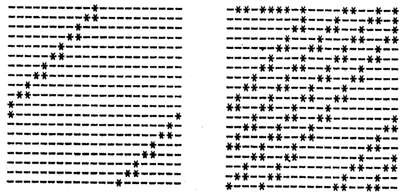
λ ratio = .5   Z = .5   0000000000111100-0000000000111100-rule   3932220
=3-rule 6 -00000110   Length=1 -14





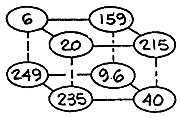


00000110	6
10011111	159
11010111	215
00010100	20

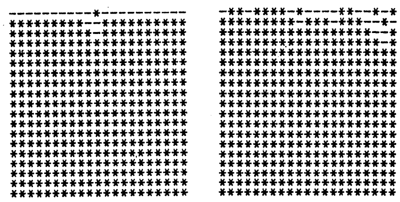


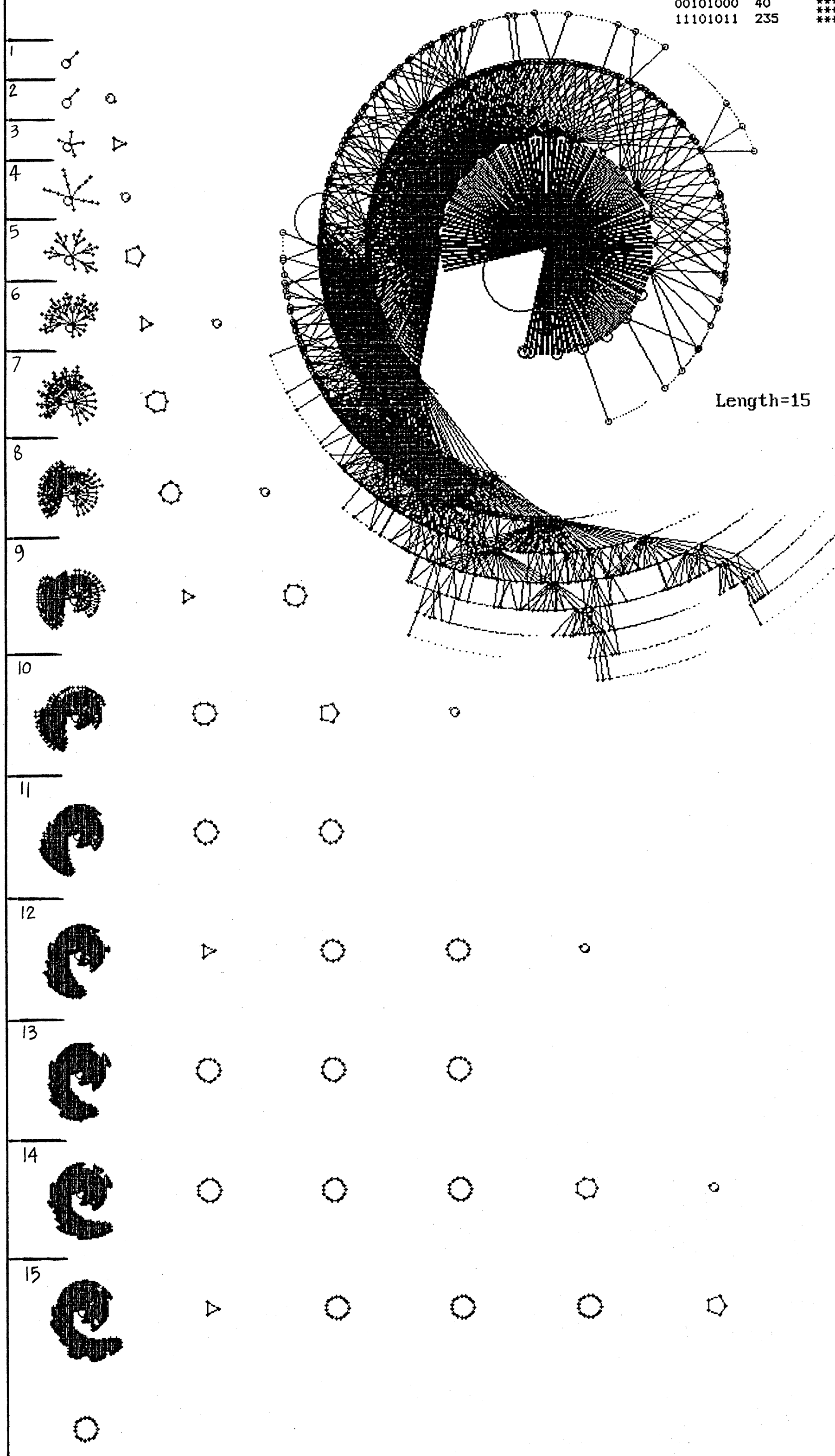
```
3-rule 6 =00000110
ty. at no(p)s g ml mp
L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)2 1 1 2
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)8 4 2 5
 total NATs = 1
L= 4
 1.0000 1(1)6 5 1 6
 2.0001 2(4)4 0 0 1
 3.0101 2(1)1 0 0 1
 total NATs = 5
L= 5
 1.00000 1(1)7 6 1 7
 2.00001 1(10)25 10 2 3
 total NATs = 2
L= 6
 1.000000 1(1)14 10 2 11
 2.000001 2(6)18 12 1 4
 3.000101 2(6)6 0 0 1
 4.010101 2(1)1 0 0 1
 total NATs = 7
L= 7
 1.0000000 1(1)16 15 1 16
 2.0000001 1(14)112 63 6 6
 total NATs = 2
L= 8
 1.00000000 1(1)22 21 1 22
 2.00000001 2(8)72 48 3 10
 3.00010001 2(4)32 20 3 4
 4.00010101 2(8)8 0 0 1
 5.00011001 1(8)8 0 0 1
 6.01010101 2(1)1 0 0 1
 total NATs = 10
L= 9
 1.000000000 1(1)35 31 2 32
 2.000000001 1(18)225 180 3 15
 3.000010001 1(18)225 117 6 6
 4.000011001 1(18)27 9 1 2
 total NATs = 4
L= 10
 1.0000000000 1(1)47 46 1 47
 2.0000000001 2(10)200 165 4 22
 3.0000010001 2(10)150 95 4 9
 4.0001000101 2(10)45 25 3 4
 5.0000011001 2(10)40 25 2 3
 6.0000100001 1(10)55 40 2 9
 7.0000110001 2(5)10 5 1 2
 8.0001001101 2(5)5 0 0 1
 9.0001010101 2(10)10 0 0 1
 10.0101010101 2(1)1 0 0 1
 total NATs = 18
L= 11
 1.00000000000 1(1)68 67 1 68
 2.00000000001 1(22)638 550 3 33
 3.00000010001 1(22)946 594 10 14
 4.00000011001 1(22)110 77 2 5
 5.00000100001 1(22)165 132 2 12
 6.00001100001 1(22)121 88 2 6
 total NATs = 6
L= 12
 1.000000000000 1(1)102 98 2 99
 2.000000000001 2(12)504 462 3 49
 3.000000010001 2(12)528 390 6 21
 4.000100010001 2(4)280 180 6 9
 5.000000011001 2(12)84 66 2 8
 6.000000100001 2(12)210 162 4 19
 7.000001100001 1(12)168 120 3 9
 8.000000110001 2(12)90 72 2 8
 9.000001000001 2(6)66 54 2 16
 10.000100010101 2(12)54 30 3 4
 11.000110010001 2(12)66 42 3 6
 12.000100110101 2(12)12 0 0 1
 13.000101000101 2(6)6 0 0 1
 14.000101010101 2(12)12 0 0 1
 15.010101010101 2(1)1 0 0 1
 total NATs = 28
L= 13
 1.0000000000000 1(1)145 144 1 145
 2.0000000000001 1(26)1638 1521 4 72
 3.0000000010001 1(26)1560 1287 5 31
 4.0000100010001 1(26)2093 1313 10 14
 5.0000000011001 1(26)325 273 2 12
 6.0000000100001 1(26)546 468 3 30
 7.0000001000001 1(26)650 533 3 26
 8.0000001100001 1(26)507 390 3 14
 9.0000000110001 1(26)260 221 2 10
 10.0000110010001 1(26)182 130 2 8
 11.0001000100011 1(26)234 182 3 12
 12.0000100011001 1(26)52 26 1 3
 total NATs = 12
L= 14
 1.00000000000000 1(1)212 211 1 212
 2.00000000000001 2(14)1288 1218 3 106
 3.00000000010001 2(14)1386 1183 6 46
 4.00000100010001 2(14)1281 917 6 21
 5.00000000011001 2(14)245 217 2 18
 6.00000000100001 2(14)427 385 3 44
 7.00000001000001 1(14)826 679 6 42
 8.00000001100001 2(14)378 315 3 22
 9.00000000110001 2(14)252 224 2 16
 10.00000001000001 2(14)420 378 3 41
 11.00010001000101 2(14)385 245 6 9
 12.00000011000001 2(7)182 154 3 19
 13.00000110010001 2(14)259 189 5 12
 14.00001000010001 2(14)364 273 4 15
 15.00001000100011 2(14)336 245 4 18
 16.00001100010001 2(14)168 140 2 16
 17.00000100010011 2(14)42 28 1 4
 18.00001100100001 2(14)49 35 1 6
 19.00010001001001 2(14)63 35 3 4
 20.00010001001101 2(14)63 35 3 4
 21.00011000100001 2(14)35 21 1 3
 22.00010011010101 2(14)14 0 0 1
 23.00010100010101 2(14)14 0 0 1
 24.00010100110101 2(7)7 0 0 1
 25.00010101010101 2(14)14 0 0 1
 26.01010101010101 2(1)1 0 0 1
 total NATs = 50
```

λ ratio = .5   Z = .5   1111111111000011-1111111111000011-rule 4291035075
=3-rule 249 -11111001   Length=1 -15




```
11111001  249
01100000  96
00101000  40
11101011  235
```

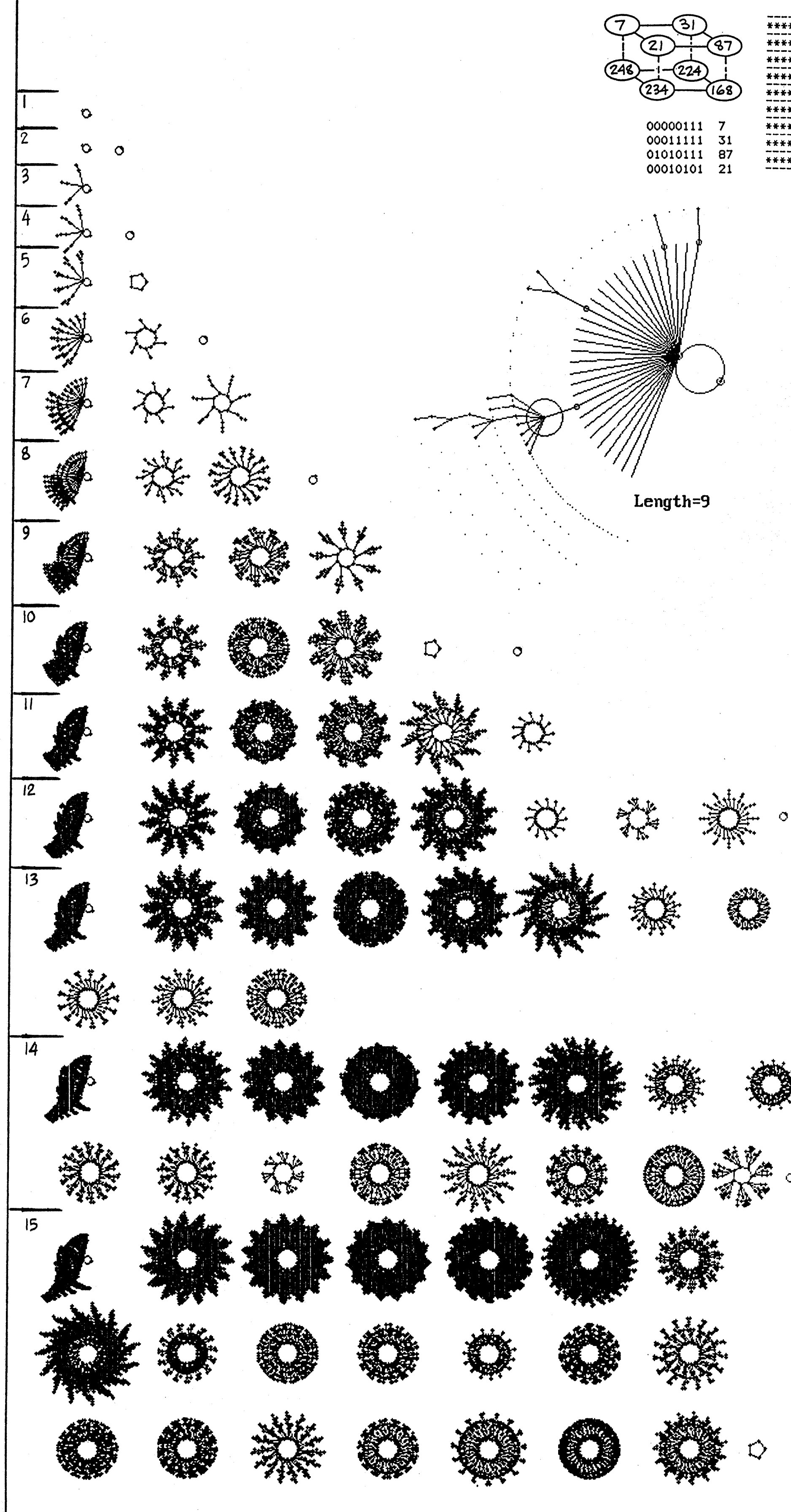


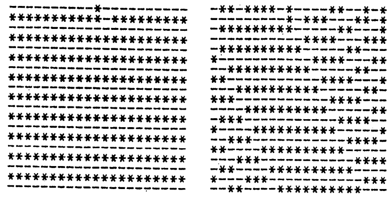


Length=15

```
3-rule 249 =11111001

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.111 1(1)5 4 1 5
 2.001 1(3)3 0 0 1
 total NATs = 2
L= 4
 1.1111 1(1)14 5 3 6
 2.0101 1(2)2 0 0 1
 total NATs = 2
L= 5
 1.11111 1(1)27 16 3 7
 2.00101 1(5)5 0 0 1
 total NATs = 2
L= 6
 1.111111 1(1)59 34 4 11
 2.001001 1(3)3 0 0 1
 3.010101 1(2)2 0 0 1
 total NATs = 3
L= 7
 1.1111111 1(1)121 78 5 16
 2.0010101 1(7)7 0 0 1
 total NATs = 2
L= 8
 1.11111111 1(1)246 157 5 22
 2.00100101 1(8)8 0 0 1
 3.01010101 1(2)2 0 0 1
 total NATs = 3
L= 9
 1.111111111 1(1)500 337 6 32
 2.001001001 1(3)3 0 0 1
 3.001010101 1(9)9 0 0 1
 total NATs = 3
L= 10
 1.1111111111 1(1)1007 716 7 47
 2.0010010101 1(10)10 0 0 1
 3.0010100101 1(5)5 0 0 1
 4.0101010101 1(2)2 0 0 1
 total NATs = 4
L= 11
 1.11111111111 1(1)2026 1508 7 68
 2.00100100101 1(11)11 0 0 1
 3.00101010101 1(11)11 0 0 1
 total NATs = 3
L= 12
 1.111111111111 1(1)4067 3134 8 99
 2.001001001001 1(3)3 0 0 1
 3.001001010101 1(12)12 0 0 1
 4.001010010101 1(12)12 0 0 1
 5.010101010101 1(2)2 0 0 1
 total NATs = 5
L= 13
 1.1111111111111 1(1)8153 6488 9 145
 2.0010010010101 1(13)13 0 0 1
 3.0010010100101 1(13)13 0 0 1
 4.0010101010101 1(13)13 0 0 1
 total NATs = 4
L= 14
 1.11111111111111 1(1)16333 13364 9 212
 2.00100100100101 1(14)14 0 0 1
 3.00100101010101 1(14)14 0 0 1
 4.00101001010101 1(14)14 0 0 1
 5.00101010010101 1(7)7 0 0 1
 6.01010101010101 1(2)2 0 0 1
 total NATs = 6
L= 15
 1.111111111111111 1(1)32700 27424 10 310
 2.001001001001001 1(3)3 0 0 1
 3.001001001010101 1(15)15 0 0 1
 4.001001010010101 1(15)15 0 0 1
 5.001001010100101 1(15)15 0 0 1
 6.001010010100101 1(5)5 0 0 1
 7.001010101010101 1(15)15 0 0 1
 total NATs = 7
```

λ ratio = .75 $Z = .75$ 0000000000111111-0000000000111111-rule 4128831 =3-rule 7 -00000111 Length=1 -15





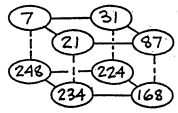
3-rule 7 =00000111

ty. at no(p)s g ml mp

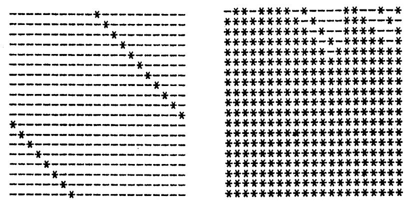
```
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(2)8 3 2 4
 total NATs = 1
L= 4
 1.0000 1(2)14 4 3 5
 2.0101 2(1)1 0 0 1
 total NATs = 3
L= 5
 1.00000 1(2)27 10 4 6
 2.00011 1(5)5 0 0 1
 total NATs = 2
L= 6
 1.000000 1(2)44 15 5 10
 2.000011 1(12)18 6 1 2
 3.010101 2(1)1 0 0 1
 total NATs = 4
L= 7
 1.0000000 1(2)79 35 6 15
 2.0000011 1(14)21 7 1 2
 3.0000111 1(7)28 7 3 2
 total NATs = 3
L= 8
 1.00000000 1(2)134 60 7 21
 2.00000011 1(16)40 16 2 3
 3.00000111 1(16)80 32 3 2
 4.01010101 2(1)1 0 0 1
 total NATs = 5
L= 9
 1.000000000 1(2)233 120 8 31
 2.000000011 1(18)81 36 3 5
 3.000000111 1(18)117 54 3 3
 4.000001111 1(9)81 27 5 2
 total NATs = 4
L= 10
 1.0000000000 1(2)397 210 9 46
 2.0000000011 1(20)160 80 4 7
 3.0000000111 1(20)220 110 3 5
 4.0000001111 1(20)240 110 5 3
 5.0001100011 1(5)5 0 0 1
 6.0101010101 2(1)1 0 0 1
 total NATs = 7
L= 11
 1.00000000000 1(2)684 396 10 67
 2.00000000011 1(22)297 154 5 10
 3.00000000111 1(22)429 231 4 7
 4.00000001111 1(22)385 198 5 5
 5.01111100000 1(11)220 99 7 3
 6.00001100011 1(22)33 11 1 2
 total NATs = 6
L= 12
 1.000000000000 1(2)1172 703 11 98
 2.000000000011 1(24)552 312 6 15
 3.000000000111 1(24)804 456 5 10
 4.000000001111 1(24)720 396 5 7
 5.011111100000 1(24)708 384 7 5
 6.000001100011 1(24)36 12 1 2
 7.000011000011 1(12)30 18 1 4
 8.000011000111 1(24)72 24 3 2
 9.010101010101 2(1)1 0 0 1
 total NATs = 10
L= 13
 1.0000000000000 1(2)2017 1287 12 144
 2.0000000000011 1(26)1027 611 7 22
 3.0000000000111 1(26)1469 871 6 15
 4.0000000001111 1(26)1352 780 5 10
 5.0111111100000 1(26)1222 728 7 7
 6.0011111100000 1(13)637 351 9 5
 7.0000001100011 1(26)65 26 2 3
 8.0000011000011 1(26)65 39 1 4
 9.0000011000111 1(26)91 39 3 2
 10.0000011100011 1(26)91 39 3 2
 11.0000110000111 1(26)156 78 3 4
 total NATs = 11
L= 14
 1.00000000000000 1(2)3467 2289 13 211
 2.00000000000011 1(28)1904 1204 8 32
 3.00000000000111 1(28)2688 1680 7 22
 4.00000000001111 1(28)2478 1498 6 15
 5.01111111100000 1(28)2240 1386 7 10
 6.00111111100000 1(28)2198 1358 9 7
 7.00000001100011 1(28)126 56 3 5
 8.00000011000011 1(28)112 70 2 6
 9.00000011000111 1(28)140 70 3 3
 10.00000011100011 1(28)140 70 3 3
 11.00000110000011 1(14)35 21 1 4
 12.00000110000111 1(28)196 112 3 4
 13.00000110001111 1(28)140 56 5 2
 14.10000011100001 1(28)196 112 3 4
 15.00000111000111 1(28)224 112 3 4
 16.10000111000011 1(7)98 49 3 4
 17.01010101010101 2(1)1 0 0 1
 total NATs = 18
L= 15
 1.000000000000000 1(2)5973 4123 14 309
 2.000000000000011 1(30)3510 2310 9 47
 3.000000000000111 1(30)4920 3210 8 32
 4.000000000001111 1(30)4485 2820 7 22
 5.011111111100000 1(30)4110 2655 7 15
 6.001111111100000 1(30)3870 2520 9 10
 7.000000001100011 1(30)240 120 4 7
 8.001111111000000 1(15)1995 1275 11 7
 9.000000011000011 1(30)225 150 3 10
 10.000000011000111 1(30)255 135 3 5
 11.000000011100011 1(30)255 135 3 5
 12.000000110000011 1(30)120 75 2 6
 13.000000110000111 1(30)300 180 3 6
 14.000000110001111 1(30)210 105 5 3
 15.100000011100001 1(30)300 180 3 6
 16.000000111000111 1(30)330 195 3 4
 17.000000111100011 1(30)210 105 5 3
 18.000001100000111 1(30)240 150 3 4
 19.000001100001111 1(30)330 180 5 4
 20.100000111000011 1(30)555 315 3 4
 21.000001110001111 1(30)330 180 5 4
 22.000110001100011 1(5)5 0 0 1
 total NATs = 22
```

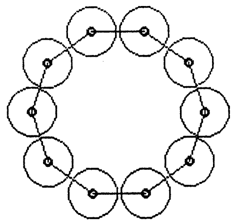
λ ratio = .75   Z = .75

1111111111000000-1111111111000000-rule 4290838464
=3-rule 248 -11111000   Length=1 -15

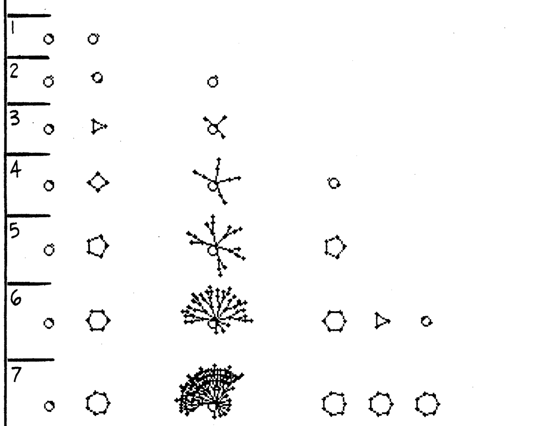


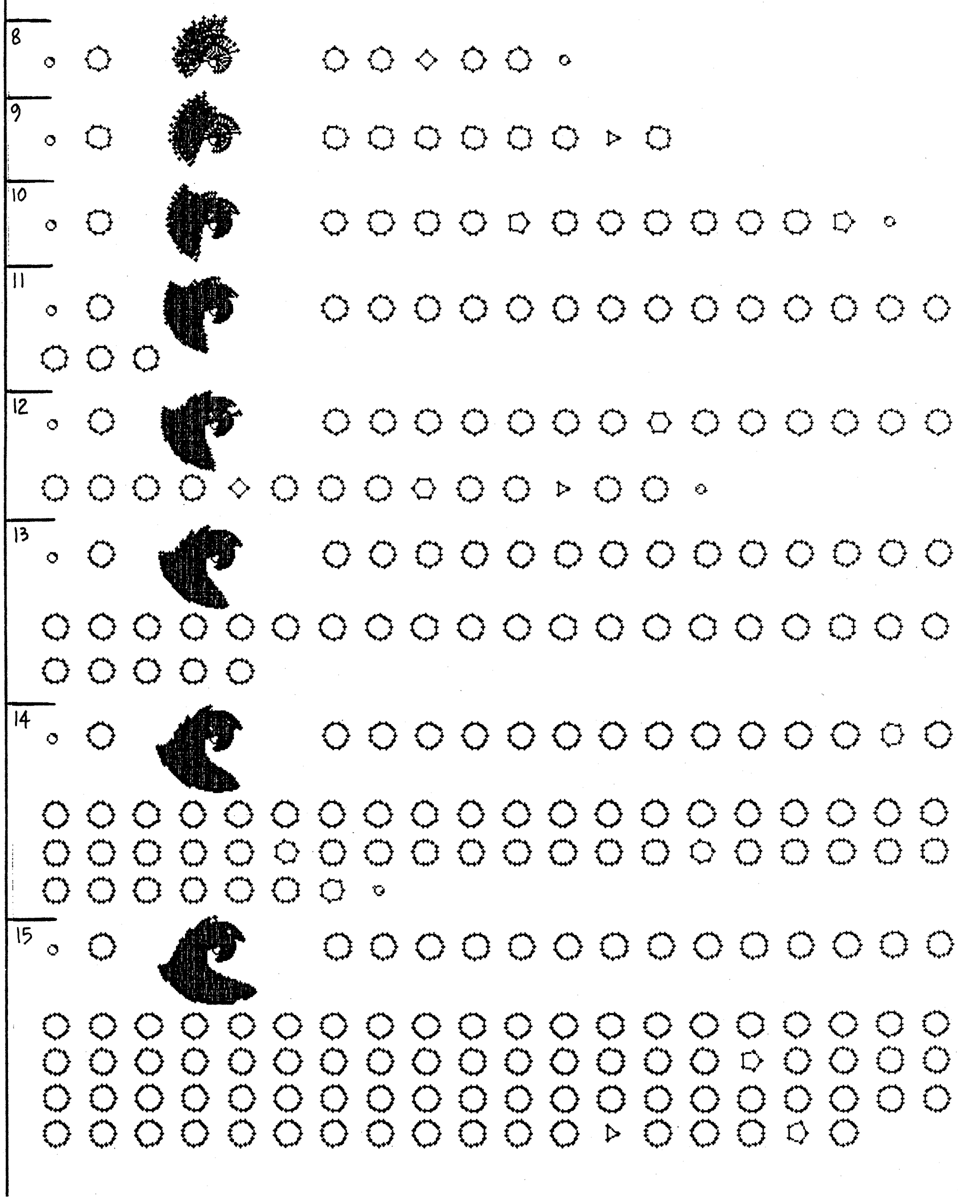
11111000 248
11100000 224
10101000 168
11101010 234





Length=10





3-rule 248 =11111000

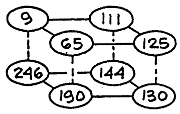
ty. at no(p)s g ml mp

L= 1
1.0 1(1)1 0 0 1
2.1 1(1)1 0 0 1
total NATs = 2
L= 2
1.00 1(1)1 0 0 1
2.01 1(2)2 0 0 1
3.11 1(1)1 0 0 1
total NATs = 3
L= 3
1.000 1(1)1 0 0 1
2.001 1(3)3 0 0 1
3.111 1(1)4 3 1 4
total NATs = 3
L= 4
1.0000 1(1)1 0 0 1
2.0001 1(4)4 0 0 1
3.1111 1(1)9 4 2 5
4.0101 1(2)2 0 0 1
total NATs = 4
L= 5
1.00000 1(1)1 0 0 1
2.00001 1(5)5 0 0 1
3.11111 1(1)21 10 3 6
4.00101 1(5)5 0 0 1
total NATs = 4
L= 6
1.000000 1(1)1 0 0 1
2.000001 1(6)6 0 0 1
3.111111 1(1)46 21 4 10
4.000101 1(6)6 0 0 1
5.001001 1(3)3 0 0 1
6.010101 1(2)2 0 0 1
total NATs = 6
L= 7
1.0000000 1(1)1 0 0 1
2.0000001 1(7)7 0 0 1
3.1111111 1(1)99 49 5 15
4.0000101 1(7)7 0 0 1
5.0001001 1(7)7 0 0 1
6.0010101 1(7)7 0 0 1
total NATs = 6
L= 8
1.00000000 1(1)1 0 0 1
2.00000001 1(8)8 0 0 1
3.11111111 1(1)209 108 6 21
4.00000101 1(8)8 0 0 1
5.00001001 1(8)8 0 0 1
6.00010001 1(4)4 0 0 1
7.00010101 1(8)8 0 0 1
8.00100101 1(8)8 0 0 1
9.01010101 1(2)2 0 0 1
total NATs = 9
L= 9
1.000000000 1(1)1 0 0 1
2.000000001 1(9)9 0 0 1
3.111111111 1(1)436 237 7 31
4.000000101 1(9)9 0 0 1
5.000001001 1(9)9 0 0 1
6.000010001 1(9)9 0 0 1
7.000010101 1(9)9 0 0 1
8.000100101 1(9)9 0 0 1
9.000101001 1(9)9 0 0 1
10.001001001 1(3)3 0 0 1
11.001010101 1(9)9 0 0 1
total NATs = 11
L= 10
1.0000000000 1(1)1 0 0 1
2.0000000001 1(10)10 0 0 1
3.1111111111 1(1)901 510 8 46
4.0000000101 1(10)10 0 0 1
5.0000001001 1(10)10 0 0 1
6.0000010001 1(10)10 0 0 1
7.0000010101 1(10)10 0 0 1
8.0000100001 1(5)5 0 0 1
9.0000100101 1(10)10 0 0 1
10.0000101001 1(10)10 0 0 1
11.0001000101 1(10)10 0 0 1
12.0001001001 1(10)10 0 0 1
13.0001010101 1(10)10 0 0 1
14.0010010101 1(10)10 0 0 1
15.0010100101 1(5)5 0 0 1
16.0101010101 1(2)2 0 0 1
total NATs = 16
L= 11
1.00000000000 1(1)1 0 0 1
2.00000000001 1(11)11 0 0 1
3.11111111111 1(1)1849 1089 9 67
4.00000000101 1(11)11 0 0 1
5.00000001001 1(11)11 0 0 1
6.00000010001 1(11)11 0 0 1
7.00000010101 1(11)11 0 0 1
8.00000100001 1(11)11 0 0 1
9.00000100101 1(11)11 0 0 1
10.00000101001 1(11)11 0 0 1
11.00001000101 1(11)11 0 0 1
12.00001001001 1(11)11 0 0 1
13.00001010001 1(11)11 0 0 1
14.00001010101 1(11)11 0 0 1
15.00010001001 1(11)11 0 0 1
16.00010010101 1(11)11 0 0 1
17.00010100101 1(11)11 0 0 1
18.00010101001 1(11)11 0 0 1
19.00100100101 1(11)11 0 0 1
20.00101010101 1(11)11 0 0 1
total NATs = 20
L= 12
1.000000000000 1(1)1 0 0 1
2.000000000001 1(12)12 0 0 1
3.111111111111 1(1)3774 2305 10 98
4.000000000101 1(12)12 0 0 1
5.000000001001 1(12)12 0 0 1
6.000000010001 1(12)12 0 0 1
7.000000010101 1(12)12 0 0 1
8.000000100001 1(12)12 0 0 1
9.000000100101 1(12)12 0 0 1
10.000000101001 1(12)12 0 0 1

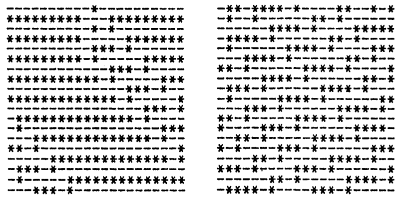
20.000010100101 1(12)12 0 0 1
21.000010101001 1(12)12 0 0 1
22.000100010001 1(4)4 0 0 1
23.000100010101 1(12)12 0 0 1
24.000100100101 1(12)12 0 0 1
25.000100101001 1(12)12 0 0 1
26.000101000101 1(6)6 0 0 1
27.000101001001 1(12)12 0 0 1
28.000101010101 1(12)12 0 0 1
29.001001001001 1(3)3 0 0 1
30.001001010101 1(12)12 0 0 1
31.001010010101 1(12)12 0 0 1
32.010101010101 1(2)2 0 0 1
total NATs = 32

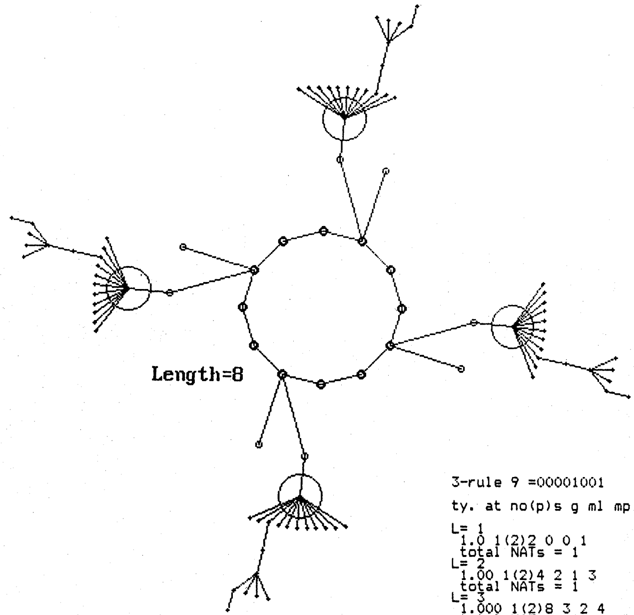
λ ratio = .5   Z = .5

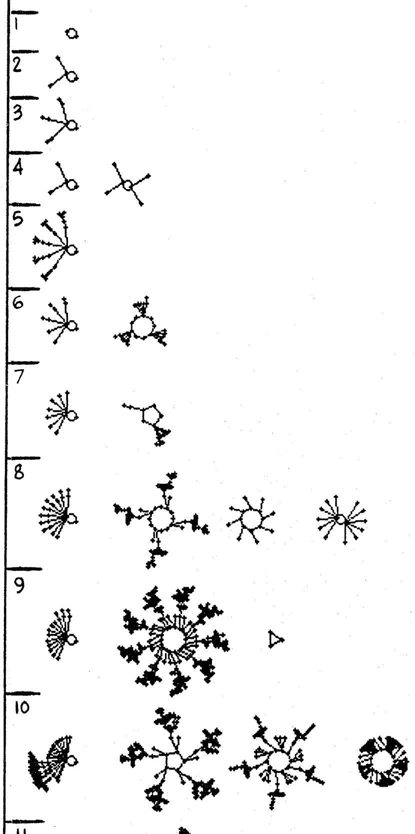
0000000011000011-0000000011000011-rule 12779715
=3-rule 9 -00001001   Length=1 -15

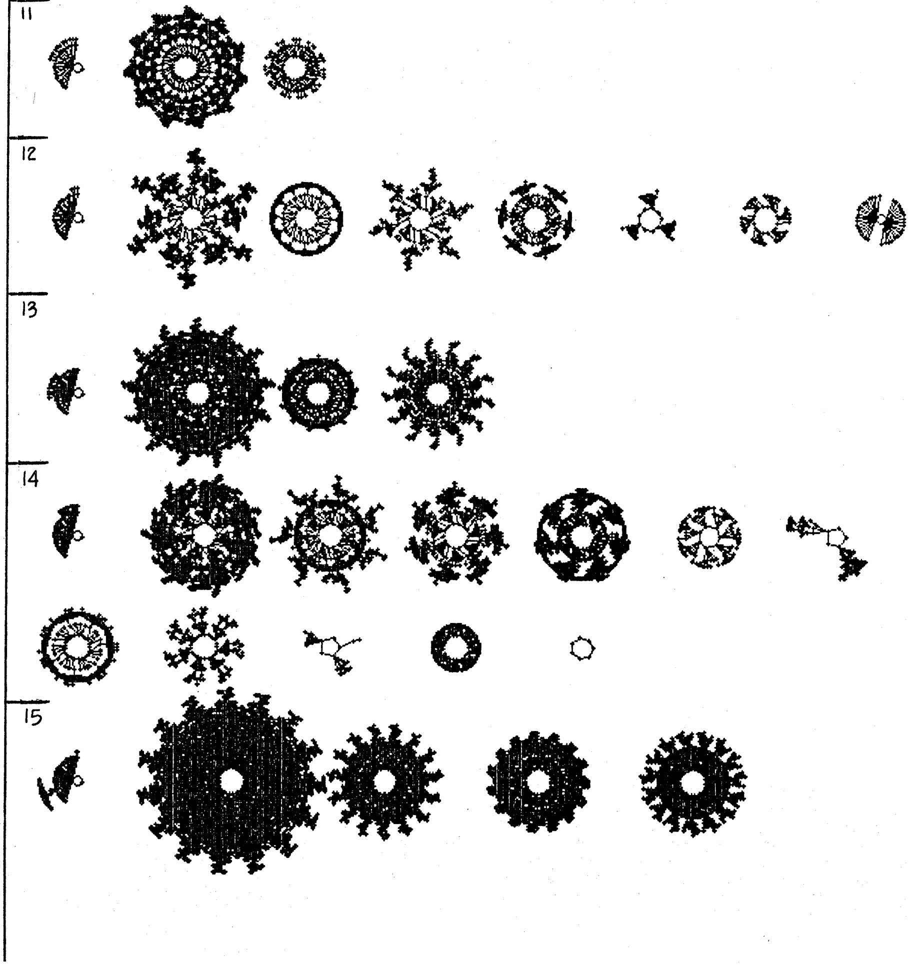


00001001 9
01101111 111
01111101 125
01000001 65



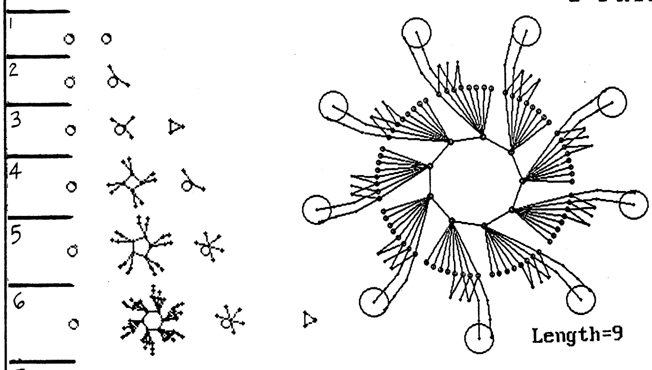


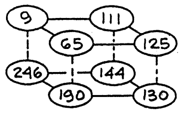




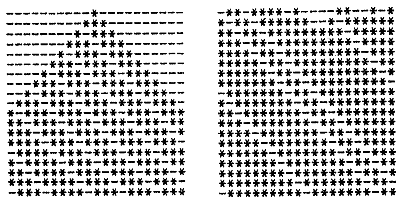

```
3-rule 9 =00001001

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)4 2 1 3
 total NATs = 1
L= 3
 1.000 1(2)8 3 2 4
 total NATs = 1
L= 4
 1.0000 1(2)4 2 1 3
 2.0001 2(2)6 4 1 3
 total NATs = 3
L= 5
 1.00000 1(2)32 15 4 6
 total NATs = 1
L= 6
 1.000000 1(2)10 5 2 6
 2.000001 2(9)27 15 2 6
 total NATs = 3
L= 7
 1.0000000 1(2)9 7 1 8
 2.0000001 7(5)17 10 2 8
 total NATs = 8
L= 8
 1.00000000 1(2)20 10 2 11
 2.01000001 2(12)96 60 7 11
 3.00001101 1(8)16 8 1 2
 4.00010001 2(2)14 12 1 7
 total NATs = 6
L= 9
 1.000000000 1(2)17 12 2 13
 2.110100001 1(18)486 333 9 16
 3.000010011 3(3)3 0 0 1
 total NATs = 5
L= 10
 1.0000000000 1(2)124 87 5 18
 2.1101000001 2(5)250 175 8 21
 3.0100001111 2(10)100 60 7 9
 4.0000010001 2(15)100 85 1 15
 total NATs = 7
L= 11
 1.00000000000 1(2)35 22 2 23
 2.11010000001 1(22)1892 1430 11 29
 3.00000001101 1(22)121 77 2 6
 total NATs = 3
L= 12
 1.000000000000 1(2)34 29 2 30
 2.110100000001 2(12)1164 888 14 39
 3.010000001111 1(12)264 228 3 17
 4.000000001101 2(12)234 156 8 8
 5.010000010001 2(18)300 252 4 29
 6.000001000001 2(9)105 93 2 32
 7.000001000111 2(18)60 42 1 7
 8.000100010001 2(2)36 34 1 18
 total NATs = 14
L= 13
 1.0000000000000 1(2)54 39 2 40
 2.1101000000001 1(26)6838 5564 17 52
 3.0100000001111 1(26)572 455 4 23
 4.0000000001101 1(26)728 533 9 11
 total NATs = 4
L= 14
 1.00000000000000 1(2)67 51 2 52
 2.11010000000001 2(14)2968 2457 11 70
 3.01000000001111 2(14)812 644 10 31
 4.11110100001111 2(14)1071 875 8 56
 5.11010000010001 2(21)1092 938 6 55
 6.01000000011111 2(7)98 77 2 7
 7.01000000100000 7(5)211 172 7 66
 8.01000001010001 2(21)532 476 4 63
 9.01000001010000 2(21)245 168 4 18
 10.00000010000011 14(5)33 25 2 14
 11.00000100010001 2(21)364 343 1 39
 12.00000101000111 2(7)7 0 0 1
 total NATs = 40
L= 15
 1.000000000000000 1(2)308 288 4 69
 2.110100000000001 1(30)26700 22770 25 93
 3.010000000001111 1(30)2235 1830 10 42
 4.000000000001101 1(30)2070 1740 7 32
 5.111101000000011 1(30)1455 1155 8 19
 total NATs = 5
```

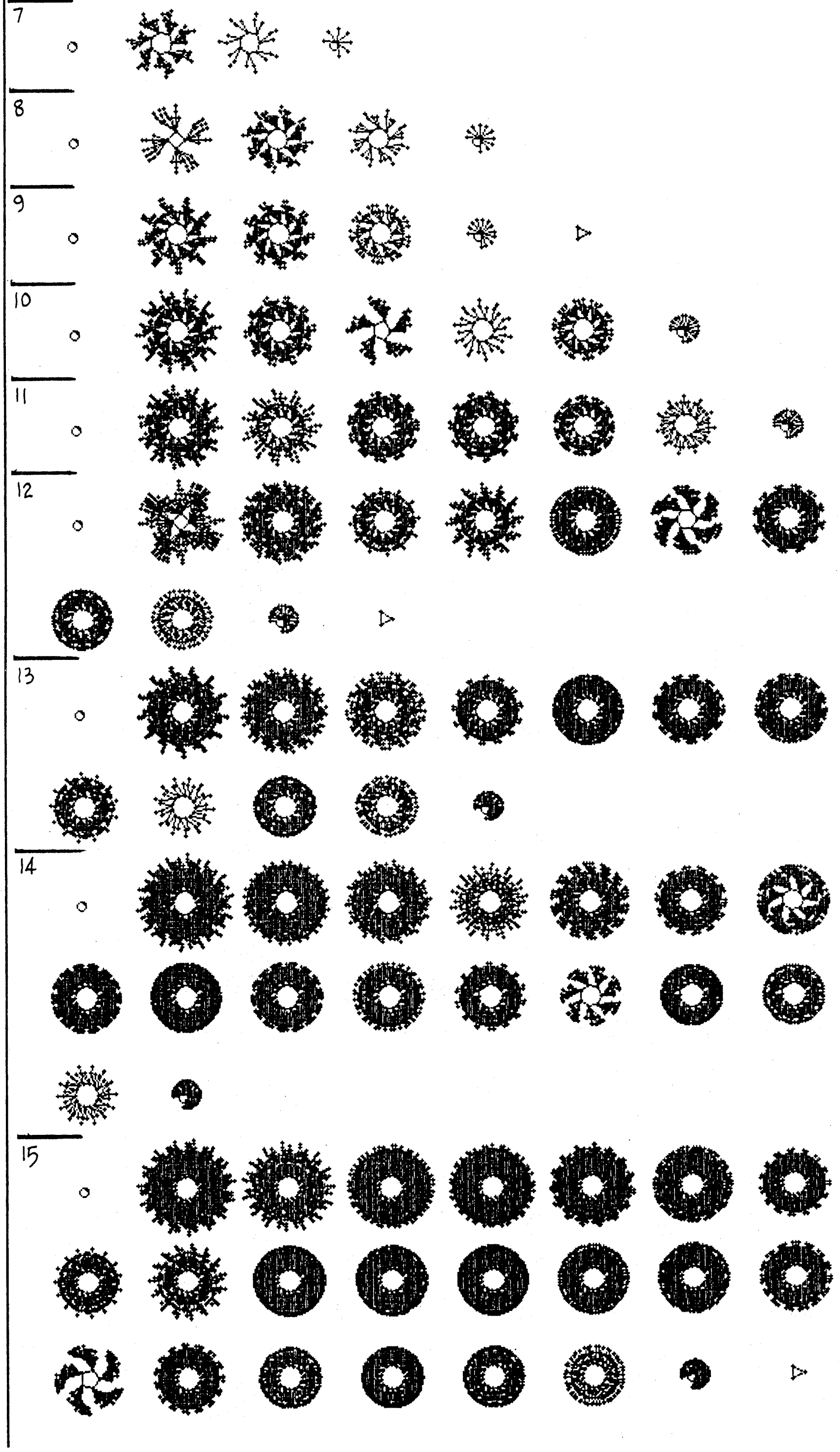
λ ratio = .5 Z = .5   1111111100111100-1111111100111100-rule 4282187580
=3-rule 246 -11110110   Length=1 -15





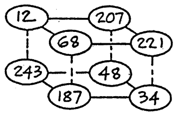
11110110	246
10010000	144
10000010	130
10111110	190



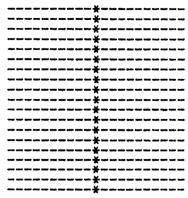


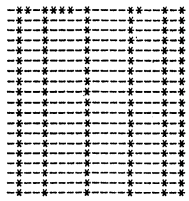

```
3-rule 246 =11110110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.11 1(1)3 2 1 3
 total NATs = 2
L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)4 3 1 4
 3.011 1(3)3 0 0 1
 total NATs = 3
L= 4
 1.0000 1(1)1 0 0 1
 2.1011 1(4)12 8 1 3
 3.1111 1(1)3 2 1 3
 total NATs = 3
L= 5
 1.00000 1(1)1 0 0 1
 2.11101 1(5)25 10 2 3
 3.11111 1(1)6 5 1 6
 total NATs = 3
L= 6
 1.000000 1(1)1 0 0 1
 2.111110 1(6)54 30 3 6
 3.111111 1(1)6 5 1 6
 4.011011 1(3)3 0 0 1
 total NATs = 4
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111110 1(7)91 56 3 8
 3.1101110 1(7)28 14 2 3
 4.1111111 1(1)8 7 1 8
 total NATs = 4
L= 8
 1.00000000 1(1)1 0 0 1
 2.11101110 1(4)60 24 3 7
 3.11111011 1(8)144 104 3 11
 4.10111101 1(8)40 24 2 4
 5.11111111 1(1)11 10 1 11
 total NATs = 5
L= 9
 1.000000000 1(1)1 0 0 1
 2.011110111 1(9)180 99 4 9
 3.111111011 1(9)234 180 3 16
 4.111110110 1(9)81 54 2 6
 5.111111111 1(1)13 12 1 13
 6.011011011 1(3)3 0 0 1
 total NATs = 6
L= 10
 1.0000000000 1(1)1 0 0 1
 2.1011111011 1(10)360 220 5 15
 3.1111110111 1(10)350 290 3 21
 4.0111101111 1(5)125 80 3 13
 5.1101110110 1(10)40 20 2 3
 6.1111101101 1(10)130 100 2 9
 7.1111111111 1(1)18 17 1 18
 total NATs = 7
L= 11
 1.00000000000 1(1)1 0 0 1
 2.10111111011 1(11)572 407 5 21
 3.10110111011 1(11)176 99 4 8
 4.01111101111 1(11)495 341 3 20
 5.11111101111 1(11)528 462 3 29
 6.11111011011 1(11)198 165 2 12
 7.10111101101 1(11)55 33 2 4
 8.11111111111 1(1)23 22 1 23
 total NATs = 8
L= 12
 1.000000000000 1(1)1 0 0 1
 2.101110111011 1(4)252 140 5 18
 3.111111101110 1(12)864 684 5 29
 4.011011101111 1(12)240 168 3 11
 5.101111011101 1(12)240 156 4 10
 6.111111011110 1(12)780 600 3 29
 7.011111011111 1(6)486 354 3 32
 8.111111011111 1(12)780 708 3 39
 9.111110110111 1(12)312 276 2 17
 10.111110110110 1(12)108 72 2 6
 11.111111111111 1(1)30 29 1 30
 12.011011011011 1(3)3 0 0 1
 total NATs = 12
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.1101111011101 1(13)1040 676 6 24
 3.1111111101110 1(13)1352 1131 5 41
 4.1011111011101 1(13)468 338 4 16
 5.0110111011111 1(13)468 338 3 17
 6.1111110111110 1(13)1521 1222 3 46
 7.1111110111111 1(13)1131 1053 3 52
 8.1111110111101 1(13)1170 975 3 40
 9.0111101111011 1(13)325 247 3 14
 10.1101110110110 1(13)52 26 2 3
 11.1111101101111 1(13)455 416 2 23
 12.1111101101101 1(13)169 130 2 9
 13.1111111111111 1(1)40 39 1 40
 total NATs = 13
L= 14
 1.00000000000000 1(1)1 0 0 1
 2.11101111101110 1(14)2016 1428 7 39
 3.11111111011101 1(14)1960 1722 5 55
 4.11011110111011 1(14)1400 1022 5 33
 5.10110111011101 1(14)224 126 4 8
 6.01111110111011 1(14)728 588 4 23
 7.01101110111111 1(14)728 588 3 25
 8.11111101111110 1(7)1183 1008 3 66
 9.11111101111011 1(14)1820 1568 3 56
 10.111111101111101 1(14)2268 1946 3 63
 11.111111011111111 1(14)1638 1554 3 70
 12.011111011111011 1(14)630 504 3 22
 13.011110111111011 1(14)630 490 3 22
 14.110111011011110 1(7)112 84 2 9
 15.111110110111111 1(14)672 630 2 31
 16.111110110101011 1(14)252 210 2 12
 17.101111101101101 1(14)70 42 2 4
 18.111111111111111 1(1)52 51 1 52
 total NATs = 18
```

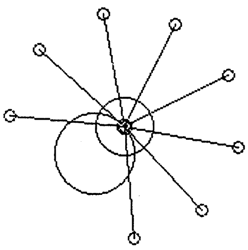
λ ratio = .5   Z = .5   0000000011110000-0000000011110000-rule 15728880
=3-rule 12 -00001100   Length=1 -15



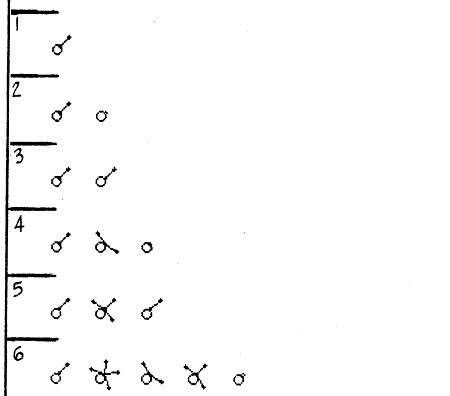
00001100	12
11001111	207
11011101	221
01000100	68

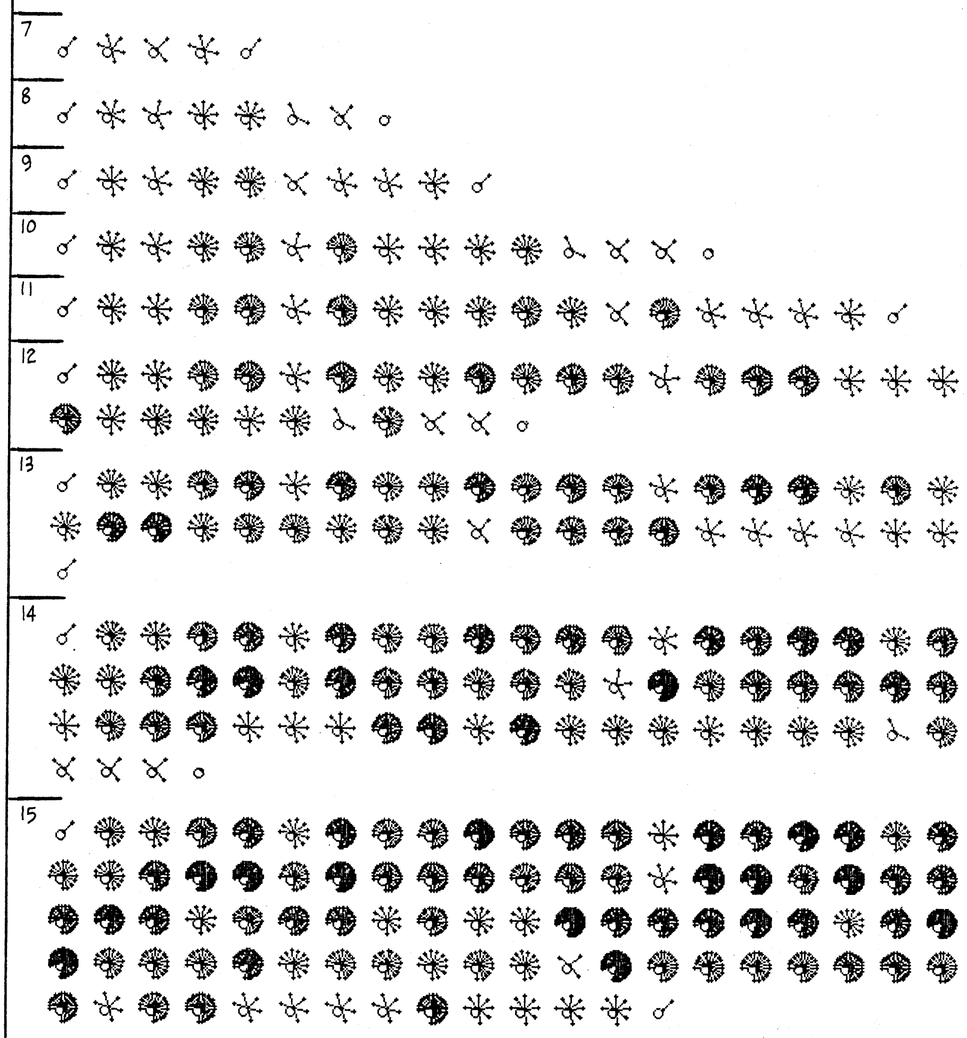






all lengths

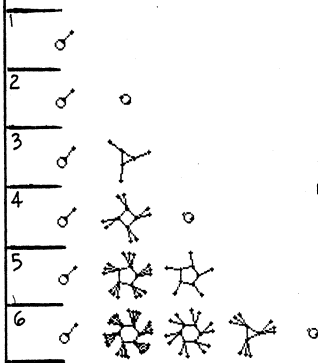


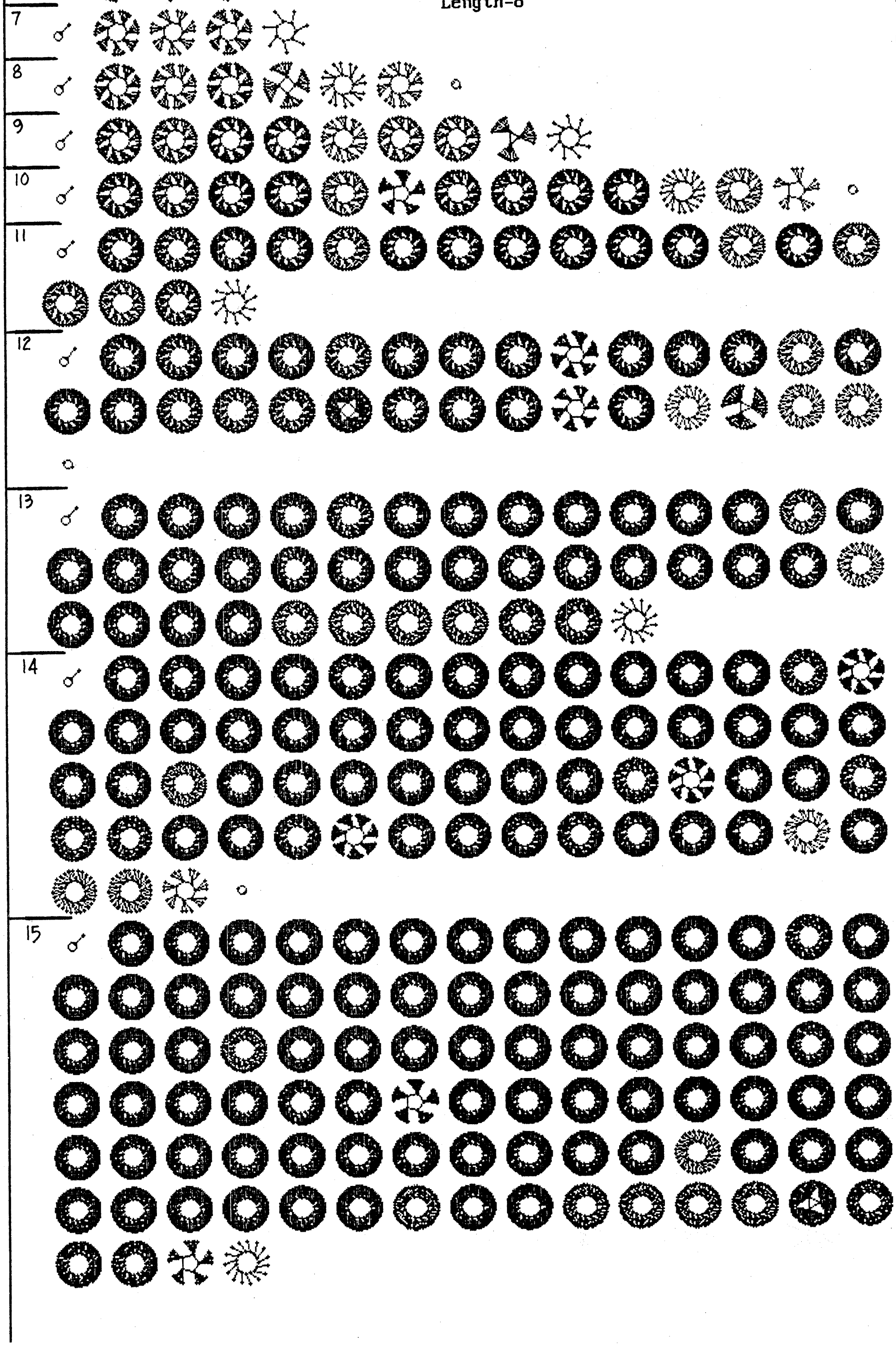


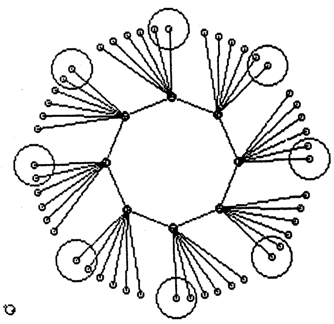

```
3-rule 12 =00001100

ty. at no(p)s g ml mp
L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)2 1 1 2
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)2 1 1 2
 2.001 3(1)2 1 1 2
 total NATs = 4
L= 4
 1.0000 1(1)2 1 1 2
 2.0001 4(1)3 2 1 3
 3.0101 2(1)1 0 0 1
 total NATs = 7
L= 5
 1.00000 1(1)2 1 1 2
 2.00001 5(1)4 3 1 4
 3.00101 5(1)2 1 1 2
 total NATs = 11
L= 6
 1.000000 1(1)2 1 1 2
 2.000001 6(1)5 4 1 5
 3.000101 6(1)3 2 1 3
 4.001001 3(1)4 3 1 4
 5.010101 2(1)1 0 0 1
 total NATs = 18
L= 7
 1.0000000 1(1)2 1 1 2
 2.0000001 7(1)6 5 1 6
 3.0000101 7(1)4 3 1 4
 4.0001001 7(1)6 5 1 6
 5.0010101 7(1)2 1 1 2
 total NATs = 29
L= 8
 1.00000000 1(1)2 1 1 2
 2.00000001 8(1)7 6 1 7
 3.00000101 8(1)5 4 1 5
 4.00001001 8(1)8 7 1 8
 5.00010001 4(1)9 8 1 9
 6.00010101 8(1)3 2 1 3
 7.00100101 8(1)4 3 1 4
 8.01010101 2(1)1 0 0 1
 total NATs = 47
L= 9
 1.000000000 1(1)2 1 1 2
 2.000000001 9(1)8 7 1 8
 3.000000101 9(1)6 5 1 6
 4.000001001 9(1)10 9 1 10
 5.000010001 9(1)12 11 1 12
 6.000010101 9(1)4 3 1 4
 7.000100101 9(1)6 5 1 6
 8.000101001 9(1)6 5 1 6
 9.001001001 3(1)8 7 1 8
 10.001010101 9(1)2 1 1 2
 total NATs = 76
L= 10
 1.0000000000 1(1)2 1 1 2
 2.0000000001 10(1)9 8 1 9
 3.0000000101 10(1)7 6 1 7
 4.0000001001 10(1)12 11 1 12
 5.0000010001 10(1)15 14 1 15
 6.0000010101 10(1)5 4 1 5
 7.0000100001 5(1)16 15 1 16
 8.0000100101 10(1)8 7 1 8
 9.0000101001 10(1)8 7 1 8
 10.0001000101 10(1)9 8 1 9
 11.0001001001 10(1)12 11 1 12
 12.0001010101 10(1)3 2 1 3
 13.0010010101 10(1)4 3 1 4
 14.0010100101 5(1)4 3 1 4
 15.0101010101 2(1)1 0 0 1
 total NATs = 123
L= 11
 1.00000000000 1(1)2 1 1 2
 2.00000000001 11(1)10 9 1 10
 3.00000000101 11(1)8 7 1 8
 4.00000001001 11(1)14 13 1 14
 5.00000010001 11(1)18 17 1 18
 6.00000010101 11(1)6 5 1 6
 7.00000100001 11(1)20 19 1 20
 8.00000100101 11(1)10 9 1 10
 9.00000101001 11(1)10 9 1 10
 10.00001000101 11(1)12 11 1 12
 11.00001001001 11(1)16 15 1 16
 12.00001010001 11(1)12 11 1 12
 13.00001010101 11(1)4 3 1 4
 14.00010001001 11(1)18 17 1 18
 15.00010010101 11(1)6 5 1 6
 16.00010100101 11(1)6 5 1 6
 17.00010101001 11(1)6 5 1 6
 18.00100100101 11(1)8 7 1 8
 19.00101010101 11(1)2 1 1 2
 total NATs = 199
L= 12
 1.000000000000 1(1)2 1 1 2
 2.000000000001 12(1)11 10 1 11
 3.000000000101 12(1)9 8 1 9
 4.000000001001 12(1)16 15 1 16
 5.000000010001 12(1)21 20 1 21
 6.000000010101 12(1)7 6 1 7
 7.000000100001 12(1)24 23 1 24
 8.000000100101 12(1)12 11 1 12
 9.000000101001 12(1)12 11 1 12
 10.000001000001 6(1)25 24 1 25
 11.000001000101 12(1)15 14 1 15
 12.000001001001 12(1)20 19 1 20
 13.000001010001 12(1)15 14 1 15
 14.000001010101 12(1)5 4 1 5
 15.000010000101 12(1)16 15 1 16
 16.000010001001 12(1)24 23 1 24
 17.000010010001 12(1)24 23 1 24
 18.000010010101 12(1)8 7 1 8
 19.000010100101 12(1)8 7 1 8
 20.000010101001 12(1)8 7 1 8
 21.000100010001 4(1)27 26 1 27
 22.000100010101 12(1)9 8 1 9
 23.000100100101 12(1)12 11 1 12
 24.000100101001 12(1)12 11 1 12
 25.000101000101 6(1)9 8 1 9
 26.000101001001 12(1)12 11 1 12
 27.000101010101 12(1)3 2 1 3
 28.001001001001 3(1)16 15 1 16
 29.001001010101 12(1)4 3 1 4
 30.001010010101 12(1)4 3 1 4
 31.010101010101 2(1)1 0 0 1
 total NATs = 322
```

λ ratio = .5 Z = .5

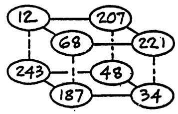
1111111100001111-1111111100001111-rule 4279238415
=3-rule 243 -11110011
Length=1 -15



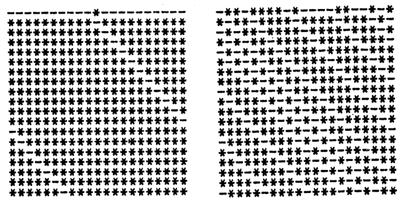




Length=8



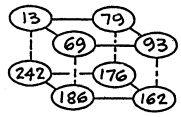
11110011	243
00110000	48
00100010	34
10111011	187



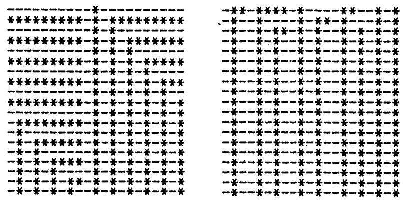
```
3-rule 243 =11110011

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.111 1(1)2 1 1 2
 2.110 1(3)6 3 1 2
 total NATs = 2
L= 4
 1.1111 1(1)2 1 1 2
 2.1110 1(4)12 8 1 3
 3.0101 1(2)2 0 0 1
 total NATs = 3
L= 5
 1.11111 1(1)2 1 1 2
 2.11110 1(5)20 15 1 4
 3.11010 1(5)10 5 1 2
 total NATs = 3
L= 6
 1.111111 1(1)2 1 1 2
 2.111110 1(6)30 24 1 5
 3.111010 1(6)18 12 1 3
 4.110110 1(3)12 9 1 4
 5.010101 1(2)2 0 0 1
 total NATs = 5
L= 7
 1.1111111 1(1)2 1 1 2
 2.1111110 1(7)42 35 1 6
 3.1111010 1(7)28 21 1 4
 4.1110110 1(7)42 35 1 6
 5.1101010 1(7)14 7 1 2
 total NATs = 5
L= 8
 1.11111111 1(1)2 1 1 2
 2.11111110 1(8)56 48 1 7
 3.11111010 1(8)40 32 1 5
 4.11110110 1(8)64 56 1 8
 5.11101110 1(4)36 32 1 9
 6.11101010 1(8)24 16 1 3
 7.11011010 1(8)32 24 1 4
 8.01010101 1(2)2 0 0 1
 total NATs = 8
L= 9
 1.111111111 1(1)2 1 1 2
 2.111111110 1(9)72 63 1 8
 3.111111010 1(9)54 45 1 6
 4.111110110 1(9)90 81 1 10
 5.111101110 1(9)108 99 1 12
 6.111101010 1(9)36 27 1 4
 7.111011010 1(9)54 45 1 6
 8.111010110 1(9)54 45 1 6
 9.110110110 1(3)24 21 1 8
 10.110101010 1(9)18 9 1 2
 total NATs = 10
L= 10
 1.1111111111 1(1)2 1 1 2
 2.1111111110 1(10)90 80 1 9
 3.1111111010 1(10)70 60 1 7
 4.1111110110 1(10)120 110 1 12
 5.1111101110 1(10)150 140 1 15
 6.1111101010 1(10)50 40 1 5
 7.1111011110 1(5)80 75 1 16
 8.1111011010 1(10)80 70 1 8
 9.1111010110 1(10)80 70 1 8
 10.1110111010 1(10)90 80 1 9
 11.1110110110 1(10)120 110 1 12
 12.1110101010 1(10)30 20 1 3
 13.1101101010 1(10)40 30 1 4
 14.1101011010 1(5)20 15 1 4
 15.0101010101 1(2)2 0 0 1
 total NATs = 15
L= 11
 1.11111111111 1(1)2 1 1 2
 2.11111111110 1(11)110 99 1 10
 3.11111111010 1(11)88 77 1 8
 4.11111110110 1(11)154 143 1 14
 5.11111101110 1(11)198 187 1 18
 6.11111101010 1(11)66 55 1 6
 7.11111011110 1(11)220 209 1 20
 8.11111011010 1(11)110 99 1 10
 9.11111010110 1(11)110 99 1 10
 10.11110111010 1(11)132 121 1 12
 11.11110110110 1(11)176 165 1 16
 12.11110101110 1(11)132 121 1 12
 13.11110101010 1(11)44 33 1 4
 14.11101110110 1(11)198 187 1 18
 15.11101101010 1(11)66 55 1 6
 16.11101011010 1(11)66 55 1 6
 17.11101010110 1(11)66 55 1 6
 18.11011011010 1(11)88 77 1 8
 19.11010101010 1(11)22 11 1 2
 total NATs = 19
L= 12
 1.111111111111 1(1)2 1 1 2
 2.111111111110 1(12)132 120 1 11
 3.111111111010 1(12)108 96 1 9
 4.111111110110 1(12)192 180 1 16
 5.111111101110 1(12)252 240 1 21
 6.111111101010 1(12)84 72 1 7
 7.111111011110 1(12)288 276 1 24
 8.111111011010 1(12)144 132 1 12
 9.111111010110 1(12)144 132 1 12
 10.111110111110 1(6)150 144 1 25
 11.111110111010 1(12)180 168 1 15
 12.111110110110 1(12)240 228 1 20
 13.111110101110 1(12)180 168 1 15
 14.111110101010 1(12)60 48 1 5
 15.111101111010 1(12)192 180 1 16
 16.111101110110 1(12)288 276 1 24
 17.111101101110 1(12)288 276 1 24
 18.111101101010 1(12)96 84 1 8
 19.111101011010 1(12)96 84 1 8
 20.111101010110 1(12)96 84 1 8
 21.111011101110 1(4)108 104 1 27
 22.111011101010 1(12)108 96 1 9
 23.111011011010 1(12)144 132 1 12
 24.111011010110 1(12)144 132 1 12
 25.111010111010 1(6)54 48 1 9
 26.111010110110 1(12)144 132 1 12
 27.111010101010 1(12)36 24 1 3
 28.110110110110 1(3)48 45 1 16
 29.110110101010 1(12)48 36 1 4
 30.110101101010 1(12)48 36 1 4
 31.010101010101 1(2)2 0 0 1
 total NATs = 31
```

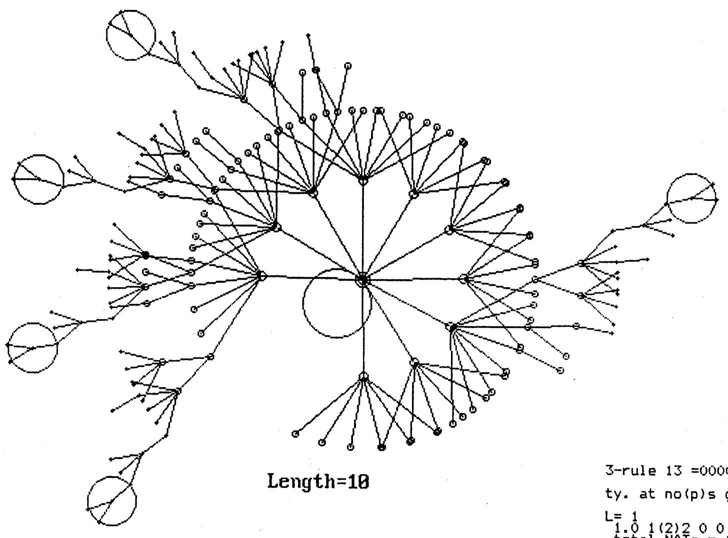
λ ratio = .75 Z = .75

00000000011110011-00000000011110011-rule 15925491
=3-rule 13 -00001101
Length=1 -15

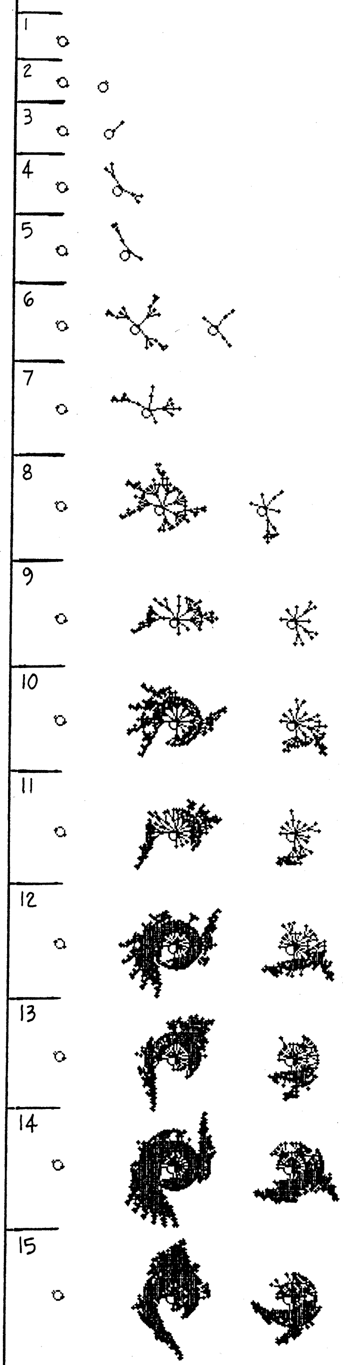


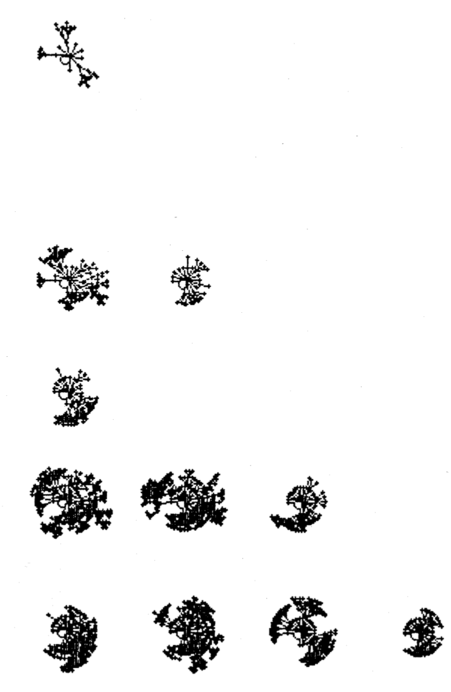
00001101 13
01001111 79
01011101 93
01000101 69





Length=10

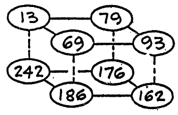




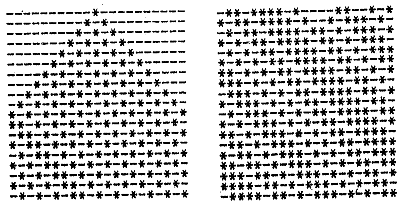
```
3-rule 13 =00001101

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(2)2 0 0 1
 2.001 3(1)2 1 1 2
 total NATs = 4
L= 4
 1.0000 1(2)2 0 0 1
 2.0101 2(1)7 4 2 3
 total NATs = 3
L= 5
 1.00000 1(2)2 0 0 1
 2.01001 5(1)6 3 3 3
 total NATs = 6
L= 6
 1.000000 1(2)2 0 0 1
 2.010101 2(1)22 12 4 4
 3.010010 3(1)6 3 2 4
 total NATs = 6
L= 7
 1.0000000 1(2)2 0 0 1
 2.0101001 7(1)18 10 5 5
 total NATs = 8
L= 8
 1.00000000 1(2)2 0 0 1
 2.01010101 2(1)67 40 6 7
 3.01010010 8(1)15 9 4 6
 total NATs = 11
L= 9
 1.000000000 1(2)2 0 0 1
 2.010101001 9(1)52 33 7 8
 3.010010010 3(1)14 10 2 8
 total NATs = 13
L= 10
 1.0000000000 1(2)2 0 0 1
 2.0101010101 2(1)196 130 8 11
 3.0101010010 10(1)44 31 6 10
 4.0101001010 5(1)38 25 4 9
 total NATs = 18
L= 11
 1.00000000000 1(2)2 0 0 1
 2.01010101001 11(1)150 106 9 13
 3.01010010010 11(1)36 27 4 12
 total NATs = 23
L= 12
 1.000000000000 1(2)2 0 0 1
 2.010101010101 2(1)568 412 10 18
 3.010101010010 12(1)126 96 8 16
 4.010101001010 12(1)112 83 6 15
 5.010010010010 3(1)34 27 2 16
 total NATs = 30
L= 13
 1.0000000000000 1(2)2 0 0 1
 2.0101010101001 13(1)432 330 11 21
 3.0101010010010 13(1)106 85 6 20
 4.0101001010010 13(1)92 72 4 18
 total NATs = 40
L= 14
 1.00000000000000 1(2)2 0 0 1
 2.01010101010101 2(1)1639 1274 12 29
 3.01010101010010 14(1)363 293 10 26
 4.01010101001010 14(1)321 254 8 24
 5.01010100101010 7(1)330 264 6 25
 6.01010010010010 14(1)87 72 4 24
 total NATs = 52
L= 15
 1.000000000000000 1(2)2 0 0 1
 2.010101010101001 15(1)1244 1007 13 34
 3.010101010010010 15(1)304 255 8 32
 4.010101001010010 15(1)271 225 6 30
 5.010101001001010 15(1)271 225 6 30
 6.010100101001010 5(1)234 189 4 27
 7.010010010010010 3(1)82 71 2 32
 total NATs = 69
```

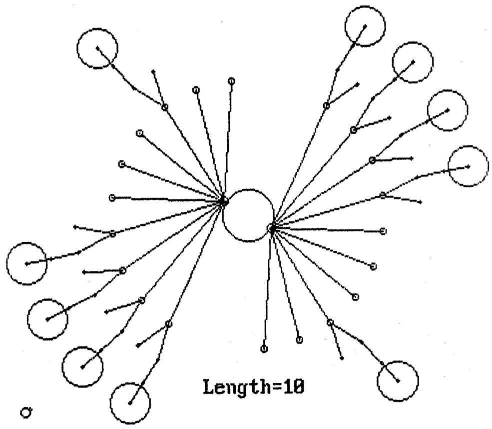
λ ratio = .75 Z = .75

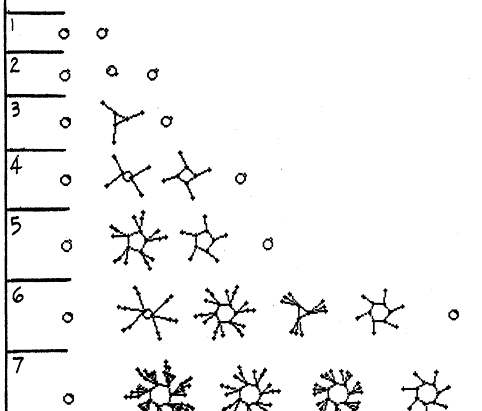
1111111100001100-1111111100001100-rule 4279041804
=3-rule 242 -11110010
Length=1 -13

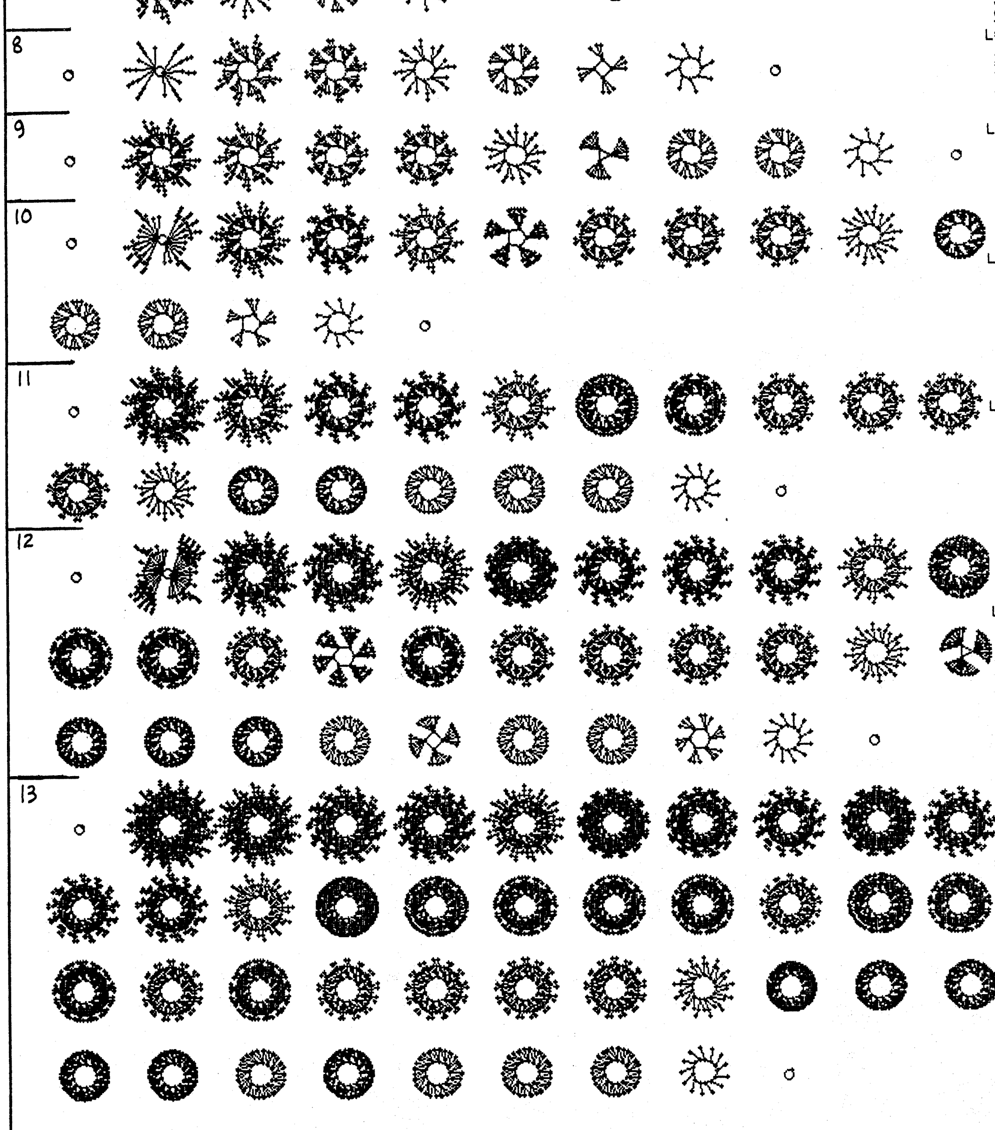


11110010 242
10110000 176
10100010 162
10111010 186



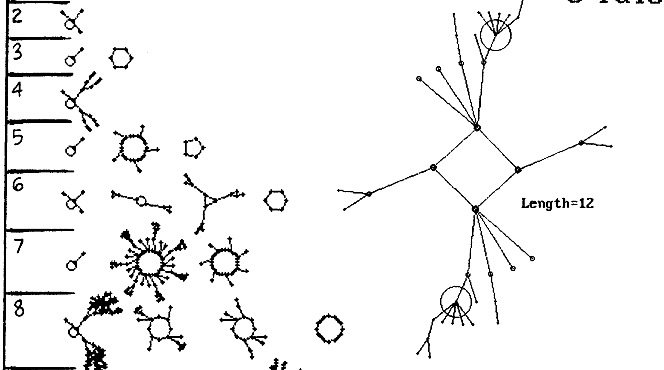


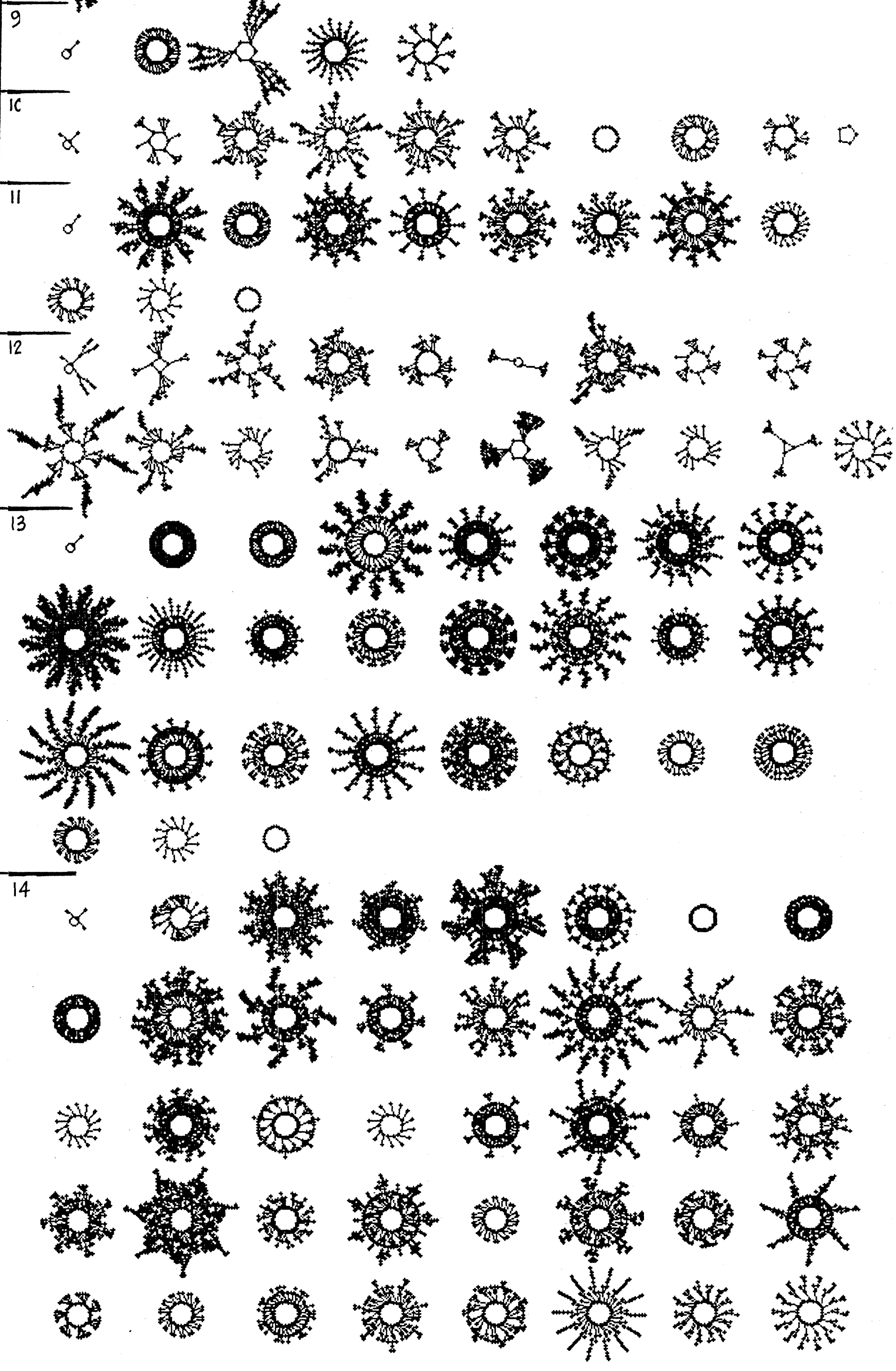


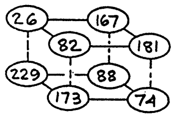



```
3-rule 242 =11110010
ty. at no(p)s g ml mp
L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 1(2)2 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)1 0 0 1
 2.110 1(3)6 3 1 2
 3.111 1(1)1 0 0 1
 total NATs = 3
L= 4
 1.0000 1(1)1 0 0 1
 2.1010 1(2)6 4 1 3
 3.1101 1(4)8 4 1 2
 4.1111 1(1)1 0 0 1
 total NATs = 4
L= 5
 1.00000 1(1)1 0 0 1
 2.01101 1(5)20 10 2 3
 3.11011 1(5)10 5 1 2
 4.11111 1(1)1 0 0 1
 total NATs = 4
L= 6
 1.000000 1(1)1 0 0 1
 2.010101 1(2)14 6 2 4
 3.111010 1(6)24 12 2 3
 4.110110 1(3)12 9 1 4
 5.110111 1(6)12 6 1 2
 6.111111 1(1)1 0 0 1
 total NATs = 6
L= 7
 1.0000000 1(1)1 0 0 1
 2.1011010 1(7)56 28 3 5
 3.1110101 1(7)28 14 2 3
 4.1101101 1(7)28 21 1 4
 5.1101111 1(7)14 7 1 2
 6.1111111 1(1)1 0 0 1
 total NATs = 6
L= 8
 1.00000000 1(1)1 0 0 1
 2.10101010 1(2)30 12 3 7
 3.01110101 1(8)64 32 3 5
 4.01101011 1(8)64 48 2 6
 5.11101011 1(8)32 16 2 3
 6.11011011 1(8)32 24 1 4
 7.11011101 1(4)16 12 1 4
 8.11011111 1(8)16 8 1 2
 9.11111111 1(1)1 0 0 1
 total NATs = 9
L= 9
 1.000000000 1(1)1 0 0 1
 2.010110101 1(9)144 81 4 8
 3.111101010 1(9)72 36 3 5
 4.111010110 1(9)72 54 2 6
 5.011010111 1(9)72 54 2 6
 6.111010111 1(9)36 18 2 3
 7.110110110 1(3)24 21 1 8
 8.110110111 1(9)36 27 1 4
 9.110111011 1(9)36 27 1 4
 10.110111111 1(9)18 9 1 2
 11.111111111 1(1)1 0 0 1
 total NATs = 11
L= 10
 1.0000000000 1(1)1 0 0 1
 2.0101010101 1(2)62 30 4 11
 3.1011101010 1(10)160 90 4 8
 4.1011010101 1(10)160 120 3 10
 5.1111010101 1(10)80 40 3 5
 6.0110101101 1(5)80 60 2 9
 7.1110101101 1(10)80 60 2 6
 8.1110101110 1(10)80 60 2 6
 9.0110101111 1(10)80 60 2 6
 10.1110101111 1(10)40 20 2 3
 11.1101101101 1(10)80 70 1 8
 12.1101101111 1(10)40 30 1 4
 13.1101110111 1(10)40 30 1 4
 14.1101111011 1(5)20 15 1 4
 15.1101111111 1(10)20 10 1 2
 16.1111111111 1(1)1 0 0 1
 total NATs = 16
L= 11
 1.00000000000 1(1)1 0 0 1
 2.10101101010 1(11)352 220 5 13
 3.01111010101 1(11)176 99 4 8
 4.01110101011 1(11)176 132 3 10
 5.10110101011 1(11)176 132 3 10
 6.11110101011 1(11)88 44 3 5
 7.11101011010 1(11)176 132 2 9
 8.01101011011 1(11)176 154 2 12
 9.11101011011 1(11)88 66 2 6
 10.11101011101 1(11)88 66 2 6
 11.11101011110 1(11)88 66 2 6
 12.01101011111 1(11)88 66 2 6
 13.11101011111 1(11)44 22 2 3
 14.11011011011 1(11)88 77 1 8
 15.11011011101 1(11)88 77 1 8
 16.11011011111 1(11)44 33 1 4
 17.11011101111 1(11)44 33 1 4
 18.11011110111 1(11)44 33 1 4
 19.11011111111 1(11)22 11 1 2
 20.11111111111 1(1)1 0 0 1
 total NATs = 20
L= 12
 1.000000000000 1(1)1 0 0 1
 2.101010101010 1(2)126 70 5 18
 3.010111010101 1(12)384 240 5 13
 4.110110101010 1(12)384 300 4 16
 5.111110101010 1(12)192 108 4 8
 6.101101010110 1(12)384 288 3 15
 7.111101010110 1(12)192 144 3 10
 8.011101010111 1(12)192 144 3 10
 9.101101010111 1(12)192 144 3 10
 10.111101010111 1(12)96 48 3 5
 11.111010110101 1(12)192 144 2 9
 12.111010110110 1(12)192 168 2 12
 13.011010110111 1(12)192 168 2 12
 14.111010110111 1(12)96 72 2 6
 15.111010111010 1(6)96 72 2 9

 30.110111110111 1(6)24 18 1 4
 31.110111111111 1(12)24 12 1 2
 32.111111111111 1(1)1 0 0 1
 total NATs = 32
```

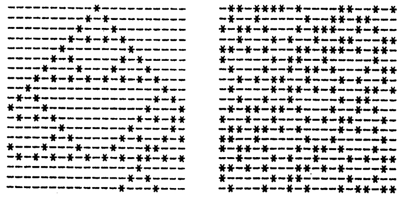
λ ratio = .75 Z = .75 0000001111001100-0000001111001100-rule 63701964 =3-rule 26 -00011010 Length=1 -14







00011010 26
10100111 167
10110101 181
01010010 82



3-rule 26 =00011010

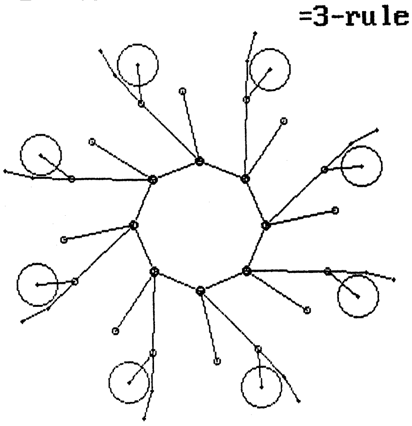
ty. at no(p)s g ml mp

L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)4 3 1 4
total NATs = 1
L= 3
1.000 1(1)2 1 1 2
2.001 1(6)6 0 0 1
total NATs = 2
L= 4
1.0000 1(1)16 5 4 4
total NATs = 1
L= 5
1.00000 1(1)2 1 1 2
2.00001 1(20)25 5 1 2
3.00111 1(5)5 0 0 1
total NATs = 3
L= 6
1.000000 1(1)4 3 1 4
2.100010 3(2)8 4 2 2
3.110010 2(3)15 6 3 2
4.001001 1(6)6 0 0 1
total NATs = 7
L= 7
1.0000000 1(1)2 1 1 2
2.0000001 1(28)91 35 4 3
3.0000011 1(28)35 7 1 2
total NATs = 3
L= 8
1.00000000 1(1)96 45 8 4
2.10001100 4(8)20 10 2 3
3.00001011 4(8)16 6 2 3
4.00100111 1(16)16 0 0 1
total NATs = 10
L= 9
1.000000000 1(1)2 1 1 2
2.000000001 1(72)126 54 1 4
3.100100100 1(6)186 81 11 4
4.000001001 1(72)153 54 3 3
5.101100100 1(18)45 18 2 2
total NATs = 5
L= 10
1.0000000000 1(1)4 3 1 4
2.1000000010 5(6)24 16 2 4
3.0011001000 2(20)90 45 5 4
4.0000001001 2(20)125 55 7 4
5.0000001011 2(20)110 50 5 4
6.0101001000 2(20)60 30 3 3
7.0000011011 2(20)20 0 0 1
8.1000110000 1(20)50 30 1 4
9.0000100001 1(20)35 15 1 4
10.0011100111 1(5)5 0 0 1
total NATs = 19
L= 11
1.00000000000 1(1)2 1 1 2
2.00000000001 1(88)473 231 8 5
3.00000000011 1(88)154 66 1 4
4.01001001000 1(44)352 187 6 6
5.00000010101 1(88)220 99 4 5
6.01100101101 1(44)187 99 4 4
7.00000100011 1(88)198 88 3 3
8.10010010010 1(44)297 143 6 4
9.00001001011 1(44)66 22 1 3
10.00001010011 1(33)55 22 1 2
11.10011011100 1(11)22 11 1 2
12.00100100111 1(22)22 0 0 1
total NATs = 12
L= 12
1.000000000000 1(1)16 5 4 4
2.010000000100 6(4)42 24 5 5
3.010010000100 4(12)75 42 6 6
4.100000001100 4(24)117 69 4 5
5.000000001011 4(24)60 30 2 5
6.010001010001 3(2)16 12 2 6
7.100000100100 4(24)159 81 9 6
8.100000110000 4(12)27 15 1 5
9.000000100001 4(12)39 24 2 6
10.011001001001 2(12)270 120 15 4
11.000000101001 4(12)81 45 6 4
12.000000101011 4(12)30 15 2 3
13.011001010001 4(24)63 27 4 3
14.000001100011 4(12)24 12 1 5
15.001001001001 1(6)174 96 4 5
16.000001101001 4(12)45 21 5 3
17.000001101011 4(12)24 12 1 3
18.110010110010 2(3)21 12 3 4
19.101101100100 1(12)48 24 2 2
total NATs = 64
L= 13
1.0000000000000 1(1)2 1 1 2
2.0000000000001 1(104)325 221 1 7
3.0000000000011 1(104)195 91 1 4
4.0110010010101 1(52)663 364 10 5
5.0000000001001 1(104)364 169 4 5
6.0000000001011 1(104)481 286 4 5
7.0100001001000 1(52)325 195 6 9
8.0000000010101 1(104)351 195 5 7
9.1000000100100 1(52)1170 585 10 8
10.0011000101000 1(104)286 104 5 5
11.0100011001000 1(52)169 104 2 6
12.0000000110101 1(104)221 104 2 3
13.1000001100100 1(52)442 273 4 5
14.1011010110010 1(52)390 234 8 6
15.0000001000101 1(104)221 104 2 5
16.1000010101100 1(52)312 169 5 6
17.0000001001011 1(52)468 234 10 4
18.0100100100101 1(52)377 234 4 6
19.0100101011000 1(52)182 78 3 5
20.0000001011001 1(52)338 156 7 6
21.0101001001000 1(52)364 208 4 4
22.1001010110010 1(52)156 78 3 5
23.0000011001011 1(52)78 26 1 3
24.0000011011001 1(52)169 91 2 3
25.0000100101001 1(52)91 39 1 3
26.0000111010011 1(13)26 13 1 2
27.0010011100111 1(26)26 0 0 1
total NATs = 27

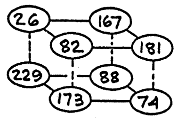
λ ratio = .75 Z = .75

1111110000110011-1111110000110011-rule =3-rule 229 -11100101

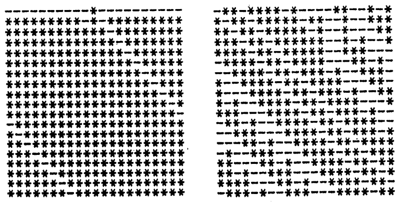
4231265331 Length=1 -13

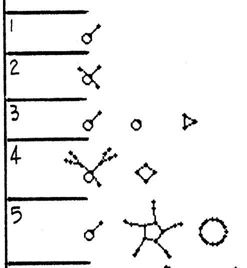


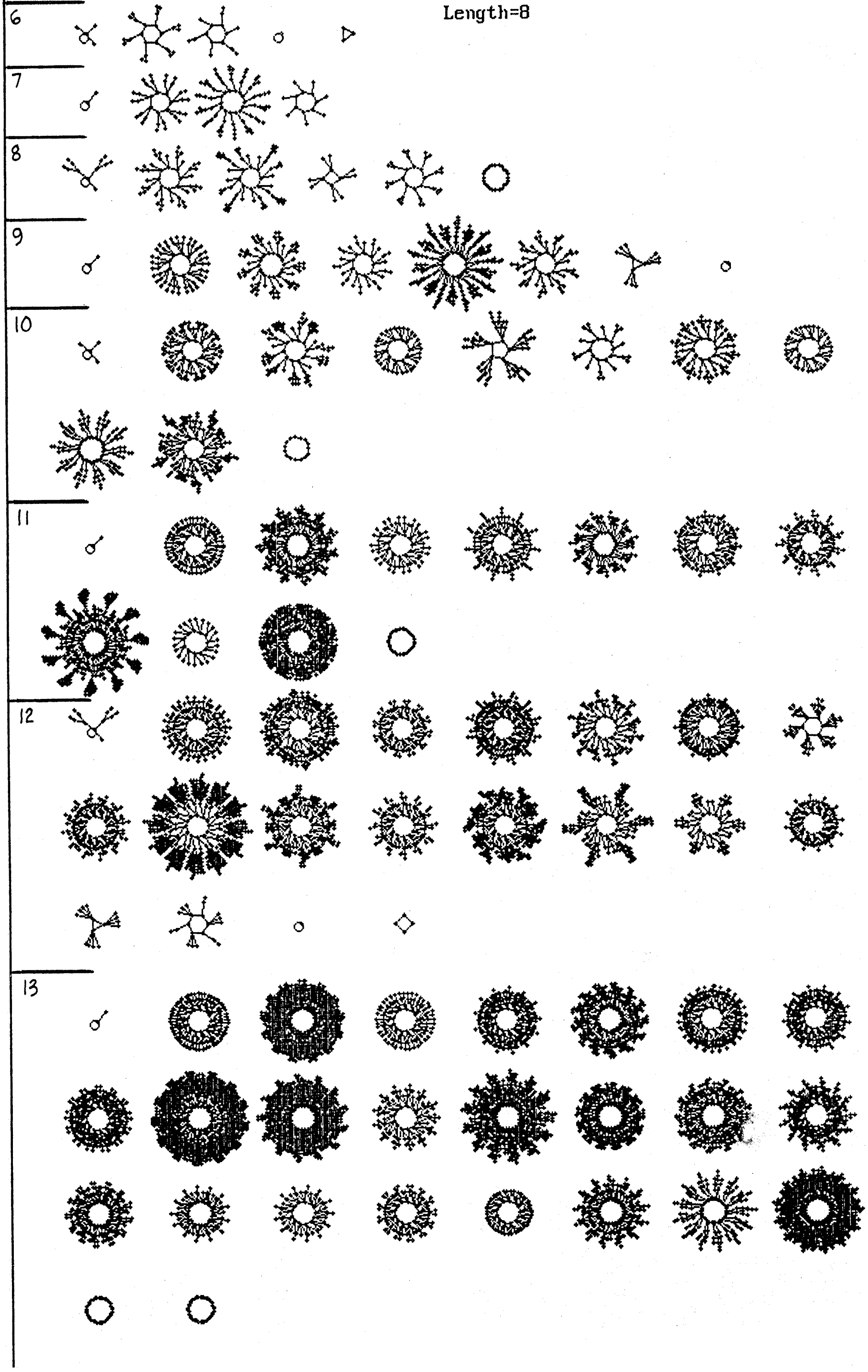
Length=8



11100101	229
01011000	88
01001010	74
10101101	173

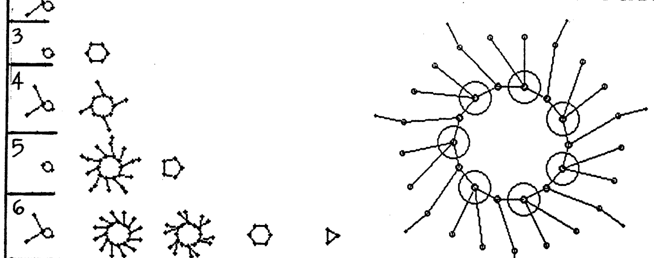


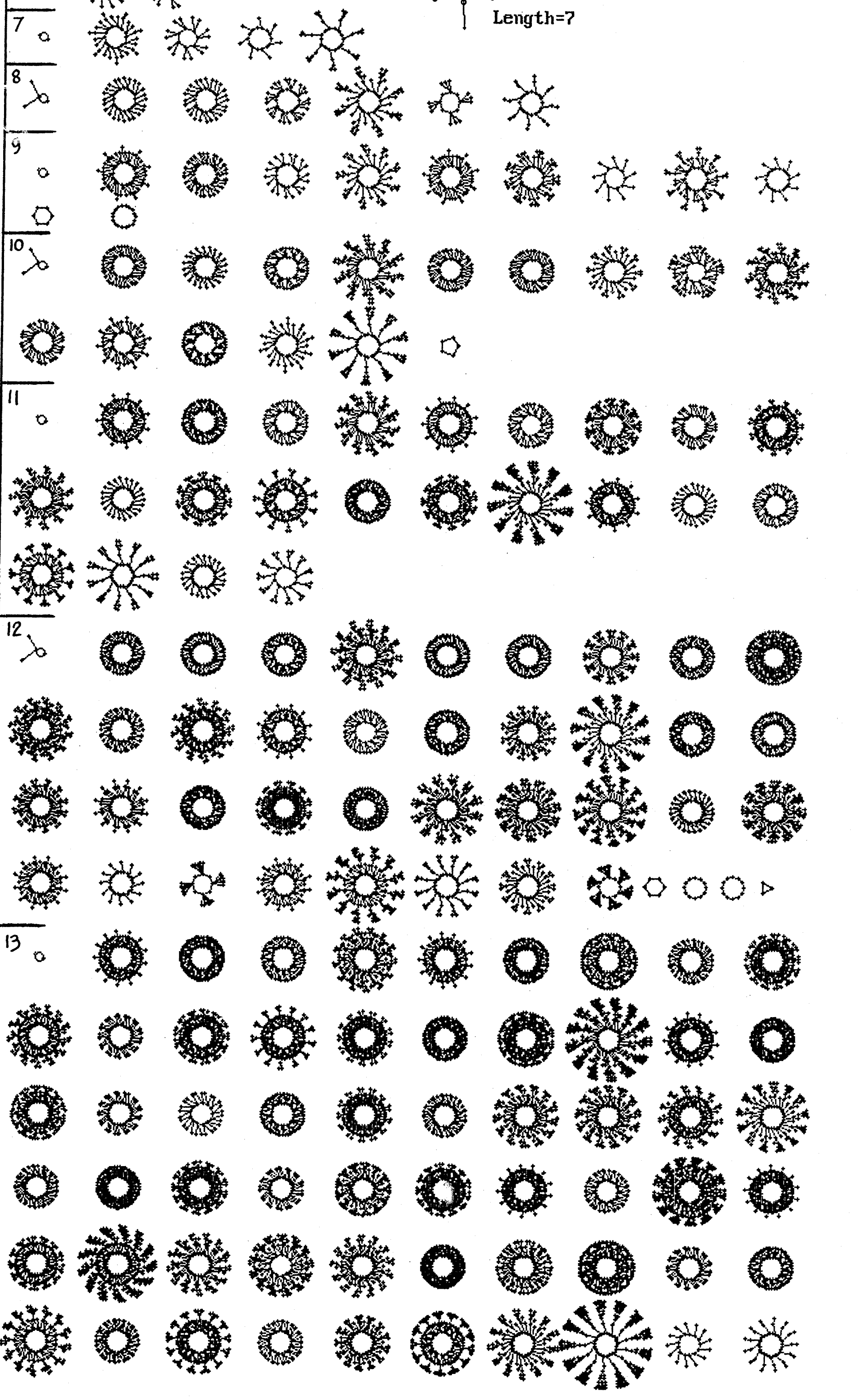


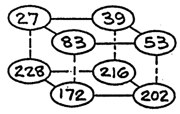



```
3-rule 229 =11100101
ty. at no(p)s g ml mp
L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)4 3 1 4
 total NATs = 1
L= 3
 1.111 1(1)2 1 1 2
 2.001 3(1)1 0 0 1
 3.011 1(3)3 0 0 1
 total NATs = 5
L= 4
 1.1111 1(1)12 5 3 4
 2.0111 1(4)4 0 0 1
 total NATs = 2
L= 5
 1.11111 1(1)2 1 1 2
 2.10111 1(5)15 5 2 2
 3.00011 1(15)15 0 0 1
 total NATs = 3
L= 6
 1.111111 1(1)4 3 1 4
 2.101111 1(6)24 12 2 2
 3.101001 2(6)15 6 2 2
 4.001001 3(1)1 0 0 1
 5.011011 1(3)3 0 0 1
 total NATs = 8
L= 7
 1.1111111 1(1)2 1 1 2
 2.1011111 1(7)35 14 2 3
 3.0111010 1(14)77 21 4 2
 4.0110111 1(7)14 7 1 2
 total NATs = 4
L= 8
 1.11111111 1(1)12 5 3 4
 2.10111111 1(8)48 24 3 3
 3.00111110 2(8)64 28 5 3
 4.01110111 1(4)12 8 1 3
 5.10110111 1(8)32 16 2 2
 6.00011001 1(24)24 0 0 1
 total NATs = 7
L= 9
 1.111111111 1(1)2 1 1 2
 2.101111111 1(9)63 27 2 4
 3.110111000 1(18)81 36 3 3
 4.011110111 1(9)36 18 2 3
 5.111111000 1(18)261 90 7 3
 6.110111110 1(9)54 27 3 3
 7.011011011 1(3)12 9 1 4
 8.001001001 3(1)1 0 0 1
 total NATs = 10
L= 10
 1.1111111111 1(1)4 3 1 4
 2.1011111111 1(10)80 50 2 4
 3.1101111000 2(10)85 40 4 4
 4.0111110111 1(10)40 30 1 4
 5.1011110111 1(5)65 25 4 4
 6.1011100011 2(10)30 15 2 2
 7.1101111110 1(10)80 40 3 3
 8.0111011011 1(10)40 30 1 4
 9.1101001000 1(30)130 30 5 2
 10.1111110001 2(10)170 85 6 4
 11.0001100011 1(15)15 0 0 1
 total NATs = 14
L= 11
 1.11111111111 1(1)2 1 1 2
 2.10111111111 1(11)99 44 2 5
 3.11011111000 1(22)319 165 5 4
 4.01111110111 1(11)66 33 2 4
 5.10111110111 1(11)121 55 4 5
 6.10111100011 1(22)121 66 3 3
 7.11011111110 1(11)110 66 3 4
 8.01111011011 1(11)99 66 3 6
 9.11110110100 1(22)550 220 9 4
 10.01110110111 1(11)33 22 1 3
 11.11110100111 1(22)495 231 4 4
 12.00011001001 1(33)33 0 0 1
 total NATs = 12
L= 12
 1.111111111111 1(1)12 5 3 4
 2.101111111111 1(12)132 72 3 5
 3.110111111000 2(12)264 138 5 5
 4.011111110111 1(12)84 60 2 5
 5.101111110111 1(12)204 96 4 6
 6.101111100011 2(12)114 66 4 3
 7.110111111110 1(12)156 96 3 4
 8.011111011111 1(6)48 36 2 6
 9.011111011011 1(12)120 84 3 6
 10.001101001110 1(12)816 288 9 4
 11.101111000111 2(12)210 120 5 5
 12.011110110111 1(12)96 60 3 5
 13.111111100011 2(12)324 186 5 5
 14.101110001111 2(12)156 78 6 4
 15.101110001101 2(12)72 36 3 3
 16.101110110111 1(12)84 60 2 6
 17.011011011011 1(3)15 12 1 5
 18.101001101001 2(6)21 12 2 4
 19.001001001001 3(1)1 0 0 1
 20.011101110111 1(4)4 0 0 1
 total NATs = 29
L= 13
 1.1111111111111 1(1)2 1 1 2
 2.1011111111111 1(13)143 65 2 6
 3.1101111111000 1(26)819 455 5 6
 4.0111111110111 1(13)104 52 2 5
 5.1011111110111 1(13)208 117 3 7
 6.1011111100011 1(26)338 208 4 4
 7.1101111111110 1(13)221 130 3 5
 8.0111111011111 1(13)182 91 3 7
 9.0111111011011 1(13)221 156 3 8
 10.1000111010011 1(26)1508 767 7 5
 11.1011111000111 1(26)871 429 6 6
 12.0111110110111 1(13)104 65 3 4
 13.1101110001111 1(26)598 286 7 6
 14.1011110001111 1(26)390 247 3 5
 15.1011110001101 1(26)273 156 4 5
 16.1101111011110 1(13)247 156 4 9
 17.1011100011011 1(26)182 104 3 4
 18.1011100011101 1(26)91 52 2 4
 19.1011101110111 1(13)65 39 2 4
 20.0111011011111 1(13)91 65 2 5
 21.0111011011011 1(13)65 52 1 5
 22.0111010011010 1(26)208 104 4 4
 23.1101001001000 1(39)169 39 5 2
 24.1111110001111 1(26)1014 520 6 5
 25.0001100011001 1(39)39 0 0 1
 26.0001100101001 1(39)39 0 0 1
 total NATs = 26
```

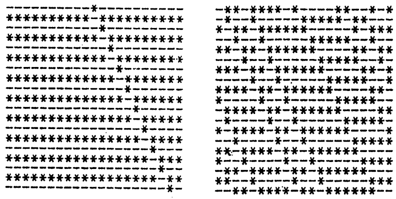
λ ratio = 1 $Z = .75$ 0000001111001111-0000001111001111-rule   63898575
=3-rule 27 -00011011   Length=1 -13







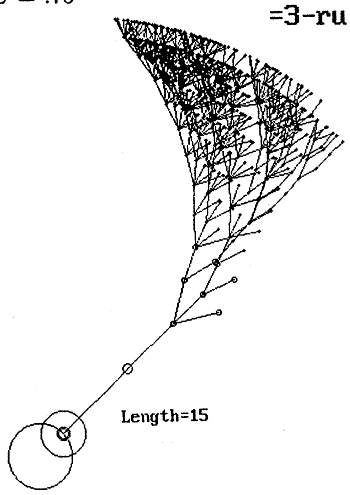
00011011 27
00100111 39
00110101 53
01010011 83

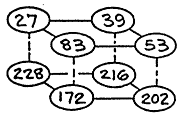


```
3-rule 27 =00011011

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)4 2 1 3
 total NATs = 1
L= 3
 1.000 1(2)2 0 0 1
 2.001 1(6)6 0 0 1
 total NATs = 2
L= 4
 1.0000 1(2)4 2 1 3
 2.0001 1(8)12 4 1 2
 total NATs = 2
L= 5
 1.00000 1(2)2 0 0 1
 2.00001 1(10)25 10 2 2
 3.00111 1(5)5 0 0 1
 total NATs = 3
L= 6
 1.000000 1(2)4 2 1 3
 2.000001 1(12)24 12 1 2
 3.111000 1(12)24 12 1 3
 4.001001 1(6)6 0 0 1
 5.001011 2(3)3 0 0 1
 total NATs = 6
L= 7
 1.0000000 1(2)2 0 0 1
 2.0000001 1(14)42 21 2 3
 3.1111000 1(7)21 14 1 3
 4.0000111 1(14)21 7 1 2
 5.0001001 1(14)42 14 3 2
 total NATs = 5
L= 8
 1.00000000 1(2)4 2 1 3
 2.00000001 1(16)40 24 1 3
 3.11111000 1(16)40 24 1 3
 4.00000111 1(16)40 24 1 4
 5.00001001 1(16)80 32 4 2
 6.00010001 1(8)20 12 1 4
 7.11100100 1(16)32 8 2 2
 total NATs = 7
L= 9
 1.000000000 1(2)2 0 0 1
 2.000000001 1(18)63 36 2 4
 3.111111000 1(18)63 45 1 4
 4.000000111 1(18)36 18 1 3
 5.000001001 1(18)72 36 3 2
 6.000010001 1(18)63 36 2 4
 7.111100100 1(18)72 45 2 3
 8.111110000 1(9)18 9 1 2
 9.011100100 1(18)72 36 3 4
 10.000100111 1(18)27 9 1 2
 11.001001001 1(6)6 0 0 1
 12.001001011 1(18)18 0 0 1
 total NATs = 12
L= 10
 1.0000000000 1(2)4 2 1 3
 2.0000000001 1(20)60 40 1 4
 3.1111111000 1(20)60 40 1 3
 4.0000000111 1(20)60 40 1 5
 5.0000001001 1(20)120 60 4 3
 6.0000010001 1(20)60 40 1 4
 7.1111100000 1(20)60 40 1 4
 8.1111100100 1(20)60 30 2 2
 9.0000100001 1(10)65 40 2 4
 10.0111100100 1(20)120 80 3 4
 11.0000100111 1(20)50 30 1 3
 12.0000111001 1(20)60 30 2 4
 13.1110111000 1(20)70 50 1 6
 14.0001000111 1(20)50 20 2 2
 15.0001001001 1(20)120 40 5 2
 16.0011100111 1(5)5 0 0 1
 total NATs = 16
L= 11
 1.00000000000 1(2)2 0 0 1
 2.00000000001 1(22)88 55 2 5
 3.11111111000 1(22)88 66 1 5
 4.00000000111 1(22)55 33 1 4
 5.00000001001 1(22)110 66 3 3
 6.00000010001 1(22)88 55 2 4
 7.11111100000 1(11)44 33 1 4
 8.11111100100 1(22)110 77 2 4
 9.11111110000 1(22)55 33 1 3
 10.00000100001 1(22)110 77 2 4
 11.01111100100 1(22)110 66 3 4
 12.00000100111 1(22)44 22 1 2
 13.11111000110 1(22)110 77 2 4
 14.00000111001 1(22)110 66 3 6
 15.11110111000 1(22)99 77 1 6
 16.00001000111 1(22)121 88 2 6
 17.00001001001 1(22)220 88 6 2
 18.11110001110 1(22)99 66 2 6
 19.00001110001 1(22)44 22 1 2
 20.00001110111 1(22)55 33 1 4
 21.00010001001 1(22)110 66 3 4
 22.01110010010 1(22)88 22 4 2
 23.11100011100 1(22)44 22 1 3
 24.11100100110 1(11)44 22 2 2
 total NATs = 24
L= 12
 1.000000000000 1(2)4 2 1 3
 2.000000000001 1(24)84 60 1 5
 3.111111111000 1(24)84 60 1 4
 4.000000000111 1(24)84 60 1 6
 5.000000001001 1(24)168 96 4 4
 6.000000010001 1(24)84 60 1 4
 7.111111100000 1(24)84 60 1 4
 8.111111100100 1(24)96 60 2 3
 9.111111110000 1(24)84 60 1 5
 10.000000100001 1(24)192 132 2 6
 11.011111100100 1(24)168 120 3 4
 12.000000100111 1(24)72 48 1 4
 13.111111000110 1(24)168 120 3 6
 14.000000111001 1(24)96 60 2 6
 15.000001000001 1(12)48 36 1 4
 16.111110111000 1(24)96 72 1 6
 17.000001000111 1(24)96 60 2 4
 18.000001001001 1(24)192 96 5 2
 19.111110001110 1(24)96 72 1 6
 20.111110000100 1(24)84 60 1 4

 35.001110010010 1(24)192 96 5 4
 36.000100100111 1(24)72 24 3 2
 37.000100111001 1(24)72 36 2 2
 38.111000111000 1(12)60 48 1 9
 39.001001001001 1(6)6 0 0 1
 40.001001001011 2(12)12 0 0 1
 41.001001011011 1(12)12 0 0 1
 42.001011001011 2(3)3 0 0 1
 total NATs = 44
```

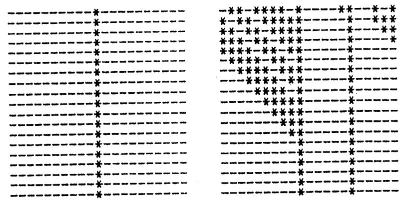
λ ratio = 1 Z = .75

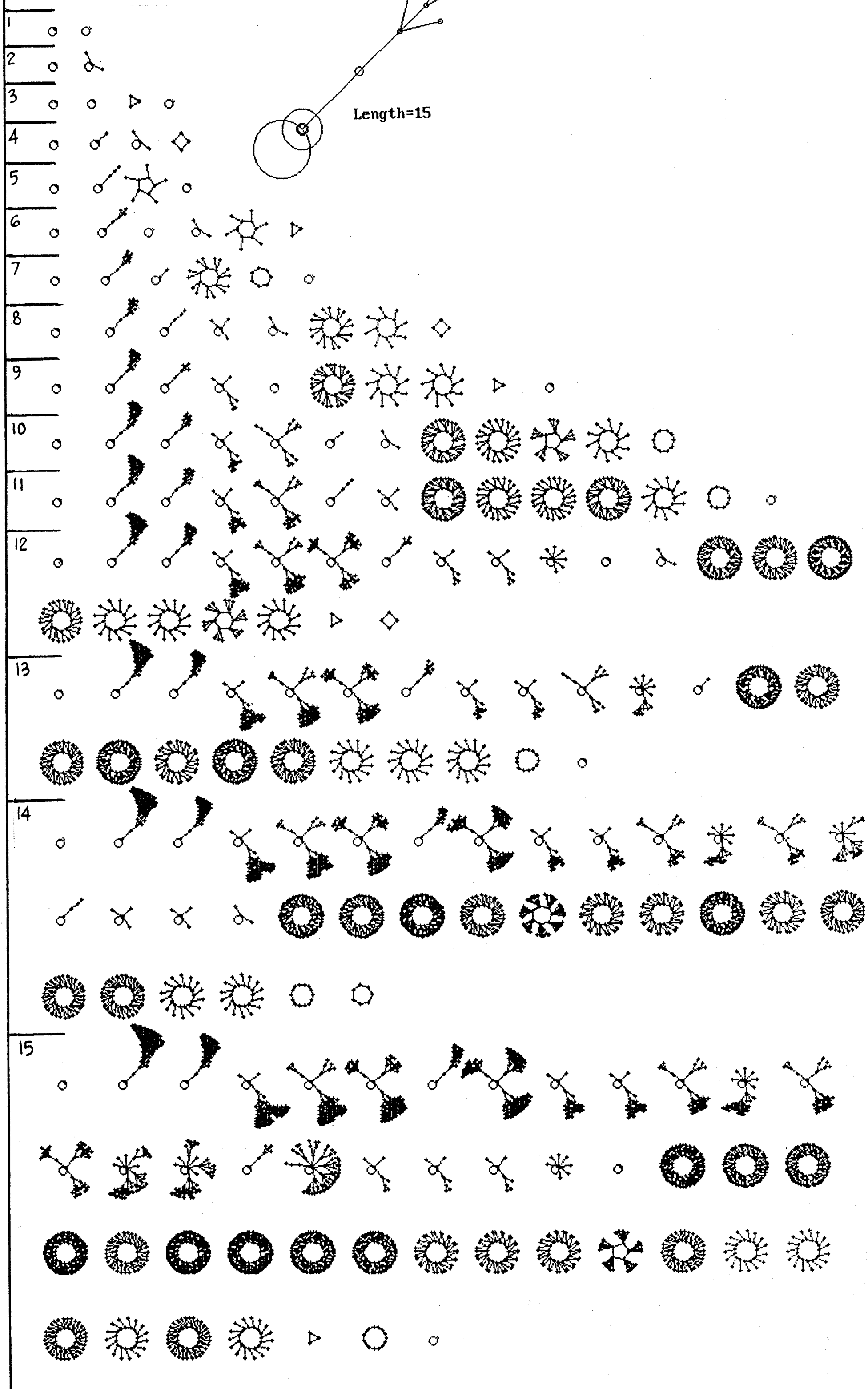
1111110000110000-1111110000110000-rule 4231068720
=3-rule 228 -11100100 Length=1 -15



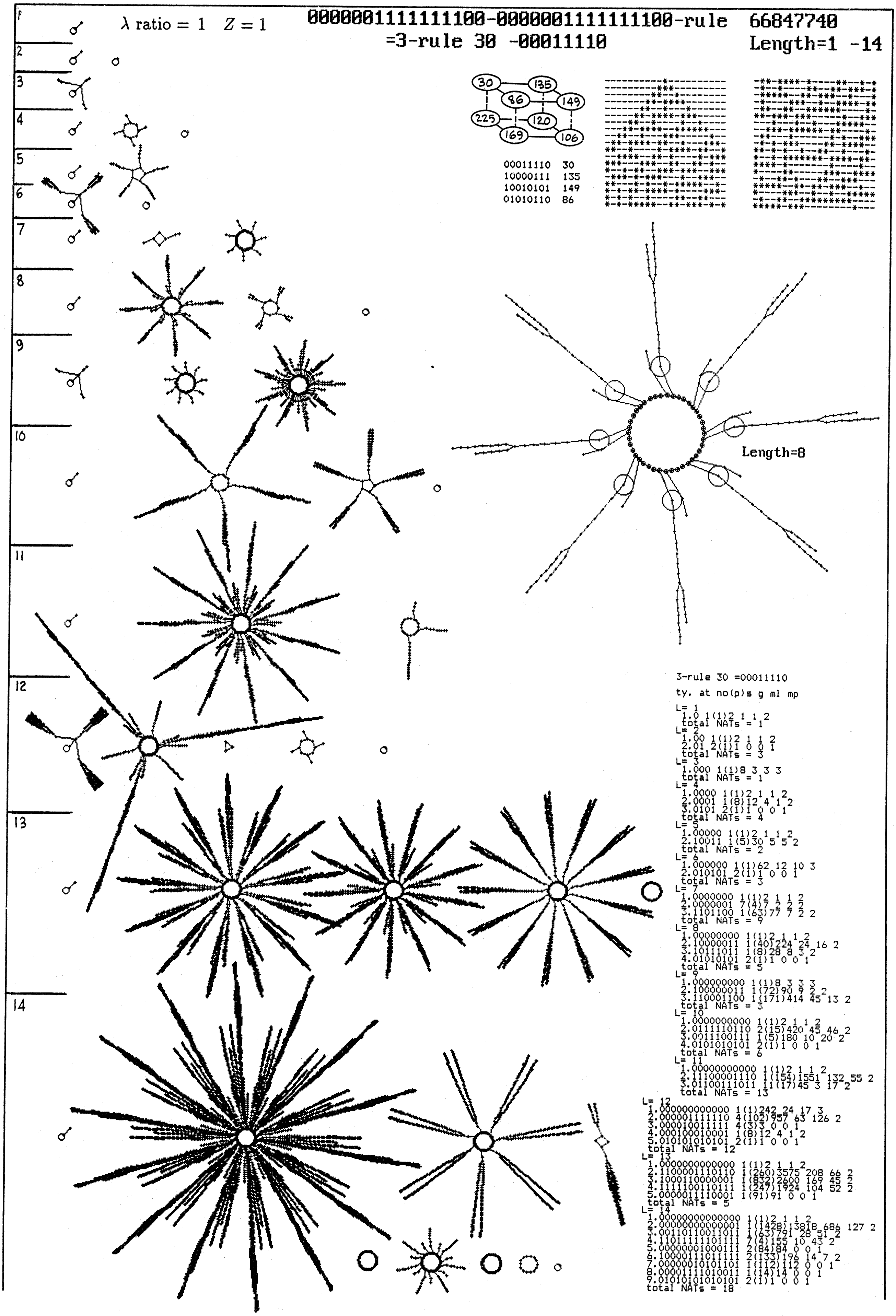


11100100 228
11011000 216
11001010 202
10101100 172



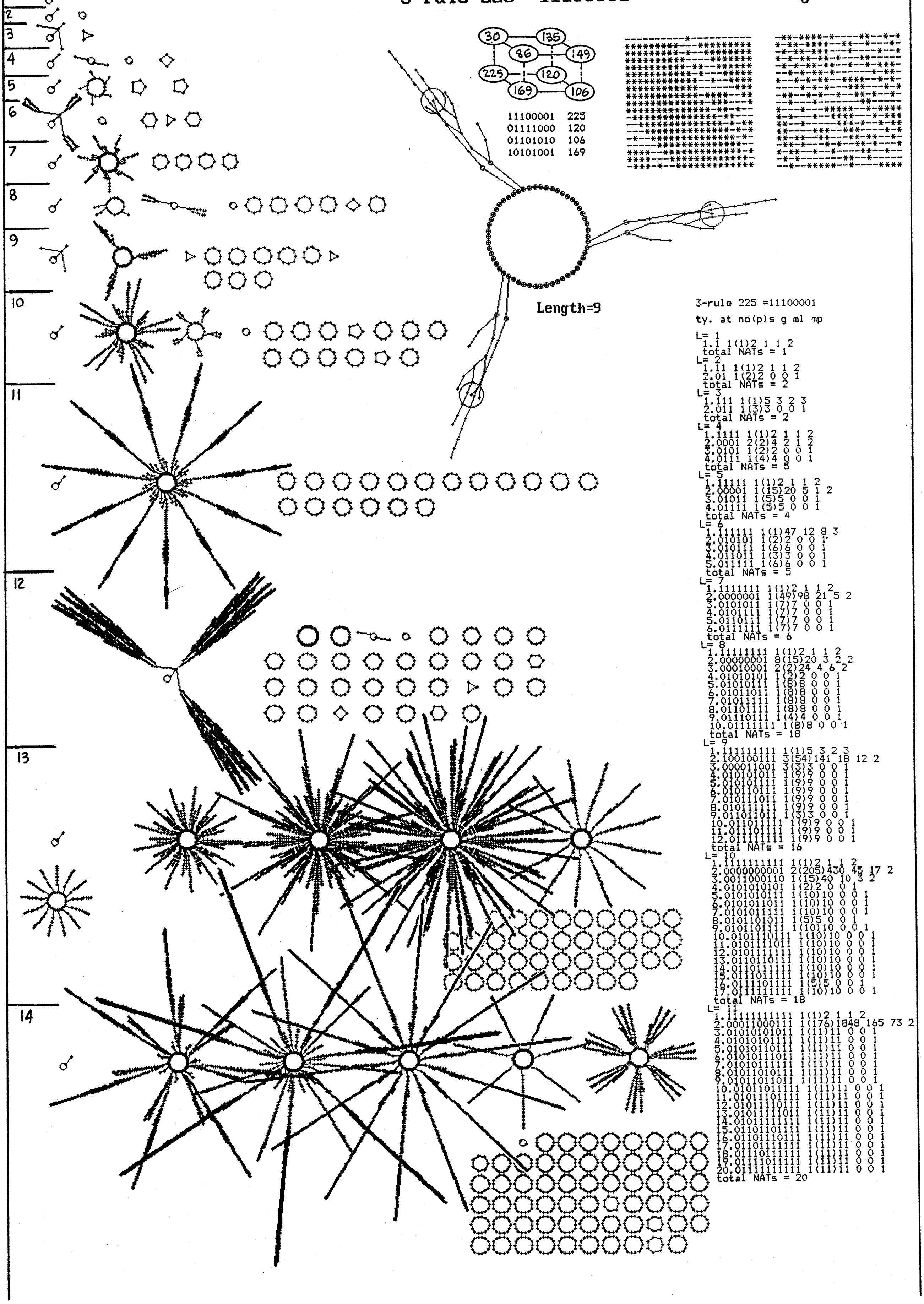



```
3-rule 228 =11100100

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.11 1(1)3 2 1 3
 total NATs = 2
L= 3
 1.000 1(1)1 0 0 1
 2.001 3(1)1 0 0 1
 3.011 1(3)3 0 0 1
 4.111 1(1)1 0 0 1
 total NATs = 6
L= 4
 1.0000 1(1)1 0 0 1
 2.0001 4(1)2 1 1 2
 3.1111 1(1)3 2 1 3
 4.0111 1(4)4 0 0 1
 total NATs = 7
L= 5
 1.00000 1(1)1 0 0 1
 2.00001 5(1)4 1 3 2
 3.11101 1(5)10 5 1 2
 4.11111 1(1)1 0 0 1
 total NATs = 8
L= 6
 1.000000 1(1)1 0 0 1
 2.000001 6(1)7 3 4 3
 3.001001 3(1)1 0 0 1
 4.111111 1(1)3 2 1 3
 5.111011 1(6)12 6 1 2
 6.011011 1(3)3 0 0 1
 total NATs = 13
L= 7
 1.0000000 1(1)1 0 0 1
 2.0000001 7(1)12 5 5 3
 3.0001001 7(1)2 1 1 2
 4.1111101 1(7)21 14 1 3
 5.0110111 1(7)7 0 0 1
 6.1111111 1(1)1 0 0 1
 total NATs = 18
L= 8
 1.00000000 1(1)1 0 0 1
 2.00000001 8(1)20 10 6 4
 3.00001001 8(1)4 1 3 2
 4.00010001 4(1)4 3 1 4
 5.11111111 1(1)3 2 1 3
 6.11111011 1(8)24 16 1 3
 7.11101101 1(8)16 8 1 2
 8.01110111 1(4)4 0 0 1
 total NATs = 25
L= 9
 1.000000000 1(1)1 0 0 1
 2.000000001 9(1)33 17 7 4
 3.000001001 9(1)7 3 4 3
 4.000010001 9(1)8 5 3 4
 5.001001001 3(1)1 0 0 1
 6.111111101 1(9)36 27 1 4
 7.111011011 1(9)18 9 1 2
 8.111011101 1(9)18 9 1 2
 9.011011011 1(3)3 0 0 1
 10.111111111 1(1)1 0 0 1
 total NATs = 36
L= 10
 1.0000000000 1(1)1 0 0 1
 2.0000000001 10(1)54 31 8 5
 3.0000001001 10(1)12 5 5 3
 4.0000010001 10(1)14 10 4 6
 5.0000100001 5(1)16 7 3 4
 6.0001001001 10(1)2 1 1 2
 7.1111111111 1(1)3 2 1 3
 8.1111111011 1(10)40 30 1 4
 9.1111101101 1(10)30 20 1 3
 10.1110111101 1(5)20 15 1 4
 11.1110111011 1(10)20 10 1 2
 12.0110110111 1(10)10 0 0 1
 total NATs = 52
L= 11
 1.00000000000 1(1)1 0 0 1
 2.00000000001 11(1)88 53 9 6
 3.00000001001 11(1)20 10 6 4
 4.00000010001 11(1)24 17 5 6
 5.00000100001 11(1)28 16 4 6
 6.00001001001 11(1)4 1 3 2
 7.00010001001 11(1)4 3 1 4
 8.11111111101 1(11)55 44 1 5
 9.11111011011 1(11)33 22 1 3
 10.11111011101 1(11)33 22 1 3
 11.11101111011 1(11)44 33 1 4
 12.11101101101 1(11)22 11 1 2
 13.01101110111 1(11)11 0 0 1
 14.11111111111 1(1)1 0 0 1
 total NATs = 74
L= 12
 1.000000000000 1(1)1 0 0 1
 2.000000000001 12(1)143 92 10 6
 3.000000001001 12(1)33 17 7 4
 4.000000010001 12(1)40 30 6 8
 5.000000100001 12(1)48 27 5 6
 6.000001000001 6(1)49 33 4 9
 7.000001001001 12(1)7 3 4 3
 8.000010001001 12(1)8 5 3 4
 9.000010010001 12(1)8 5 3 4
 10.000100010001 4(1)8 7 1 8
 11.001001001001 3(1)1 0 0 1
 12.111111111111 1(1)3 2 1 3
 13.111111111011 1(12)60 48 1 5
 14.111111101101 1(12)48 36 1 4
 15.111110111101 1(12)72 60 1 6
 16.111110111011 1(12)36 24 1 3
 17.111011011011 1(12)24 12 1 2
 18.111011011101 1(12)24 12 1 2
 19.111011111011 1(6)24 18 1 4
 20.111011101101 1(12)24 12 1 2
 21.011011011011 1(3)3 0 0 1
 22.011101110111 1(4)4 0 0 1
 total NATs = 109
```


λ ratio = 1 Z = 1
0000001111111100-0000001111111100-rule 66847740
=3-rule 30 -00011110
Length=1 -14
30
135
86
149
225
120
169
106
00011110 30
10000111 135
10010101 149
01010110 86
Length=8
1
2
3
4
5
6
7
8
9
10
11
12
13
14
3-rule 30 =00011110
ty. at no(p)s g ml mp
L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)2 1 1 2
2.01 2(1)1 0 0 1
total NATs = 3
L= 3
1.000 1(1)8 3 3 3
total NATs = 1
L= 4
1.0000 1(1)2 1 1 2
2.0001 1(8)12 4 1 2
3.0101 2(1)1 0 0 1
total NATs = 4
L= 5
1.00000 1(1)2 1 1 2
2.10011 1(5)30 5 5 2
total NATs = 2
L= 6
1.000000 1(1)62 12 10 3
2.010101 2(1)1 0 0 1
total NATs = 3
L= 7
1.0000000 1(1)2 1 1 2
2.0000001 7(4)7 2 2 2
3.1101100 1(63)77 7 2 2
total NATs = 9
L= 8
1.00000000 1(1)2 1 1 2
2.10000011 1(40)224 24 16 2
3.10111011 1(8)28 8 3 2
4.01010101 2(1)1 0 0 1
total NATs = 5
L= 9
1.000000000 1(1)8 3 3 3
2.100000011 1(72)90 9 2 2
3.110001100 1(171)414 45 13 2
total NATs = 3
L= 10
1.0000000000 1(1)2 1 1 2
2.0111110110 2(15)420 45 46 2
3.0011100111 1(5)180 10 20 2
4.0101010101 2(1)1 0 0 1
total NATs = 6
L= 11
1.00000000000 1(1)2 1 1 2
2.11100001110 1(154)1551 132 55 2
3.01100111011 11(17)45 3 17 2
total NATs = 13
L= 12
1.000000000000 1(1)242 24 17 3
2.000001111110 4(102)957 63 126 2
3.000010011111 4(3)3 0 0 1
4.000100010001 1(8)12 4 1 2
5.010101010101 2(1)1 0 0 1
total NATs = 12
L= 13
1.0000000000000 1(1)2 1 1 2
2.1100001110110 1(260)3575 208 66 2
3.1000110000001 1(832)2600 169 45 2
4.1111100110111 1(247)1924 104 52 2
5.0000011110001 1(91)91 0 0 1
total NATs = 5
L= 14
1.00000000000000 1(1)2 1 1 2
2.00000000000001 1(1428)13818 686 127 2
3.00110110011011 1(63)791 28 51 2
4.11011111101111 7(4)155 10 43 2
5.00000001000111 2(84)84 0 0 1
6.10000111011111 2(133)196 14 7 2
7.00000010101101 1(112)112 0 0 1
8.00001111010011 1(14)14 0 0 1
9.01010101010101 2(1)1 0 0 1
total NATs = 18

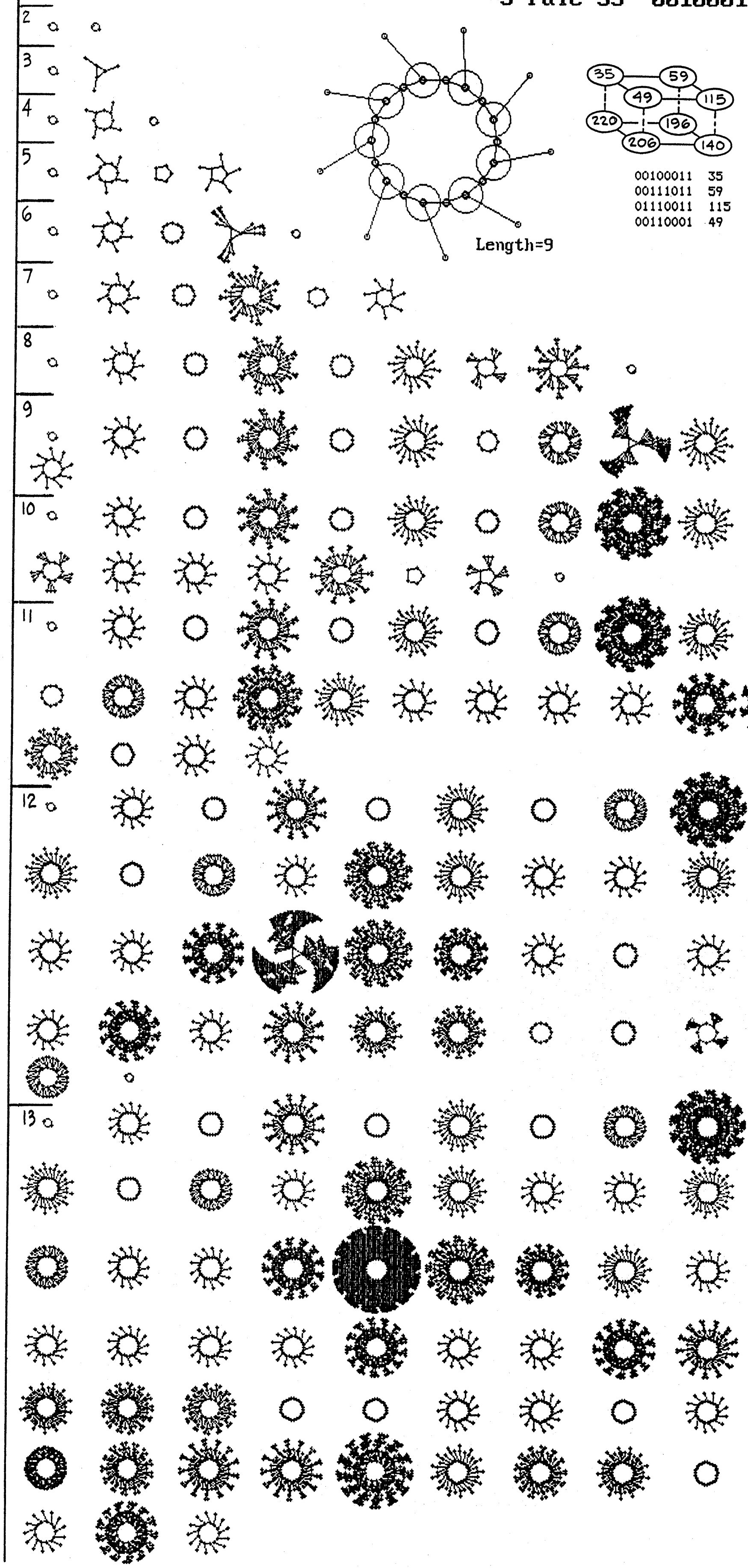
λ ratio = 1   Z = 1

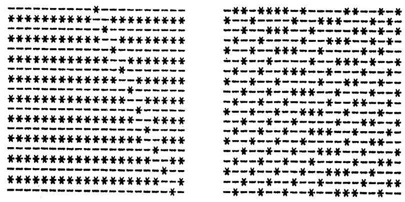
1111110000000011-1111110000000011-rule 4228119555
=3-rule 225 -11100001
Length=1 -14




```
3-rule 225 =11100001

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.111 1(1)5 3 2 3
 2.011 1(3)3 0 0 1
 total NATs = 2
L= 4
 1.1111 1(1)2 1 1 2
 2.0001 2(2)4 2 1 2
 3.0101 1(2)2 0 0 1
 4.0111 1(4)4 0 0 1
 total NATs = 5
L= 5
 1.11111 1(1)2 1 1 2
 2.00001 1(15)20 5 1 2
 3.01011 1(5)5 0 0 1
 4.01111 1(5)5 0 0 1
 total NATs = 4
L= 6
 1.111111 1(1)47 12 8 3
 2.010101 1(2)2 0 0 1
 3.010111 1(6)6 0 0 1
 4.011011 1(3)3 0 0 1
 5.011111 1(6)6 0 0 1
 total NATs = 5
L= 7
 1.1111111 1(1)2 1 1 2
 2.0000001 1(49)98 21 5 2
 3.0101011 1(7)7 0 0 1
 4.0101111 1(7)7 0 0 1
 5.0110111 1(7)7 0 0 1
 6.0111111 1(7)7 0 0 1
 total NATs = 6
L= 8
 1.11111111 1(1)2 1 1 2
 2.00000001 8(15)20 3 2 2
 3.00010001 2(2)24 4 6 2
 4.01010101 1(2)2 0 0 1
 5.01010111 1(8)8 0 0 1
 6.01011011 1(8)8 0 0 1
 7.01011111 1(8)8 0 0 1
 8.01101111 1(8)8 0 0 1
 9.01110111 1(4)4 0 0 1
 10.01111111 1(8)8 0 0 1
 total NATs = 18
L= 9
 1.111111111 1(1)5 3 2 3
 2.100100111 3(54)141 18 12 2
 3.000011001 3(3)3 0 0 1
 4.010101011 1(9)9 0 0 1
 5.010101111 1(9)9 0 0 1
 6.010110111 1(9)9 0 0 1
 7.010111011 1(9)9 0 0 1
 8.010111111 1(9)9 0 0 1
 9.011011011 1(3)3 0 0 1
 10.011011111 1(9)9 0 0 1
 11.011101111 1(9)9 0 0 1
 12.011111111 1(9)9 0 0 1
 total NATs = 16
L= 10
 1.1111111111 1(1)2 1 1 2
 2.0000000001 2(205)430 45 17 2
 3.0011000110 1(15)40 10 3 2
 4.0101010101 1(2)2 0 0 1
 5.0101010111 1(10)10 0 0 1
 6.0101011011 1(10)10 0 0 1
 7.0101011111 1(10)10 0 0 1
 8.0101101011 1(5)5 0 0 1
 9.0101101111 1(10)10 0 0 1
 10.0101110111 1(10)10 0 0 1
 11.0101111011 1(10)10 0 0 1
 12.0101111111 1(10)10 0 0 1
 13.0110110111 1(10)10 0 0 1
 14.0110111111 1(10)10 0 0 1
 15.0111011111 1(10)10 0 0 1
 16.0111101111 1(5)5 0 0 1
 17.0111111111 1(10)10 0 0 1
 total NATs = 18
L= 11
 1.11111111111 1(1)2 1 1 2
 2.00011000111 1(176)1848 165 73 2
 3.01010101011 1(11)11 0 0 1
 4.01010101111 1(11)11 0 0 1
 5.01010110111 1(11)11 0 0 1
 6.01010111011 1(11)11 0 0 1
 7.01010111111 1(11)11 0 0 1
 8.01011010111 1(11)11 0 0 1
 9.01011011011 1(11)11 0 0 1
 10.01011011111 1(11)11 0 0 1
 11.01011101111 1(11)11 0 0 1
 12.01011110111 1(11)11 0 0 1
 13.01011111011 1(11)11 0 0 1
 14.01011111111 1(11)11 0 0 1
 15.01101101111 1(11)11 0 0 1
 16.01101110111 1(11)11 0 0 1
 17.01101111111 1(11)11 0 0 1
 18.01110111111 1(11)11 0 0 1
 19.01111011111 1(11)11 0 0 1
 20.01111111111 1(11)11 0 0 1
 total NATs = 20
```

λ ratio = .75 Z = .625 0000110000001111-0000110000001111-rule 202312719
=3-rule 35 -00100011 Length=1 -13





3-rule 35 =00100011

ty. at no(p)s g ml mp

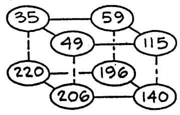
L= 1
1.0 1(2)2 0 0 1
total NATs = 1
L= 2
1.00 1(2)2 0 0 1
2.01 1(2)2 0 0 1
total NATs = 2
L= 3
1.000 1(2)2 0 0 1
2.001 1(3)6 3 1 2
total NATs = 2
L= 4
1.0000 1(2)2 0 0 1
2.0001 1(8)12 4 1 2
3.0101 1(2)2 0 0 1
total NATs = 3
L= 5
1.00000 1(2)2 0 0 1
2.00001 1(10)15 5 1 2
3.00011 1(5)5 0 0 1
4.00101 1(5)10 5 1 2
total NATs = 4
L= 6
1.000000 1(2)2 0 0 1
2.000001 1(12)18 6 1 2
3.000011 1(12)12 0 0 1
4.100100 1(3)30 15 3 4
5.010101 1(2)2 0 0 1
total NATs = 5
L= 7
1.0000000 1(2)2 0 0 1
2.0000001 1(14)21 7 1 2
3.0000011 1(14)14 0 0 1
4.1000100 1(14)70 35 3 4
5.0000111 1(7)7 0 0 1
6.0010101 1(7)14 7 1 2
total NATs = 6
L= 8
1.00000000 1(2)2 0 0 1
2.00000001 1(16)24 8 1 2
3.00000011 1(16)16 0 0 1
4.10000100 1(16)80 40 3 4
5.00000111 1(16)16 0 0 1
6.10001001 1(16)40 16 2 2
7.00010001 1(8)20 12 1 4
8.10010100 1(8)56 32 3 4
9.01010101 1(2)2 0 0 1
total NATs = 9
L= 9
1.000000000 1(2)2 0 0 1
2.000000001 1(18)27 9 1 2
3.000000011 1(18)18 0 0 1
4.100000100 1(18)90 45 3 4
5.000000111 1(18)18 0 0 1
6.100001001 1(18)45 18 2 2
7.000001111 1(9)9 0 0 1
8.000010001 1(18)45 27 1 4
9.010010010 1(3)168 102 5 8
10.100010011 1(18)45 18 2 2
11.000100011 1(18)27 9 1 2
12.001010101 1(9)18 9 1 2
total NATs = 12
L= 10
1.0000000000 1(2)2 0 0 1
2.0000000001 1(20)30 10 1 2
3.0000000011 1(20)20 0 0 1
4.1000000100 1(20)100 50 3 4
5.0000000111 1(20)20 0 0 1
6.1000001001 1(20)50 20 2 2
7.0000001111 1(20)20 0 0 1
8.0000010001 1(20)50 30 1 4
9.0100010010 1(20)420 260 5 8
10.1000010011 1(20)50 20 2 2
11.0000011001 1(20)50 20 2 2
12.0000100001 1(10)25 15 1 4
13.0000100011 1(20)30 10 1 2
14.0000110001 1(20)30 10 1 2
15.0000110011 1(20)30 10 1 2
16.1001010100 1(10)70 40 3 4
17.0001100011 1(5)5 0 0 1
18.0010100101 1(5)20 15 1 4
19.0101010101 1(2)2 0 0 1
total NATs = 19
L= 11
1.00000000000 1(2)2 0 0 1
2.00000000001 1(22)33 11 1 2
3.00000000011 1(22)22 0 0 1
4.10000000100 1(22)110 55 3 4
5.00000000111 1(22)22 0 0 1
6.10000001001 1(22)55 22 2 2
7.00000001111 1(22)22 0 0 1
8.00000010001 1(22)55 33 1 4
9.01000010010 1(22)462 286 5 8
10.10000010011 1(22)55 22 2 2
11.00000011001 1(22)55 22 2 2
12.00000011111 1(11)11 0 0 1
13.00000100001 1(22)55 33 1 4
14.00000100011 1(22)33 11 1 2
15.11000100100 1(22)209 110 4 4
16.10000100111 1(22)55 22 2 2
17.00000110001 1(22)33 11 1 2
18.00000110011 1(22)33 11 1 2
19.00001000011 1(22)33 11 1 2
20.00001000111 1(22)33 11 1 2
21.10001001000 1(22)187 132 3 8
22.01001010010 1(11)330 220 5 8
23.10001001100 1(11)66 44 2 4
24.00001100011 1(22)22 0 0 1
25.00001100111 1(22)33 11 1 2
26.00101010101 1(11)22 11 1 2
total NATs = 26
L= 12
1.000000000000 1(2)2 0 0 1
2.000000000001 1(24)36 12 1 2
3.000000000011 1(24)24 0 0 1
4.100000000100 1(24)120 60 3 4
5.000000000111 1(24)24 0 0 1

20.000001000001 1(12)30 18 1 4
21.000001000011 1(24)36 12 1 2
22.000001000111 1(24)36 12 1 2
23.100001001000 1(24)204 144 3 8
24.001001001001 1(3)990 687 7 16
25.110001001001 1(24)228 120 4 4

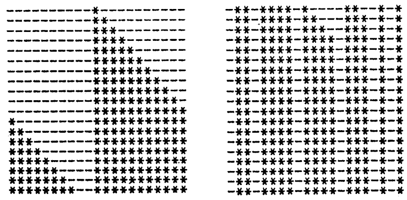
40.100101010100 1(12)84 48 3 4
41.001010010101 1(12)48 36 1 4
42.010101010101 1(2)2 0 0 1
total NATs = 42

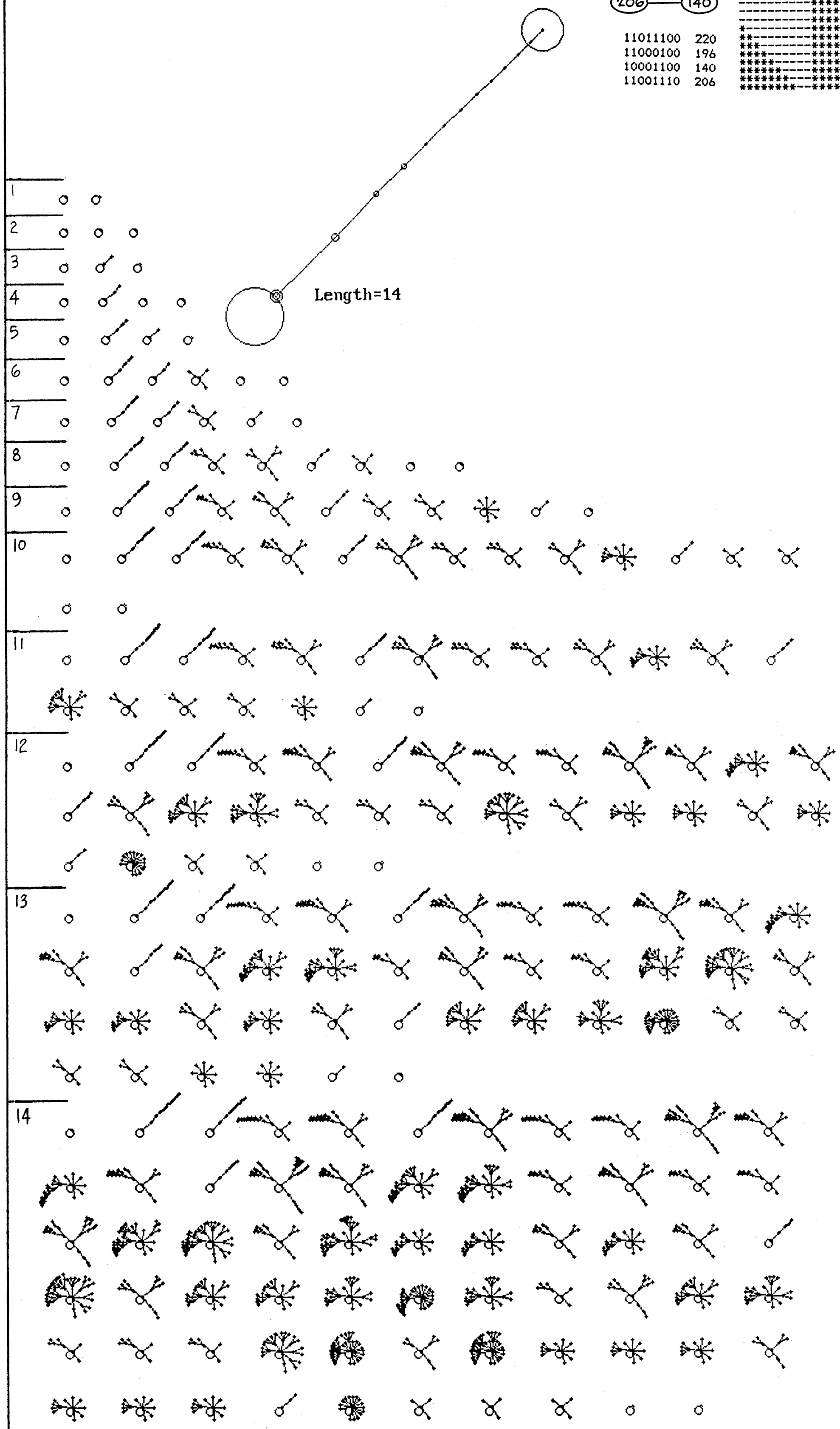
λ ratio = .75   Z = .625

1111001111110000-1111001111110000-rule 4092654576
=3-rule 220 -11011100   Length=1 -14



11011100	220
11000100	196
10001100	140
11001110	206



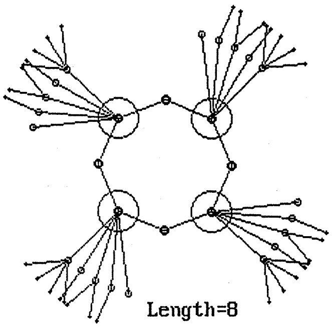


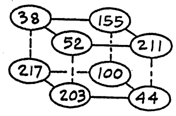

```
3-rule 220 =11011100

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 2(1)1 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 4
L= 3
 1.000 1(1)1 0 0 1
 2.101 3(1)2 1 1 2
 3.111 1(1)1 0 0 1
 total NATs = 5
L= 4
 1.0000 1(1)1 0 0 1
 2.1101 4(1)3 1 2 2
 3.0101 2(1)1 0 0 1
 4.1111 1(1)1 0 0 1
 total NATs = 8
L= 5
 1.00000 1(1)1 0 0 1
 2.11101 5(1)4 1 3 2
 3.10101 5(1)2 1 1 2
 4.11111 1(1)1 0 0 1
 total NATs = 12
L= 6
 1.000000 1(1)1 0 0 1
 2.111101 6(1)5 1 4 2
 3.110101 6(1)3 1 2 2
 4.101101 3(1)4 3 1 4
 5.010101 2(1)1 0 0 1
 6.111111 1(1)1 0 0 1
 total NATs = 19
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111101 7(1)6 1 5 2
 3.1110101 7(1)4 1 3 2
 4.1101101 7(1)6 4 2 4
 5.1010101 7(1)2 1 1 2
 6.1111111 1(1)1 0 0 1
 total NATs = 30
L= 8
 1.00000000 1(1)1 0 0 1
 2.11111101 8(1)7 1 6 2
 3.11110101 8(1)5 1 4 2
 4.11101101 8(1)8 5 3 4
 5.11011101 4(1)9 5 2 4
 6.11010101 8(1)3 1 2 2
 7.10110101 8(1)4 3 1 4
 8.01010101 2(1)1 0 0 1
 9.11111111 1(1)1 0 0 1
 total NATs = 48
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111101 9(1)8 1 7 2
 3.111110101 9(1)6 1 5 2
 4.111101101 9(1)10 6 4 4
 5.111011101 9(1)12 6 3 4
 6.111010101 9(1)4 1 3 2
 7.110110101 9(1)6 4 2 4
 8.110101101 9(1)6 4 2 4
 9.101101101 3(1)8 7 1 8
 10.101010101 9(1)2 1 1 2
 11.111111111 1(1)1 0 0 1
 total NATs = 77
L= 10
 1.0000000000 1(1)1 0 0 1
 2.1111111101 10(1)9 1 8 2
 3.1111110101 10(1)7 1 6 2
 4.1111101101 10(1)12 7 5 4
 5.1111011101 10(1)15 7 4 4
 6.1111010101 10(1)5 1 4 2
 7.1110111101 5(1)16 7 3 4
 8.1110110101 10(1)8 5 3 4
 9.1110101101 10(1)8 5 3 4
 10.1101110101 10(1)9 5 2 4
 11.1101101101 10(1)12 10 2 8
 12.1101010101 10(1)3 1 2 2
 13.1011010101 10(1)4 3 1 4
 14.1010110101 5(1)4 3 1 4
 15.0101010101 2(1)1 0 0 1
 16.1111111111 1(1)1 0 0 1
 total NATs = 124
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111101 11(1)10 1 9 2
 3.11111110101 11(1)8 1 7 2
 4.11111101101 11(1)14 8 6 4
 5.11111011101 11(1)18 8 5 4
 6.11111010101 11(1)6 1 5 2
 7.11110111101 11(1)20 8 4 4
 8.11110110101 11(1)10 6 4 4
 9.11110101101 11(1)10 6 4 4
 10.11101110101 11(1)12 6 3 4
 11.11101101101 11(1)16 13 3 8
 12.11101011101 11(1)12 6 3 4
 13.11101010101 11(1)4 1 3 2
 14.11011101101 11(1)18 14 2 8
 15.11011010101 11(1)6 4 2 4
 16.11010110101 11(1)6 4 2 4
 17.11010101101 11(1)6 4 2 4
 18.10110110101 11(1)8 7 1 8
 19.10101010101 11(1)2 1 1 2
 20.11111111111 1(1)1 0 0 1
 total NATs = 200
L= 12
 1.000000000000 1(1)1 0 0 1
 2.111111111101 12(1)11 1 10 2
 3.111111110101 12(1)9 1 8 2
 4.111111101101 12(1)16 9 7 4
 5.111111011101 12(1)21 9 6 4
 6.111111010101 12(1)7 1 6 2
 7.111110111101 12(1)24 9 5 4
 8.111110110101 12(1)12 7 5 4
 9.111110101101 12(1)12 7 5 4
 10.111101111101 6(1)25 9 4 4

 25.110101110101 6(1)9 5 2 4
 26.110101101101 12(1)12 10 2 8
 27.110101010101 12(1)3 1 2 2
 28.101101101101 3(1)16 15 1 16
 29.101101010101 12(1)4 3 1 4
 30.101011010101 12(1)4 3 1 4
 31.010101010101 2(1)1 0 0 1
 32.111111111111 1(1)1 0 0 1
 total NATs = 323
```

λ ratio = .75 $Z = .75$

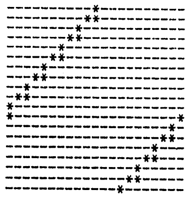
0000110000111100-0000110000111100-rule 205261884

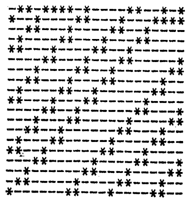
=3-rule 38 -00100110 Length=1 -14

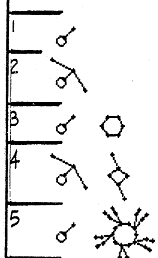


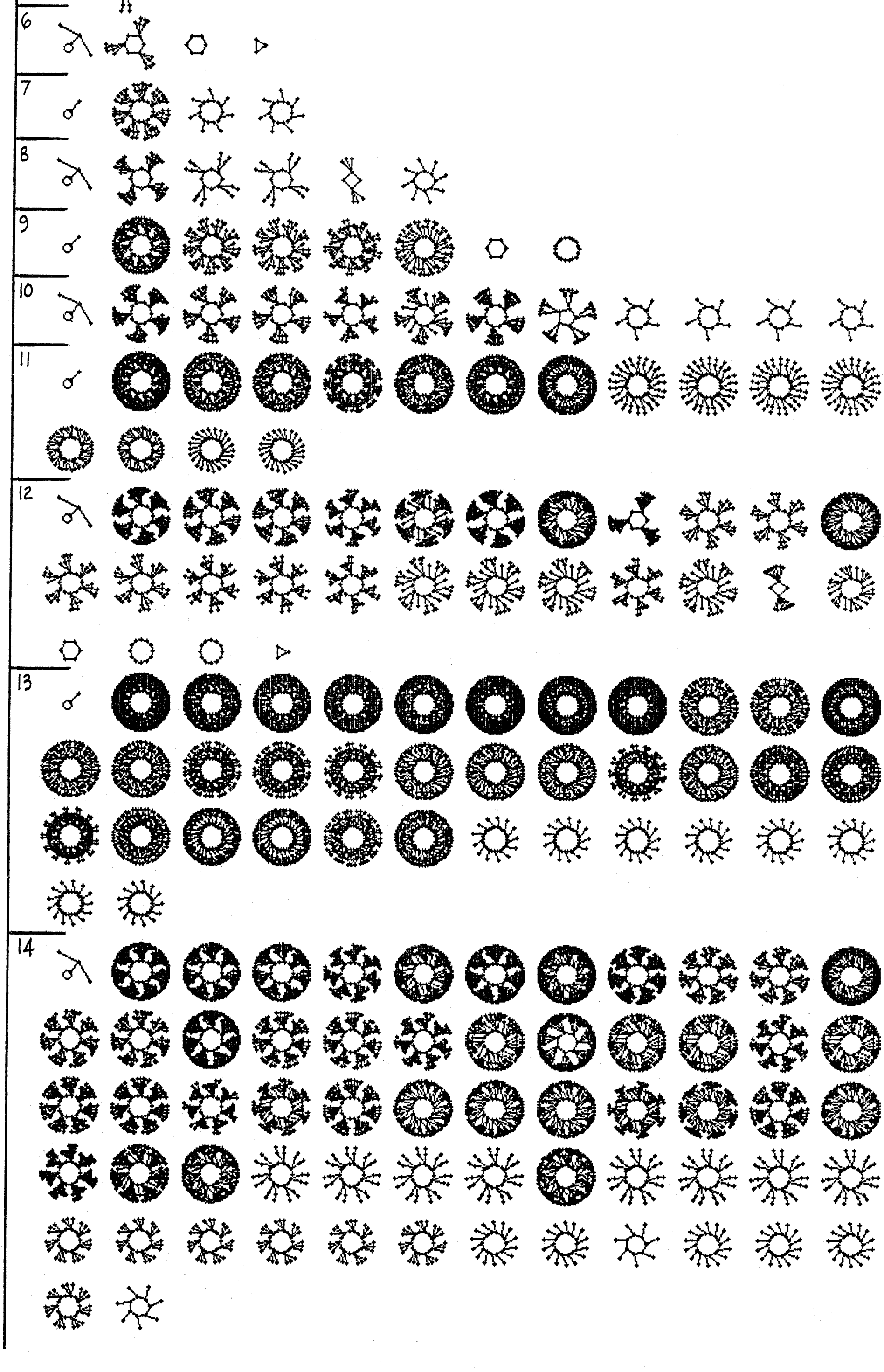


00100110 38
10011011 155
11010011 211
00110100 52









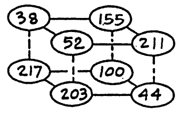
3-rule 38 =00100110

ty. at no(p)s g ml mp

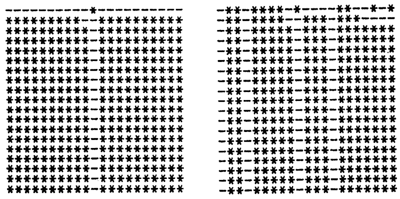
L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)4 2 2 2
total NATs = 1
L= 3
1.000 1(1)2 1 1 2
2.001 1(6)6 0 0 1
total NATs = 2
L= 4
1.0000 1(1)4 2 2 2
2.0001 2(4)6 2 1 2
total NATs = 3
L= 5
1.00000 1(1)2 1 1 2
2.00001 1(10)30 10 2 3
total NATs = 2
L= 6
1.000000 1(1)4 2 2 2
2.000001 2(6)24 12 2 4
3.001001 1(6)6 0 0 1
4.001101 2(3)3 0 0 1
total NATs = 6
L= 7
1.0000000 1(1)2 1 1 2
2.0000001 1(14)84 42 2 5
3.0001001 1(14)21 7 1 2
4.0001101 1(14)21 7 1 2
total NATs = 4
L= 8
1.00000000 1(1)4 2 2 2
2.00000001 2(8)60 36 2 6
3.00001001 2(8)24 8 2 3
4.00001101 2(8)24 8 2 3
5.00010001 2(4)10 6 1 4
6.00011001 1(8)16 8 1 2
total NATs = 10
L= 9
1.000000000 1(1)2 1 1 2
2.000000001 1(18)180 108 2 7
3.000001001 1(18)72 36 2 4
4.000001101 1(18)72 36 2 4
5.000010001 1(18)81 45 2 6
6.000011001 1(18)81 45 2 3
7.001001001 1(6)6 0 0 1
8.001001101 1(18)18 0 0 1
total NATs = 8
L= 10
1.0000000000 1(1)4 2 2 2
2.0000000001 2(10)120 80 2 8
3.0000001001 2(10)60 30 2 5
4.0000001101 2(10)60 30 2 5
5.0000010001 2(10)60 40 2 8
6.0000011001 2(10)60 40 2 4
7.0000100001 1(10)90 40 2 9
8.0000110001 2(5)45 30 2 3
9.0001001001 2(10)15 5 1 2
10.0001001101 2(10)15 5 1 2
11.0001101001 2(10)15 5 1 2
12.0001101101 2(10)15 5 1 2
total NATs = 22
L= 11
1.00000000000 1(1)2 1 1 2
2.00000000001 1(22)330 220 2 9
3.00000001001 1(22)165 99 2 6
4.00000001101 1(22)165 99 2 6
5.00000010001 1(22)187 121 2 10
6.00000011001 1(22)209 143 2 6
7.00000100001 1(22)264 154 2 12
8.00000110001 1(22)264 198 2 6
9.00001001001 1(22)66 22 2 3
10.00001001101 1(22)66 22 2 3
11.00001101001 1(22)66 22 2 3
12.00001101101 1(22)66 22 2 3
13.00010001001 1(22)55 33 1 4
14.00010001101 1(22)55 33 1 4
15.00010011001 1(22)44 22 1 2
16.00011001001 1(22)44 22 1 2
total NATs = 16
L= 12
1.000000000000 1(1)4 2 2 2
2.000000000001 2(12)210 150 2 10
3.000000001001 2(12)120 72 2 7
4.000000001101 2(12)120 72 2 7
5.000000010001 2(12)126 90 2 12
6.000000011001 2(12)144 108 2 8
7.000000100001 2(12)216 120 2 15
8.000000110001 2(12)216 168 2 9
9.000001000001 2(6)96 66 2 16
10.000001001001 2(12)48 24 2 4
11.000001001101 2(12)48 24 2 4
12.000001100001 1(12)192 156 2 8
13.000001101001 2(12)48 24 2 4
14.000001101101 2(12)48 24 2 4
15.000010001001 2(12)54 30 2 6
16.000010001101 2(12)54 30 2 6
17.000010010001 2(12)54 30 2 6
18.000010011001 2(12)54 30 2 3
19.000011001001 2(12)54 30 2 3
20.000011001101 2(12)54 30 2 3
21.000011010001 2(12)54 30 2 6
22.000011011001 2(12)54 30 2 3
23.000100010001 2(4)18 14 1 8
24.000100011001 2(12)36 24 1 4
25.001001001001 1(6)6 0 0 1
26.001001001101 2(12)12 0 0 1
27.001001101101 1(12)12 0 0 1
28.001101001101 2(3)3 0 0 1
total NATs = 52

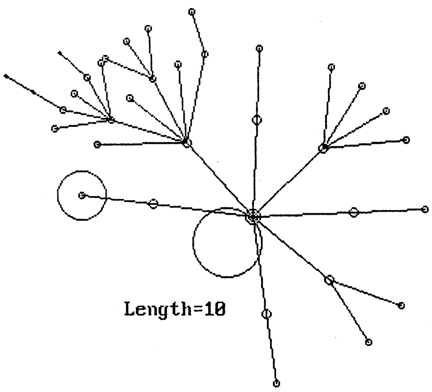
λ ratio = .75 Z = .75

1111001111000011-1111001111000011-rule 4089705411
=3-rule 217 -11011001 Length=1 -15

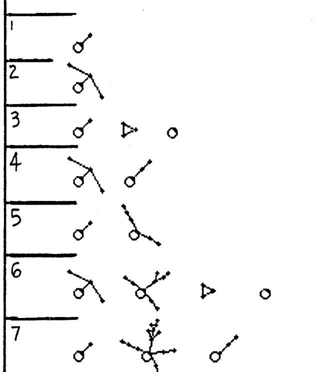


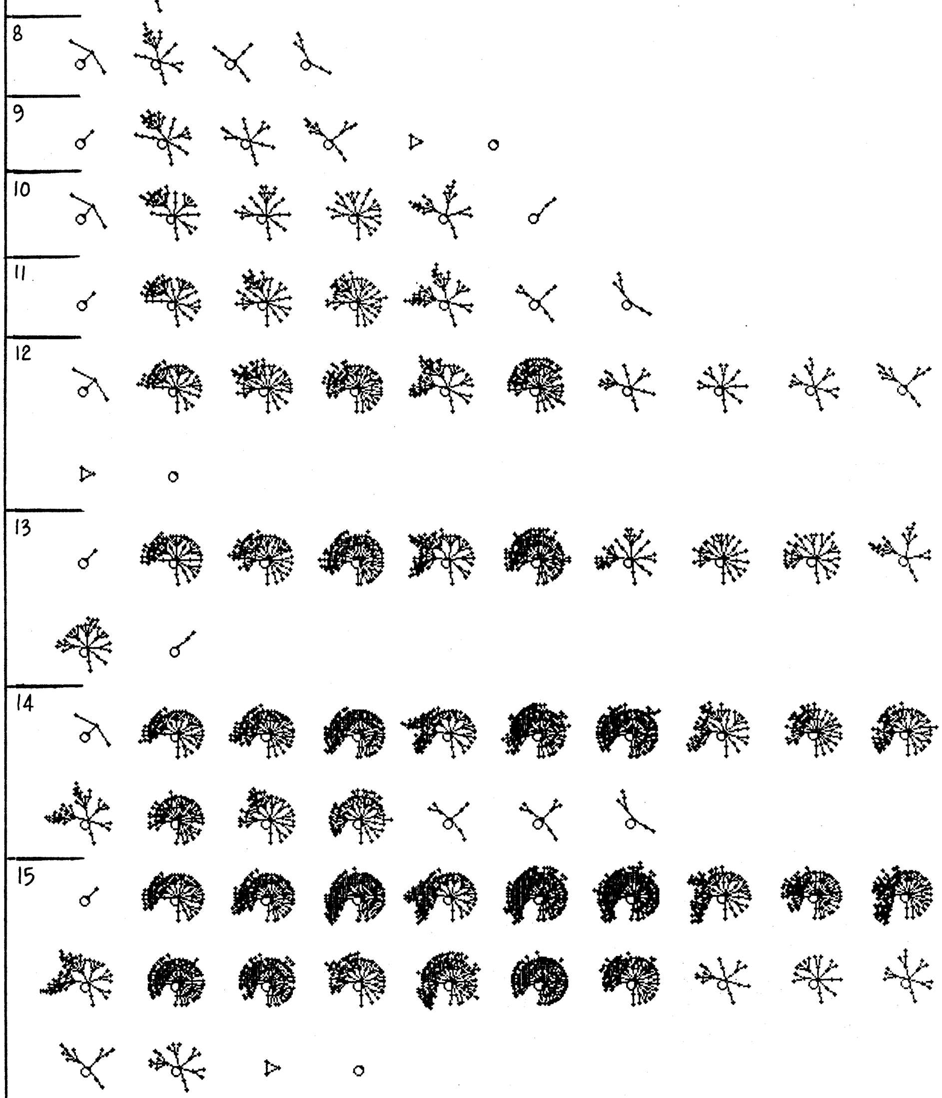
11011001 217
01100100 100
00101100 44
11001011 203





Length=10



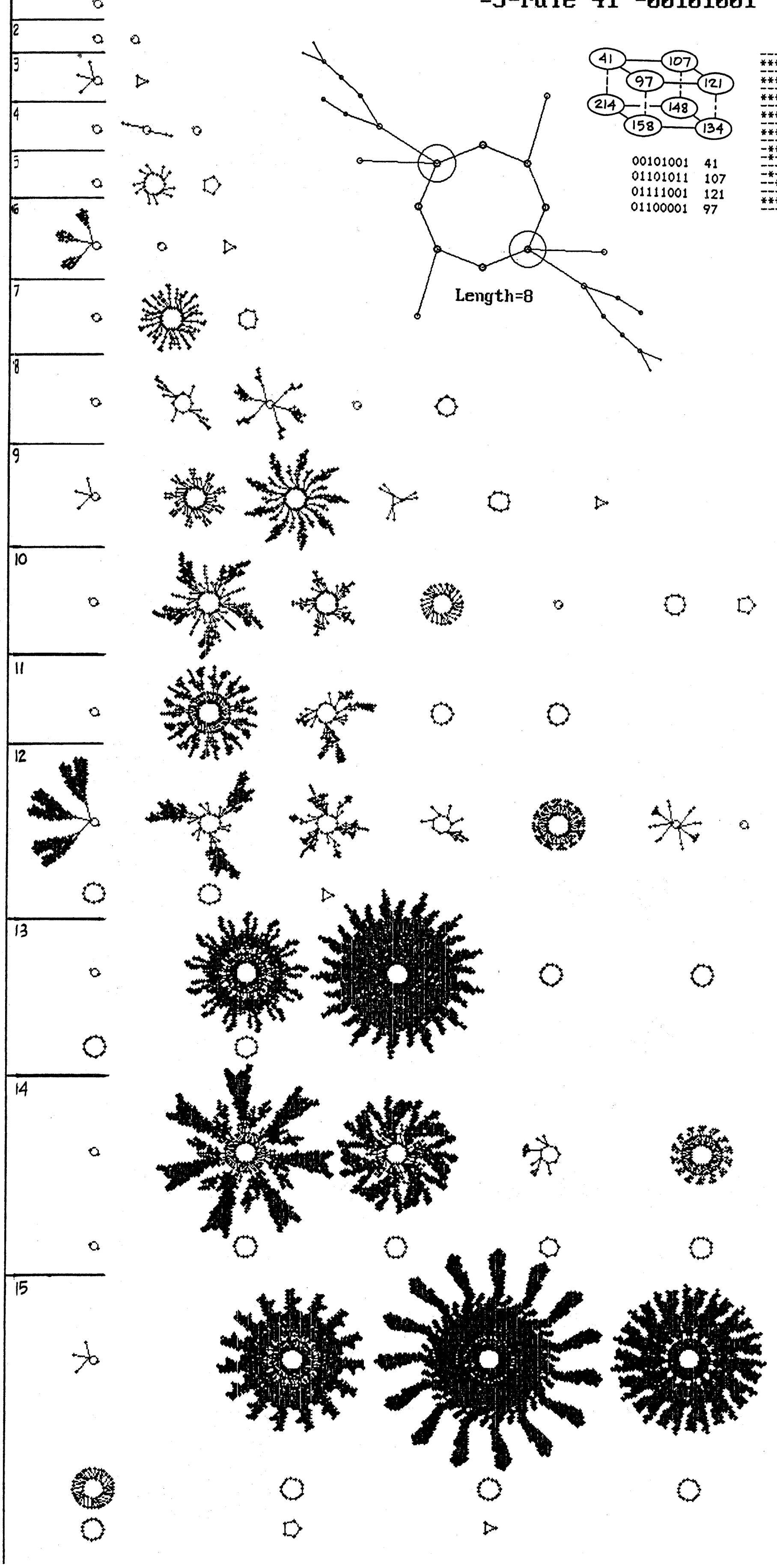



```
3-rule 217 =11011001

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.111 1(1)2 1 1 2
 2.001 1(3)3 0 0 1
 3.011 3(1)1 0 0 1
 total NATs = 5
L= 4
 1.1111 1(1)4 2 2 2
 2.1110 4(1)3 1 2 2
 total NATs = 5
L= 5
 1.11111 1(1)2 1 1 2
 2.11110 5(1)6 2 3 3
 total NATs = 6
L= 6
 1.111111 1(1)4 2 2 2
 2.111110 6(1)9 4 3 4
 3.001001 1(3)3 0 0 1
 4.011011 3(1)1 0 0 1
 total NATs = 11
L= 7
 1.1111111 1(1)2 1 1 2
 2.1111110 7(1)15 7 4 5
 3.1101110 7(1)3 1 2 2
 total NATs = 15
L= 8
 1.11111111 1(1)4 2 2 2
 2.11111110 8(1)21 11 5 6
 3.11101110 4(1)7 3 2 4
 4.11011110 8(1)7 3 3 3
 total NATs = 21
L= 9
 1.111111111 1(1)2 1 1 2
 2.111111110 9(1)28 16 4 7
 3.111101110 9(1)14 7 3 6
 4.111011011 9(1)14 7 4 4
 5.001001001 1(3)3 0 0 1
 6.011011011 3(1)1 0 0 1
 total NATs = 32
L= 10
 1.1111111111 1(1)4 2 2 2
 2.1111111110 10(1)36 22 5 8
 3.1111101110 10(1)24 13 3 8
 4.1111011110 5(1)28 16 3 9
 5.1111011011 10(1)25 14 5 5
 6.1101101110 10(1)3 1 2 2
 total NATs = 46
L= 11
 1.11111111111 1(1)2 1 1 2
 2.11111111110 11(1)45 29 4 9
 3.11111101110 11(1)38 22 4 10
 4.11111011110 11(1)48 30 3 12
 5.11111011011 11(1)40 24 6 6
 6.11101101110 11(1)8 4 2 4
 7.11011011110 11(1)7 3 3 3
 total NATs = 67
L= 12
 1.111111111111 1(1)4 2 2 2
 2.111111111110 12(1)55 37 5 10
 3.111111101110 12(1)55 34 5 12
 4.111111011110 12(1)76 50 4 15
 5.111111011011 12(1)58 38 6 7
 6.111110111110 6(1)83 54 3 16
 7.111101101110 12(1)18 10 3 6
 8.111011101110 4(1)18 10 2 8
 9.111011011110 12(1)17 10 3 6
 10.111011011011 12(1)14 7 4 4
 11.001001001001 1(3)3 0 0 1
 12.011011011011 3(1)1 0 0 1
 total NATs = 99
L= 13
 1.1111111111111 1(1)2 1 1 2
 2.1111111111110 13(1)66 46 4 11
 3.1111111101110 13(1)75 49 5 14
 4.1111111011110 13(1)110 77 5 18
 5.1111111011011 13(1)82 56 7 8
 6.1111110111110 13(1)129 88 4 20
 7.1111101101110 13(1)35 21 4 8
 8.1111011101110 13(1)36 22 3 12
 9.1111011011110 13(1)38 24 3 9
 10.1111011011011 13(1)25 14 5 5
 11.1110111011011 13(1)31 20 3 8
 12.1101101101110 13(1)3 1 2 2
 total NATs = 144
L= 14
 1.11111111111111 1(1)4 2 2 2
 2.11111111111110 14(1)78 56 5 12
 3.11111111101110 14(1)98 67 5 16
 4.11111111011110 14(1)151 111 5 21
 5.11111111011011 14(1)109 78 8 9
 6.11111110111110 14(1)186 133 5 24
 7.11111101111110 7(1)199 141 4 25
 8.11111101101110 14(1)60 38 5 10
 9.11111011101110 14(1)63 41 3 16
 10.11111011011110 14(1)73 49 4 12
 11.11111011011011 14(1)41 25 6 6
 12.11110111101110 14(1)72 48 3 18
 13.11110111011011 14(1)51 35 4 10
 14.11110110111110 14(1)69 47 3 12
 15.11101101101110 14(1)8 4 2 4
 16.11011101101110 7(1)9 5 2 4
 17.11011011011110 14(1)7 3 3 3
 total NATs = 211
L= 15
 1.111111111111111 1(1)2 1 1 2
 2.111111111111110 15(1)91 67 4 13
 3.111111111101110 15(1)124 88 5 18
 4.111111111011110 15(1)199 152 5 24
 5.111111111011011 15(1)140 104 7 10
 6.111111110111110 15(1)254 189 5 28
 7.111111101111110 15(1)284 210 5 30
 8.111111101101110 15(1)94 63 6 12
 9.111111011101110 15(1)100 68 4 20
 10.111111011011110 15(1)123 86 5 15

 20.111011011011110 15(1)17 10 3 6
 21.111011011011011 15(1)14 7 4 4
 22.110111101101110 15(1)21 13 3 6
 23.001001001001001 1(3)3 0 0 1
 24.011011011011011 3(1)1 0 0 1
 total NATs = 310
```

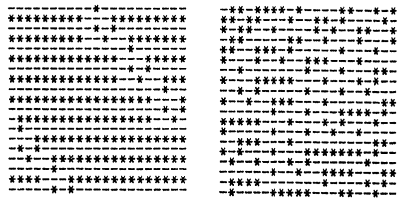
λ ratio = .75 Z = .75

00001100011000011-00001100011000011-rule 214109379
=3-rule 41 -00101001

Length=1 -15



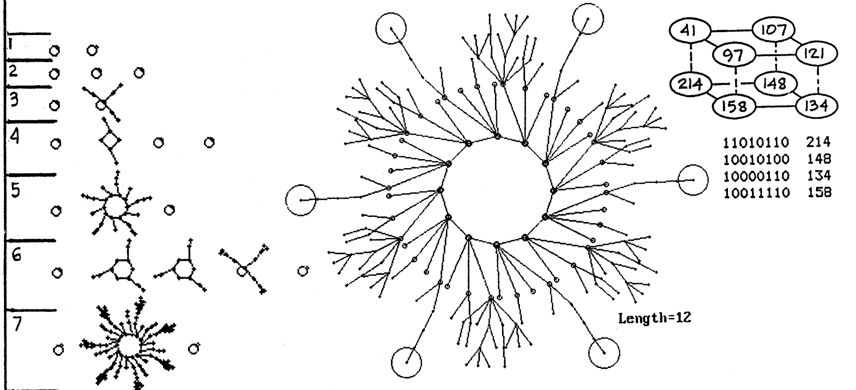
00101001 41
01101011 107
01111001 121
01100001 97

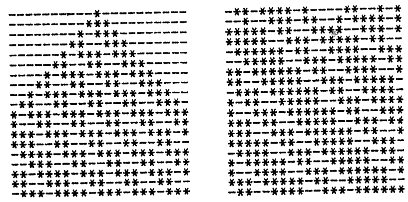


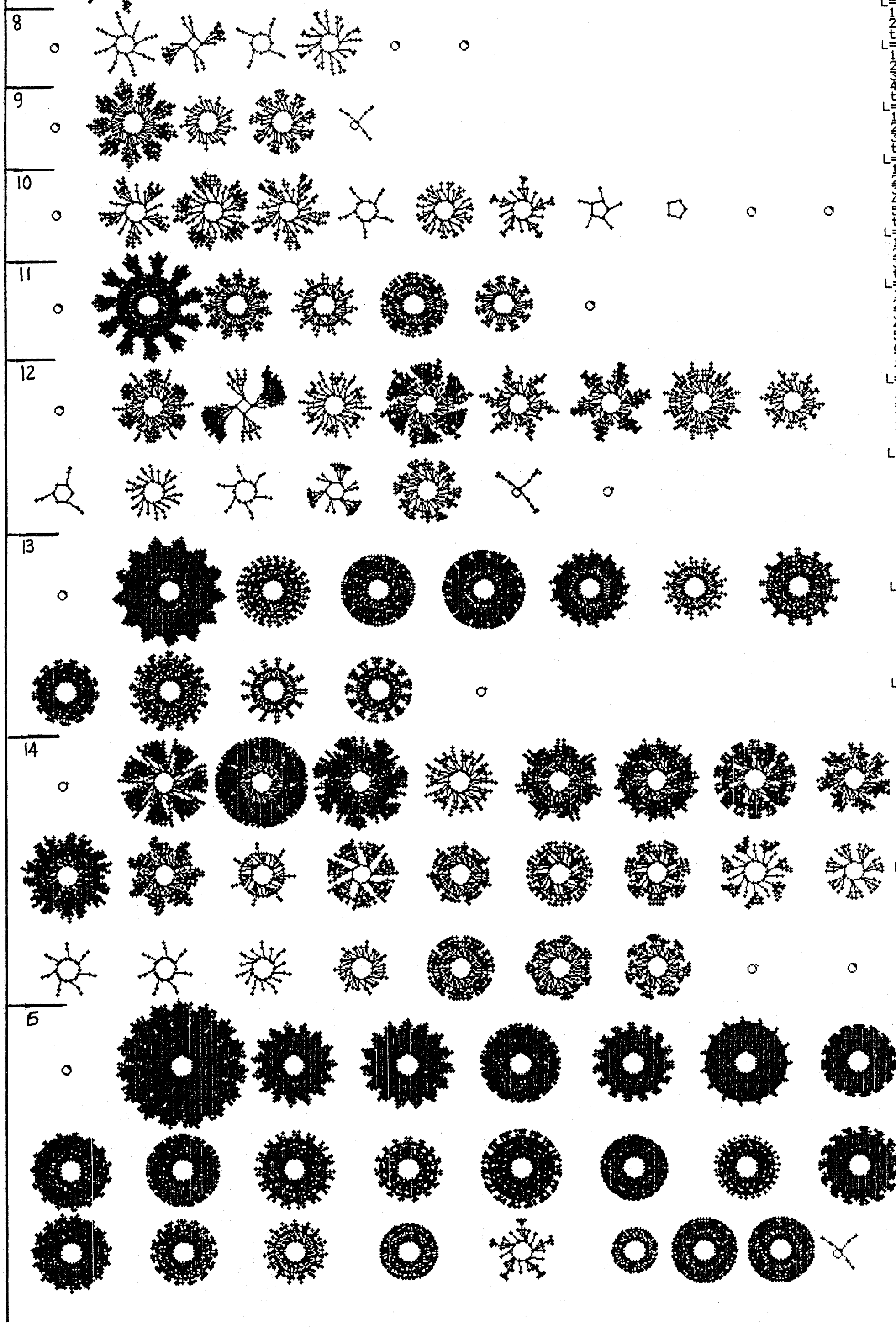
```
3-rule 41 =00101001

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.000 1(2)5 3 1 4
 2.011 1(3)3 0 0 1
 total NATs = 2
L= 4
 1.0000 1(2)2 0 0 1
 2.0001 2(2)6 2 2 2
 3.0101 1(2)2 0 0 1
 total NATs = 4
L= 5
 1.00000 1(2)2 0 0 1
 2.00001 1(15)25 10 1 2
 3.01011 1(5)5 0 0 1
 total NATs = 3
L= 6
 1.000000 1(2)59 18 6 4
 2.010101 1(2)2 0 0 1
 3.011011 1(3)3 0 0 1
 total NATs = 3
L= 7
 1.0000000 1(2)2 0 0 1
 2.0000001 1(28)119 42 4 3
 3.0101011 1(7)7 0 0 1
 total NATs = 3
L= 8
 1.00000000 1(2)2 0 0 1
 2.00000001 4(8)28 10 5 3
 3.01000100 2(2)66 34 5 4
 4.01010101 1(2)2 0 0 1
 5.01011011 1(8)8 0 0 1
 total NATs = 9
L= 9
 1.000000000 1(2)5 3 1 4
 2.000000001 1(36)99 36 3 3
 3.001000001 1(36)369 162 11 4
 4.000010011 3(3)9 6 1 3
 5.010101011 1(9)9 0 0 1
 6.011011011 1(3)3 0 0 1
 total NATs = 8
L= 10
 1.0000000000 1(2)2 0 0 1
 2.0000000001 2(20)380 170 13 4
 3.0000001001 2(40)100 40 4 3
 4.0000100001 1(15)45 30 1 4
 5.0101010101 1(2)2 0 0 1
 6.0101011011 1(10)10 0 0 1
 7.0101101011 1(5)5 0 0 1
 total NATs = 9
L= 11
 1.00000000000 1(2)2 0 0 1
 2.00000000001 1(44)451 176 9 4
 3.00000000011 11(11)143 74 10 6
 4.01010101011 1(11)11 0 0 1
 5.01011011011 1(11)11 0 0 1
 total NATs = 15
L= 12
 1.000000000000 1(2)1235 642 18 6
 2.000000000001 4(12)486 249 16 6
 3.010000000100 4(12)123 57 7 6
 4.010000001001 12(7)18 8 3 5
 5.000000100011 1(36)132 84 2 4
 6.010001000100 2(2)24 14 2 5
 7.010101010101 1(2)2 0 0 1
 8.010101011011 1(12)12 0 0 1
 9.010101101011 1(12)12 0 0 1
 10.011011011011 1(3)3 0 0 1
 total NATs = 28
L= 13
 1.0000000000000 1(2)2 0 0 1
 2.0000000000001 1(52)1456 689 15 7
 3.0100000011000 1(117)6682 3627 28 7
 4.0000011101001 1(13)13 0 0 1
 5.0101010101011 1(13)13 0 0 1
 6.0101011011011 1(13)13 0 0 1
 7.0101101011011 1(13)13 0 0 1
 total NATs = 7
L= 14
 1.00000000000000 1(2)2 0 0 1
 2.00000000000001 2(28)6867 3780 31 8
 3.01000000100000 1(28)2149 1288 14 9
 4.00000000111001 14(7)21 12 2 6
 5.00000010000101 1(28)154 98 3 5
 6.01010101010101 1(2)2 0 0 1
 7.01010101011011 1(14)14 0 0 1
 8.01010101101011 1(14)14 0 0 1
 9.01010110101011 1(7)7 0 0 1
 10.01011011011011 1(14)14 0 0 1
 total NATs = 24
L= 15
 1.000000000000000 1(2)5 3 1 4
 2.000000000000001 1(60)4680 2895 21 9
 3.000001000000100 1(60)20460 11580 45 10
 4.010000001000001 1(60)7500 4470 23 9
 5.000010000100001 1(15)55 40 1 5
 6.010101010101011 1(15)15 0 0 1
 7.010101011011011 1(15)15 0 0 1
 8.010101101011011 1(15)15 0 0 1
 9.010101101101011 1(15)15 0 0 1
 10.010110101101011 1(5)5 0 0 1
 11.011011011011011 1(3)3 0 0 1
 total NATs = 11
```

λ ratio = .75   Z = .75

1111001100111100-1111001100111100-rule 4080857916
=3-rule 214 -11010110   Length=1 -15



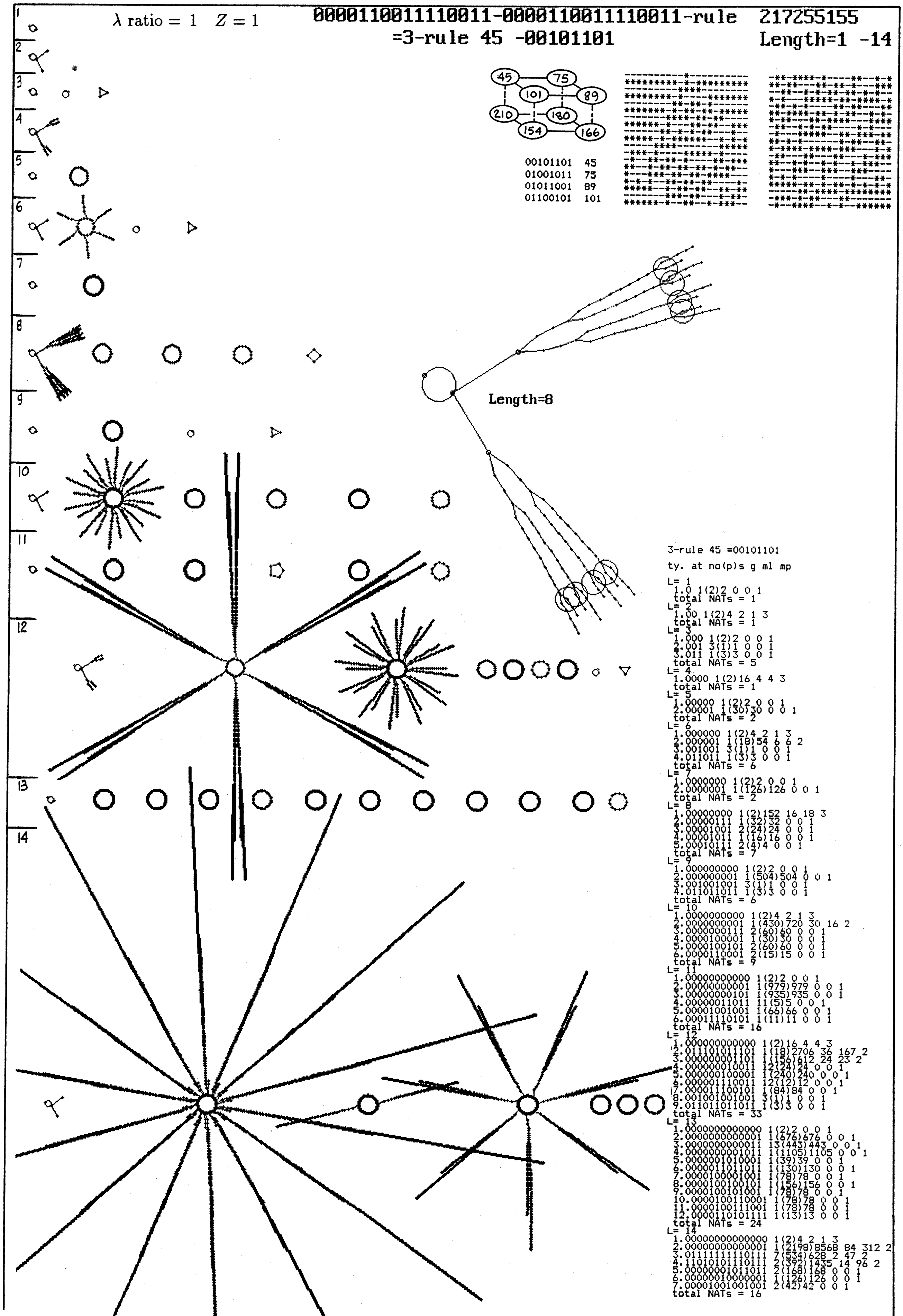




3-rule 214 =11010110

ty. at no(p)s g ml mp

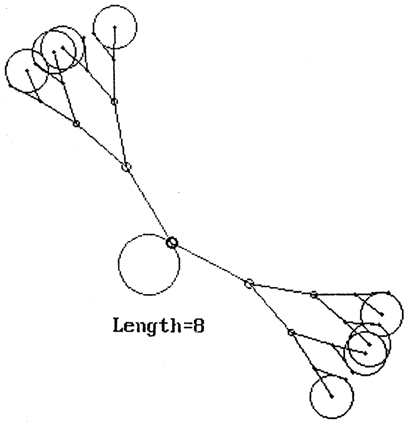
L= 1
1.0 1(1)1 0 0 1
2.1 1(1)1 0 0 1
total NATs = 2
L= 2
1.00 1(1)1 0 0 1
2.01 2(1)1 0 0 1
3.11 1(1)1 0 0 1
total NATs = 4
L= 3
1.000 1(1)1 0 0 1
2.111 1(1)7 3 2 4
total NATs = 2
L= 4
1.0000 1(1)1 0 0 1
2.1011 2(4)6 2 1 2
3.0101 2(1)1 0 0 1
4.1111 1(1)1 0 0 1
total NATs = 6
L= 5
1.00000 1(1)1 0 0 1
2.10011 1(10)30 10 3 2
3.11111 1(1)1 0 0 1
total NATs = 3
L= 6
1.000000 1(1)1 0 0 1
2.110101 2(6)12 3 2 2
3.111100 2(6)12 3 2 2
4.111111 1(1)13 6 3 4
5.010101 2(1)1 0 0 1
total NATs = 8
L= 7
1.0000000 1(1)1 0 0 1
2.1111100 1(14)126 42 7 3
3.1111111 1(1)1 0 0 1
total NATs = 3
L= 8
1.00000000 1(1)1 0 0 1
2.11101100 1(8)32 8 3 2
3.11001100 2(4)42 22 3 4
4.11010100 2(8)16 4 2 2
5.10011111 2(8)52 24 3 3
6.01010101 2(1)1 0 0 1
7.11111111 1(1)1 0 0 1
total NATs = 11
L= 9
1.000000000 1(1)1 0 0 1
2.111001100 1(18)333 126 7 4
3.110110011 1(18)54 27 2 3
4.111100111 1(18)117 63 3 3
5.111111111 1(1)7 3 2 4
total NATs = 5
L= 10
1.0000000000 1(1)1 0 0 1
2.0111010111 2(10)90 35 4 4
3.0011110011 2(10)185 85 5 4
4.1111111101 2(10)115 45 5 4
5.1101010100 2(10)20 5 2 2
6.1101100111 2(10)60 35 2 4
7.1001110011 1(10)50 30 3 4
8.1001111011 2(5)10 5 1 2
9.0011011101 2(5)5 0 0 1
10.0101010101 2(1)1 0 0 1
11.1111111111 1(1)1 0 0 1
total NATs = 19
L= 11
1.00000000000 1(1)1 0 0 1
2.00111110011 1(22)1408 671 11 6
3.11101100111 1(22)220 110 4 4
4.01111001111 1(22)99 44 3 5
5.10011111111 1(22)231 110 3 4
6.11110011100 1(22)88 55 2 3
7.11111111111 1(1)1 0 0 1
total NATs = 7
L= 12
1.000000000000 1(1)1 0 0 1
2.001110110011 2(12)204 108 5 5
3.001100110011 2(4)444 224 7 6
4.001101010011 2(12)102 42 4 4
5.111001100111 2(12)540 264 7 6
6.111110110011 2(12)108 60 5 5
7.111110011100 2(12)222 120 5 6
8.011110011111 1(12)168 60 5 4
9.111110011110 2(12)78 48 3 5
10.110101110101 2(6)12 3 2 2
11.110101110100 2(12)42 18 2 3
12.110101010100 2(12)24 6 2 2
13.111100111100 2(6)54 39 2 4
14.011111111111 2(12)126 72 3 4
15.111111111111 1(1)13 6 3 4
16.010101010101 2(1)1 0 0 1
total NATs = 29
L= 13
1.0000000000000 1(1)1 0 0 1
2.0011100110011 1(26)3653 1781 11 7
3.1111011001100 1(26)325 182 4 6
4.1110011001110 1(26)533 312 4 7
5.1111110011001 1(26)1274 728 5 6
6.0011110011111 1(26)676 377 5 5
7.1110110011100 1(26)143 78 3 4
8.1111101100111 1(26)416 234 5 6
9.0111100111111 1(26)390 221 3 6
10.1111100111111 1(26)403 195 5 5
11.1111100111101 1(26)143 65 3 4
12.1001110011111 1(26)234 143 3 6
13.1111111111111 1(1)1 0 0 1
total NATs = 13
L= 14
1.00000000000000 1(1)1 0 0 1
2.11011101011101 2(14)777 399 7 7
3.11001111001100 2(14)1932 1043 7 8
4.01111111110111 2(14)1176 679 8 8
5.00110101010011 2(14)119 49 4 4
6.11110110011001 2(14)497 266 6 7
7.11011101100111 2(14)616 357 6 7
8.11100110011100 2(14)581 350 5 8
9.11100110011110 2(14)210 133 4 5
10.01111100111110 1(14)896 462 7 9
11.11111011001111 2(14)273 161 5 6
12.11101100111101 2(14)70 42 3 6
13.11111001111110 2(7)168 98 4 6
14.01111001110110 2(14)140 91 3 5
15.01111001111111 2(14)175 91 3 8
16.11111001111011 2(14)154 84 3 7
17.11001101110100 2(14)119 63 4 5
18.11010111010100 2(7)56 28 2 4
19.11010100110100 2(14)28 7 2 2
20.11010101010100 2(14)28 7 2 2
21.11010101110100 2(14)35 14 2 2
22.11011001110011 2(14)56 35 2 4
23.10011111111111 2(14)217 112 3 5
24.11110011110011 2(14)168 112 3 5
25.01111111101111 2(14)147 105 3 5
26.01010101010101 2(1)1 0 0 1
27.11111111111111 1(1)1 0 0 1
total NATs = 51

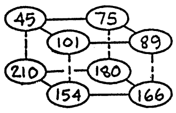



```
3-rule 45 =00101101
ty. at no(p)s g ml mp
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)4 2 1 3
 total NATs = 1
L= 3
 1.000 1(2)2 0 0 1
 2.001 3(1)1 0 0 1
 3.011 1(3)3 0 0 1
 total NATs = 5
L= 4
 1.0000 1(2)16 4 4 3
 total NATs = 1
L= 5
 1.00000 1(2)2 0 0 1
 2.00001 1(30)30 0 0 1
 total NATs = 2
L= 6
 1.000000 1(2)4 2 1 3
 2.000001 1(18)54 6 6 2
 3.001001 3(1)1 0 0 1
 4.011011 1(3)3 0 0 1
 total NATs = 6
L= 7
 1.0000000 1(2)2 0 0 1
 2.0000001 1(126)126 0 0 1
 total NATs = 2
L= 8
 1.00000000 1(2)152 16 18 3
 2.00000111 1(32)32 0 0 1
 3.00001001 2(24)24 0 0 1
 4.00001011 1(16)16 0 0 1
 5.00010111 2(4)4 0 0 1
 total NATs = 7
L= 9
 1.000000000 1(2)2 0 0 1
 2.000000001 1(504)504 0 0 1
 3.001001001 3(1)1 0 0 1
 4.011011011 1(3)3 0 0 1
 total NATs = 6
L= 10
 1.0000000000 1(2)4 2 1 3
 2.0000000001 1(430)720 30 16 2
 3.0000000111 2(60)60 0 0 1
 4.0000100001 1(30)30 0 0 1
 5.0000100101 2(60)60 0 0 1
 6.0000110001 2(15)15 0 0 1
 total NATs = 9
L= 11
 1.00000000000 1(2)2 0 0 1
 2.00000000001 1(979)979 0 0 1
 3.00000000101 1(935)935 0 0 1
 4.00000011011 11(5)5 0 0 1
 5.00001001001 1(66)66 0 0 1
 6.00011110101 1(11)11 0 0 1
 total NATs = 16
L= 12
 1.000000000000 1(2)16 4 4 3
 2.011101011101 1(18)2706 36 167 2
 3.000000001101 1(156)612 24 23 2
 4.000000010011 12(24)24 0 0 1
 5.000000100001 1(240)240 0 0 1
 6.000001110011 12(12)12 0 0 1
 7.000011100101 1(84)84 0 0 1
 8.001001001001 3(1)1 0 0 1
 9.011011011011 1(3)3 0 0 1
 total NATs = 33
L= 13
 1.0000000000000 1(2)2 0 0 1
 2.0000000000001 1(676)676 0 0 1
 3.0000000000011 13(443)443 0 0 1
 4.0000000001011 1(1105)1105 0 0 1
 5.0000001010001 1(39)39 0 0 1
 6.0000011011011 1(130)130 0 0 1
 7.0000100001001 1(78)78 0 0 1
 8.0000100100101 1(156)156 0 0 1
 9.0000100101001 1(78)78 0 0 1
 10.0000100110001 1(78)78 0 0 1
 11.0000100111001 1(78)78 0 0 1
 12.0000110101111 1(13)13 0 0 1
 total NATs = 24
L= 14
 1.00000000000000 1(2)4 2 1 3
 2.00000000000001 1(2198)8568 84 312 2
 3.01111111110111 7(534)628 2 47 2
 4.11010101110111 2(392)1435 14 96 2
 5.00000001011011 2(168)168 0 0 1
 6.00000010000001 1(126)126 0 0 1
 7.00001001001001 2(42)42 0 0 1
 total NATs = 16
```

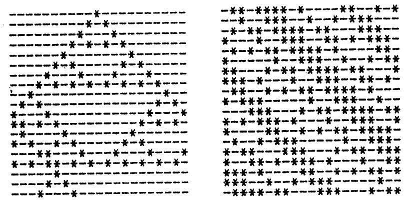
λ ratio = 1   Z = 1

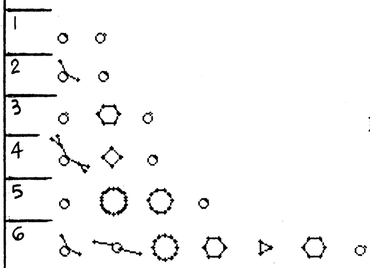
1111001100001100-1111001100001100-rule 4077712140
=3-rule 210 -11010010   **Length=1 -14**

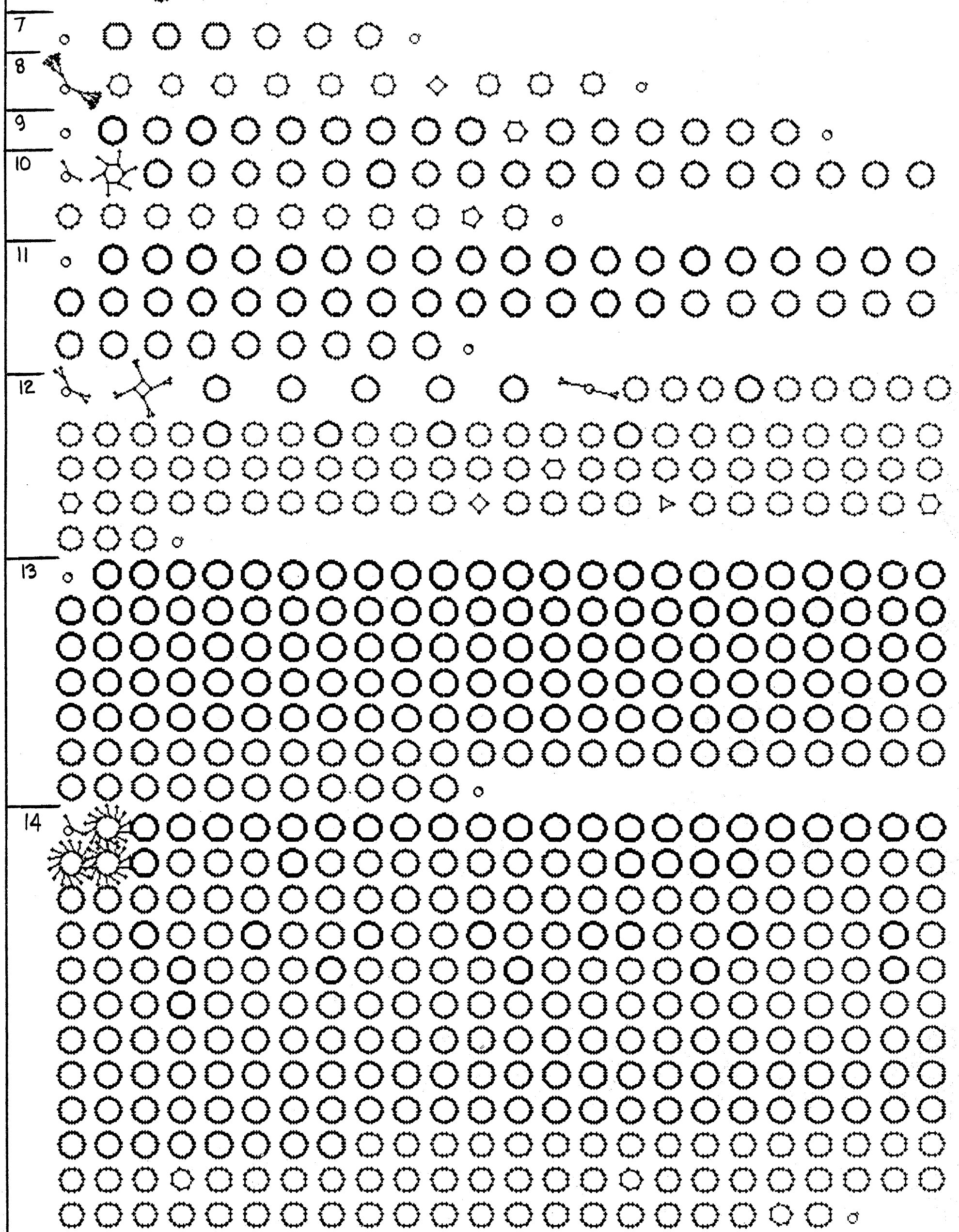




11010010 210
10110100 180
10100110 166
10011010 154





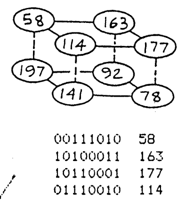


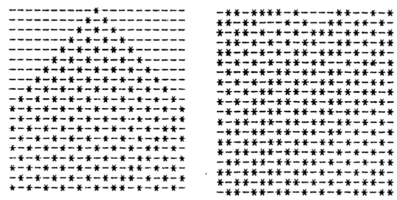
3-rule 210 =11010010

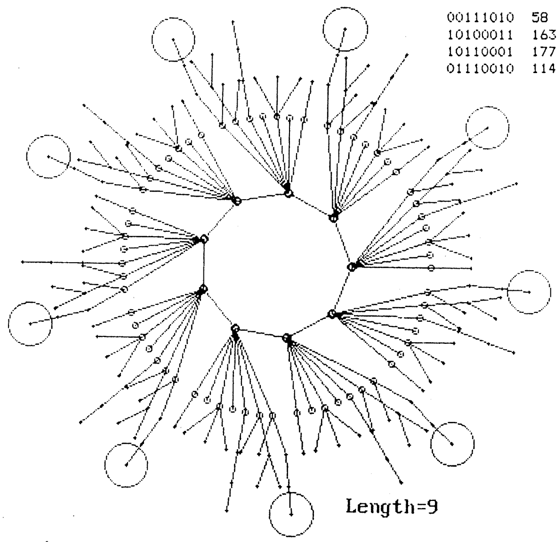
ty. at no(p)s g ml mp

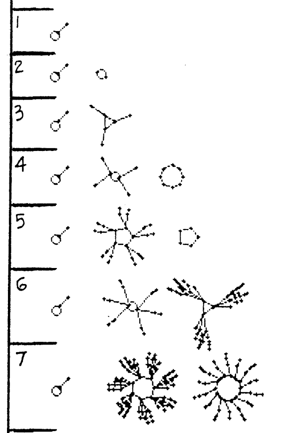
L= 1
1.0 1(1)1 0 0 1
2.1 1(1)1 0 0 1
total NATs = 2
L= 2
1.00 1(1)3 2 1 3
2.11 1(1)1 0 0 1
total NATs = 2
L= 3
1.000 1(1)1 0 0 1
2.001 1(6)6 0 0 1
3.111 1(1)1 0 0 1
total NATs = 3
L= 4
1.0000 1(1)7 4 2 3
2.0011 2(4)4 0 0 1
3.1111 1(1)1 0 0 1
total NATs = 4
L= 5
1.00000 1(1)1 0 0 1
2.00001 1(20)20 0 0 1
3.00111 1(10)10 0 0 1
4.11111 1(1)1 0 0 1
total NATs = 4
L= 6
1.000000 1(1)3 2 1 3
2.100010 3(2)4 2 1 2
3.000011 2(12)12 0 0 1
4.001001 1(6)6 0 0 1
5.001101 2(3)3 0 0 1
6.001111 2(6)6 0 0 1
7.111111 1(1)1 0 0 1
total NATs = 12
L= 7
1.0000000 1(1)1 0 0 1
2.0000001 1(28)28 0 0 1
3.0000011 1(28)28 0 0 1
4.0000111 1(28)28 0 0 1
5.0010011 1(14)14 0 0 1
6.0011011 1(14)14 0 0 1
7.0011111 1(14)14 0 0 1
8.1111111 1(1)1 0 0 1
total NATs = 8
L= 8
1.00000000 1(1)31 16 4 3
2.00000011 4(8)8 0 0 1
3.00000111 4(8)8 0 0 1
4.00001001 4(8)8 0 0 1
5.00001101 4(8)8 0 0 1
6.00001111 4(8)8 0 0 1
7.00100111 2(8)8 0 0 1
8.00110011 2(4)4 0 0 1
9.00110111 2(8)8 0 0 1
10.00111011 1(8)8 0 0 1
11.00111111 2(8)8 0 0 1
12.11111111 1(1)1 0 0 1
total NATs = 31
L= 9
1.000000000 1(1)1 0 0 1
2.000000001 1(72)72 0 0 1
3.000000111 1(36)36 0 0 1
4.000001001 1(72)72 0 0 1
5.000001111 1(36)36 0 0 1
6.000010011 1(36)36 0 0 1
7.000011001 1(36)36 0 0 1
8.000011011 1(36)36 0 0 1
9.000011101 1(36)36 0 0 1
10.000011111 1(36)36 0 0 1
11.001001001 1(6)6 0 0 1
12.001001101 1(18)18 0 0 1
13.001001111 1(18)18 0 0 1
14.001100111 1(18)18 0 0 1
15.001101111 1(18)18 0 0 1
16.001110111 1(18)18 0 0 1
17.001111111 1(18)18 0 0 1
18.111111111 1(1)1 0 0 1
total NATs = 18
L= 10
1.0000000000 1(1)3 2 1 3
2.1000000010 5(6)12 6 1 2
3.0000000011 2(40)40 0 0 1
4.0000001001 2(20)20 0 0 1
5.0000001101 2(20)20 0 0 1
6.0000001111 2(20)20 0 0 1
7.0000010011 2(20)20 0 0 1
8.0000011001 2(40)40 0 0 1
9.0000011011 2(20)20 0 0 1
10.0000011111 2(20)20 0 0 1
11.0000100001 1(20)20 0 0 1
12.0000100101 2(20)20 0 0 1
13.0000100111 2(20)20 0 0 1
14.0000110001 1(20)20 0 0 1
15.0000110011 2(20)20 0 0 1
16.0000110111 2(20)20 0 0 1
17.0000111001 2(20)20 0 0 1
18.0000111011 2(20)20 0 0 1
19.0000111101 2(20)20 0 0 1
20.0000111111 2(20)20 0 0 1
21.0010010011 2(10)10 0 0 1
22.0010011011 2(10)10 0 0 1
23.0010011101 2(10)10 0 0 1
24.0010011111 2(10)10 0 0 1
25.0011001101 2(10)10 0 0 1
26.0011001111 2(10)10 0 0 1
27.0011011111 2(10)10 0 0 1
28.0011100111 1(10)10 0 0 1
29.0011101111 2(10)10 0 0 1
30.0011110111 2(5)5 0 0 1
31.0011111111 2(10)10 0 0 1
32.1111111111 1(1)1 0 0 1
total NATs = 62

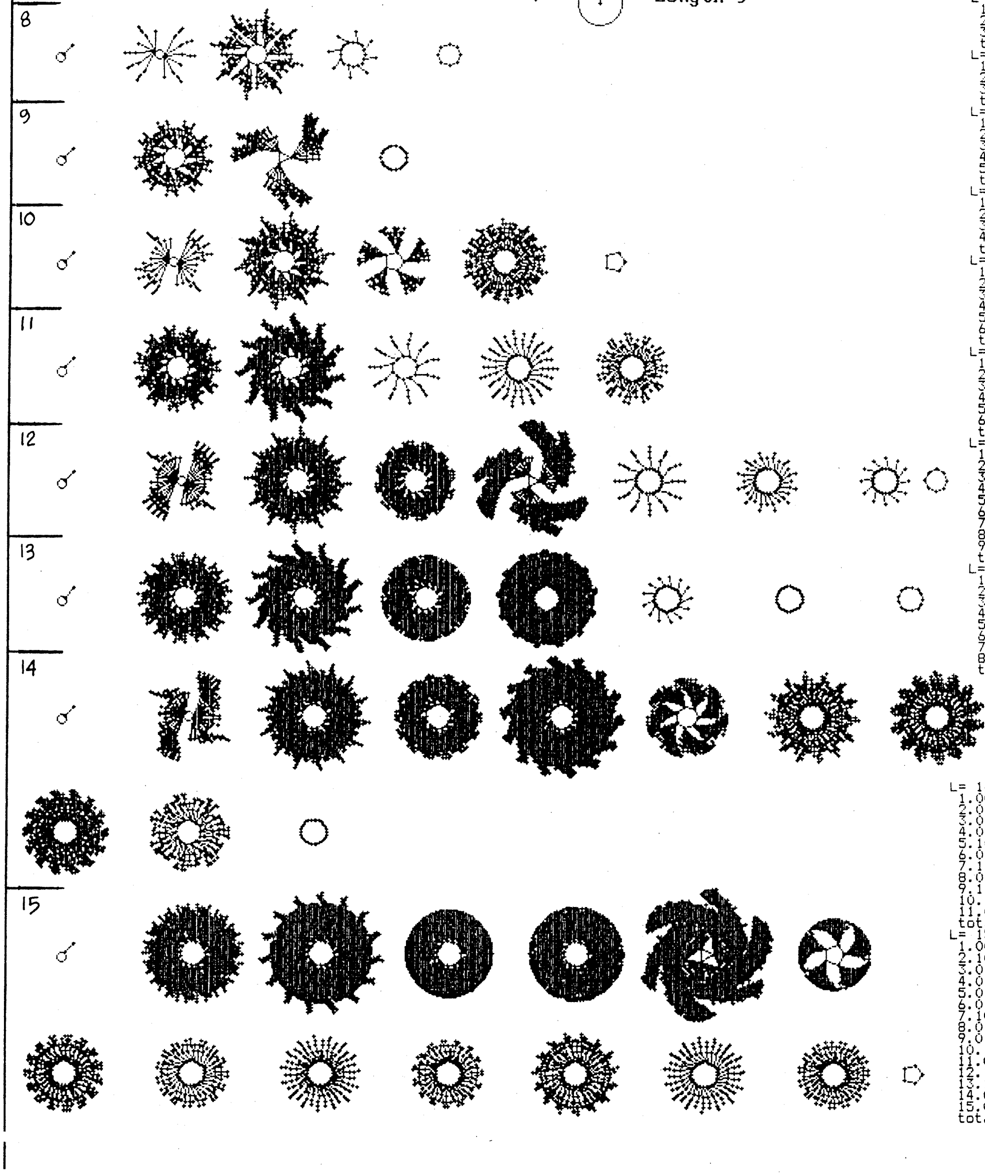
λ ratio = 1 $Z = .75$ 0000111111001100-0000111111001100-rule 265031628
=3-rule 58 -00111010 Length=1 -15





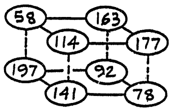




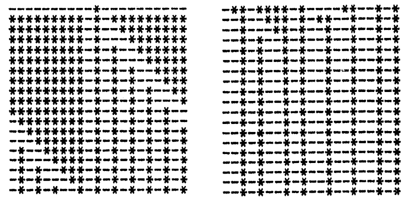


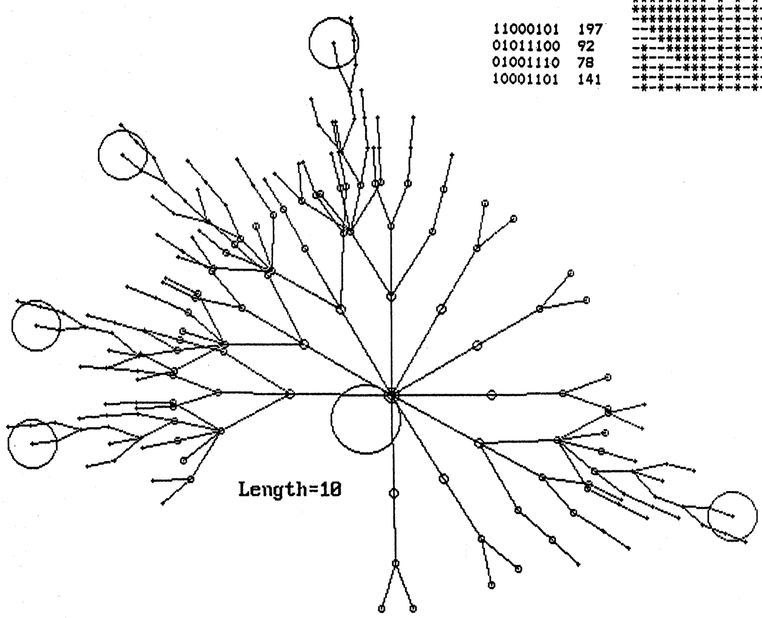

```
3-rule 58 =00111010

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.000 1(1)2 1 1 2
 2.110 1(3)6 3 1 2
 total NATs = 2
L= 4
 1.0000 1(1)2 1 1 2
 2.1010 1(2)6 4 1 3
 3.0011 1(8)8 0 0 1
 total NATs = 3
L= 5
 1.00000 1(1)2 1 1 2
 2.01101 1(5)25 10 2 3
 3.00111 1(5)5 0 0 1
 total NATs = 3
L= 6
 1.000000 1(1)2 1 1 2
 2.010101 1(2)14 6 2 4
 3.101101 1(3)48 15 5 4
 total NATs = 3
L= 7
 1.0000000 1(1)2 1 1 2
 2.1011010 1(7)77 35 3 5
 3.0111011 1(14)49 14 3 2
 total NATs = 3
L= 8
 1.00000000 1(1)2 1 1 2
 2.10101010 1(2)30 12 3 7
 3.11011010 1(8)192 88 6 6
 4.11011100 1(16)24 8 1 2
 5.00110011 1(8)8 0 0 1
 total NATs = 5
L= 9
 1.000000000 1(1)2 1 1 2
 2.010110101 1(9)207 108 4 8
 3.011011011 1(3)285 129 8 8
 4.001100111 1(18)18 0 0 1
 total NATs = 4
L= 10
 1.0000000000 1(1)2 1 1 2
 2.0101010101 1(2)62 30 4 11
 3.0110110101 1(10)480 240 7 10
 4.1011010110 1(5)165 110 3 9
 5.0110110011 1(20)310 130 5 4
 6.0011100111 1(5)5 0 0 1
 total NATs = 6
L= 11
 1.00000000000 1(1)2 1 1 2
 2.10101101010 1(11)517 297 5 13
 3.10110110101 1(11)1265 704 9 12
 4.01110110011 1(11)44 11 3 2
 5.11001100110 1(22)99 22 4 2
 6.01101110011 1(22)121 55 3 4
 total NATs = 6
L= 12
 1.000000000000 1(1)2 1 1 2
 2.101010101010 1(2)126 70 5 18
 3.101101101010 1(12)1152 636 8 16
 4.101011010110 1(12)804 552 4 15
 5.110110110110 1(3)1848 999 10 16
 6.011101110011 1(24)60 12 3 2
 7.110111001110 1(24)60 24 2 2
 8.110111001100 1(24)36 12 1 2
 9.001100110011 1(8)8 0 0 1
 total NATs = 9
L= 13
 1.0000000000000 1(1)2 1 1 2
 2.0101011010101 1(13)1235 767 6 21
 3.1101101101010 1(13)2977 1807 10 20
 4.0110101101101 1(13)1794 1196 5 18
 5.1101110110110 1(26)2106 1066 7 8
 6.1101110011100 1(26)39 13 1 2
 7.0011001100111 1(26)26 0 0 1
 8.0011001110111 1(13)13 0 0 1
 total NATs = 8
L= 14
 1.00000000000000 1(1)2 1 1 2
 2.01010101010101 1(2)254 154 6 29
 3.01011011010101 1(14)2688 1624 9 26
 4.01010110101101 1(14)1890 1358 5 24
 5.10110110110110 1(14)8666 5432 11 24
 6.01011010101101 1(7)959 679 4 25
 7.11011101100110 1(28)420 182 6 4
 8.01101100110011 1(28)574 238 6 4
 9.11011011100110 1(28)714 392 5 8
 10.11001101100110 1(14)189 84 4 4
 11.00110011100111 1(28)28 0 0 1
 total NATs = 11
L= 15
 1.000000000000000 1(1)2 1 1 2
 2.101010110101010 1(15)2865 1905 7 34
 3.011011011010101 1(15)6855 4470 11 32
 4.010110101101101 1(15)4230 2940 5 30
 5.010110110101101 1(15)4140 2910 6 30
 6.011011011011011 1(3)12351 7578 13 32
 7.101101011010110 1(5)925 745 3 27
 8.011011001110011 1(30)345 165 4 4
 9.011101100110011 1(30)135 45 3 2
 10.110011011100110 1(30)165 60 4 4
 11.011101101110011 1(30)165 90 3 4
 12.110011001110110 1(30)285 105 5 4
 13.110011001100110 1(30)135 30 4 2
 14.011011100110011 1(30)165 75 3 4
 15.001110011100111 1(5)5 0 0 1
 total NATs = 15
```

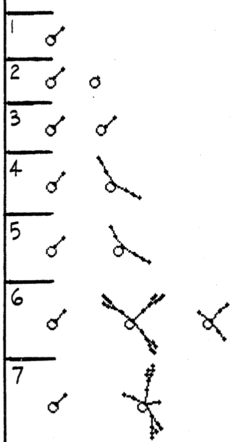
λ ratio = 1 $Z = .75$ 1111000000110011-1111000000110011-rule 4029935667
=3-rule 197 -11000101 Length=1 -15

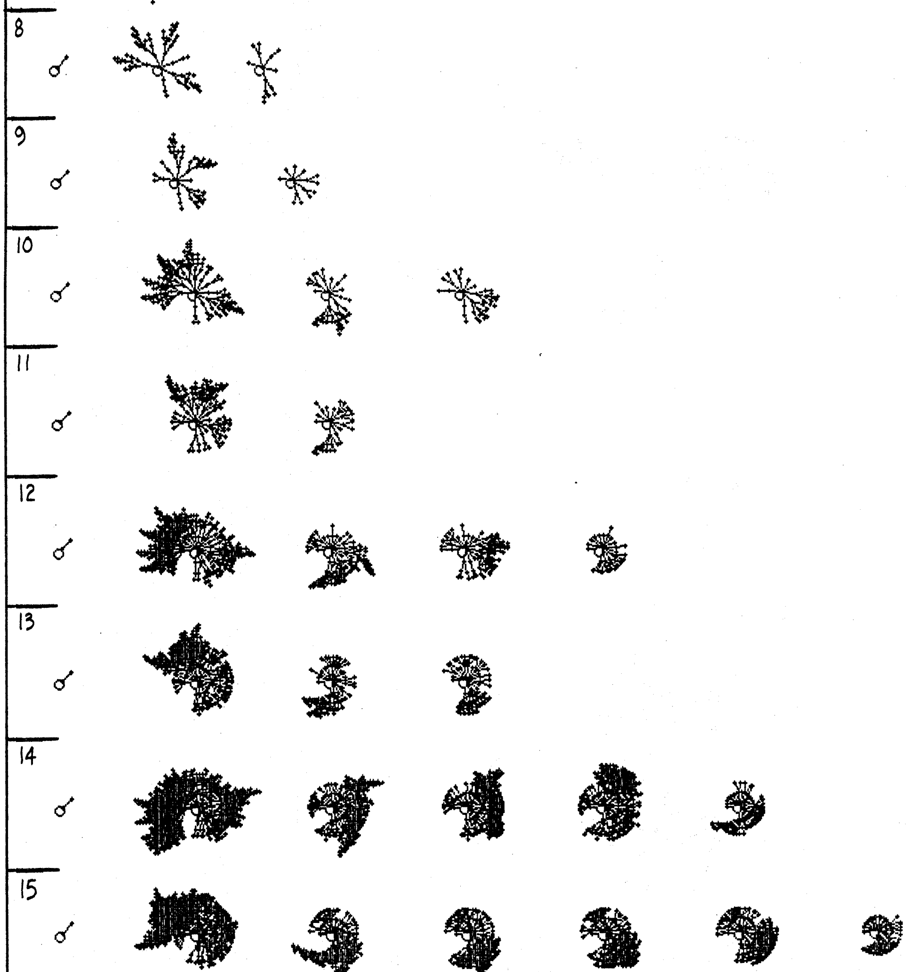



```
11000101  197
01011100  92
01001110  78
10001101  141
```





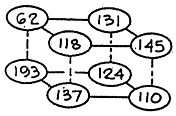




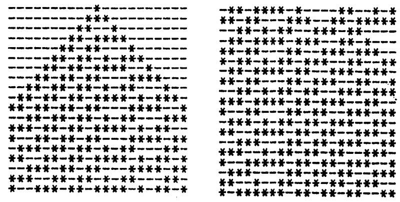

```
3-rule 197 =11000101

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.111 1(1)2 1 1 2
 2.001 3(1)2 1 1 2
 total NATs = 4
L= 4
 1.1111 1(1)2 1 1 2
 2.0101 2(1)7 2 3 3
 total NATs = 3
L= 5
 1.11111 1(1)2 1 1 2
 2.00101 5(1)6 2 3 3
 total NATs = 6
L= 6
 1.111111 1(1)2 1 1 2
 2.010101 2(1)22 6 5 4
 3.001001 3(1)6 3 2 4
 total NATs = 6
L= 7
 1.1111111 1(1)2 1 1 2
 2.0100101 7(1)18 7 5 5
 total NATs = 8
L= 8
 1.11111111 1(1)2 1 1 2
 2.01010101 2(1)67 22 7 7
 3.01001001 8(1)15 8 4 6
 total NATs = 11
L= 9
 1.111111111 1(1)2 1 1 2
 2.010100101 9(1)52 23 7 8
 3.001001001 3(1)14 10 2 8
 total NATs = 13
L= 10
 1.1111111111 1(1)2 1 1 2
 2.0101010101 2(1)196 75 9 11
 3.0101001001 10(1)44 26 6 10
 4.0100101001 5(1)38 20 4 9
 total NATs = 18
L= 11
 1.11111111111 1(1)2 1 1 2
 2.01010100101 11(1)150 74 9 13
 3.01001001001 11(1)36 25 4 12
 total NATs = 23
L= 12
 1.111111111111 1(1)2 1 1 2
 2.010101010101 2(1)568 248 11 18
 3.010101001001 12(1)126 79 8 16
 4.010100101001 12(1)112 65 6 15
 5.001001001001 3(1)34 27 2 16
 total NATs = 30
L= 13
 1.1111111111111 1(1)2 1 1 2
 2.0101010100101 13(1)432 233 11 21
 3.0101001001001 13(1)106 77 6 20
 4.0100101001001 13(1)92 63 4 18
 total NATs = 40
L= 14
 1.11111111111111 1(1)2 1 1 2
 2.01010101010101 2(1)1639 798 13 29
 3.01010101001001 14(1)363 240 10 26
 4.01010100101001 14(1)321 198 8 24
 5.01010010101001 7(1)330 207 6 25
 6.01001001001001 14(1)87 69 4 24
 total NATs = 52
L= 15
 1.111111111111111 1(1)2 1 1 2
 2.010101010100101 15(1)1244 723 13 34
 3.010101001001001 15(1)304 228 8 32
 4.010100101001001 15(1)271 195 6 30
 5.010100100101001 15(1)271 195 6 30
 6.010010100101001 5(1)234 158 4 27
 7.001001001001001 3(1)82 71 2 32
 total NATs = 69
```

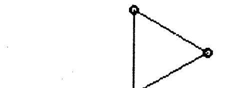
λ ratio = .75 Z = .75

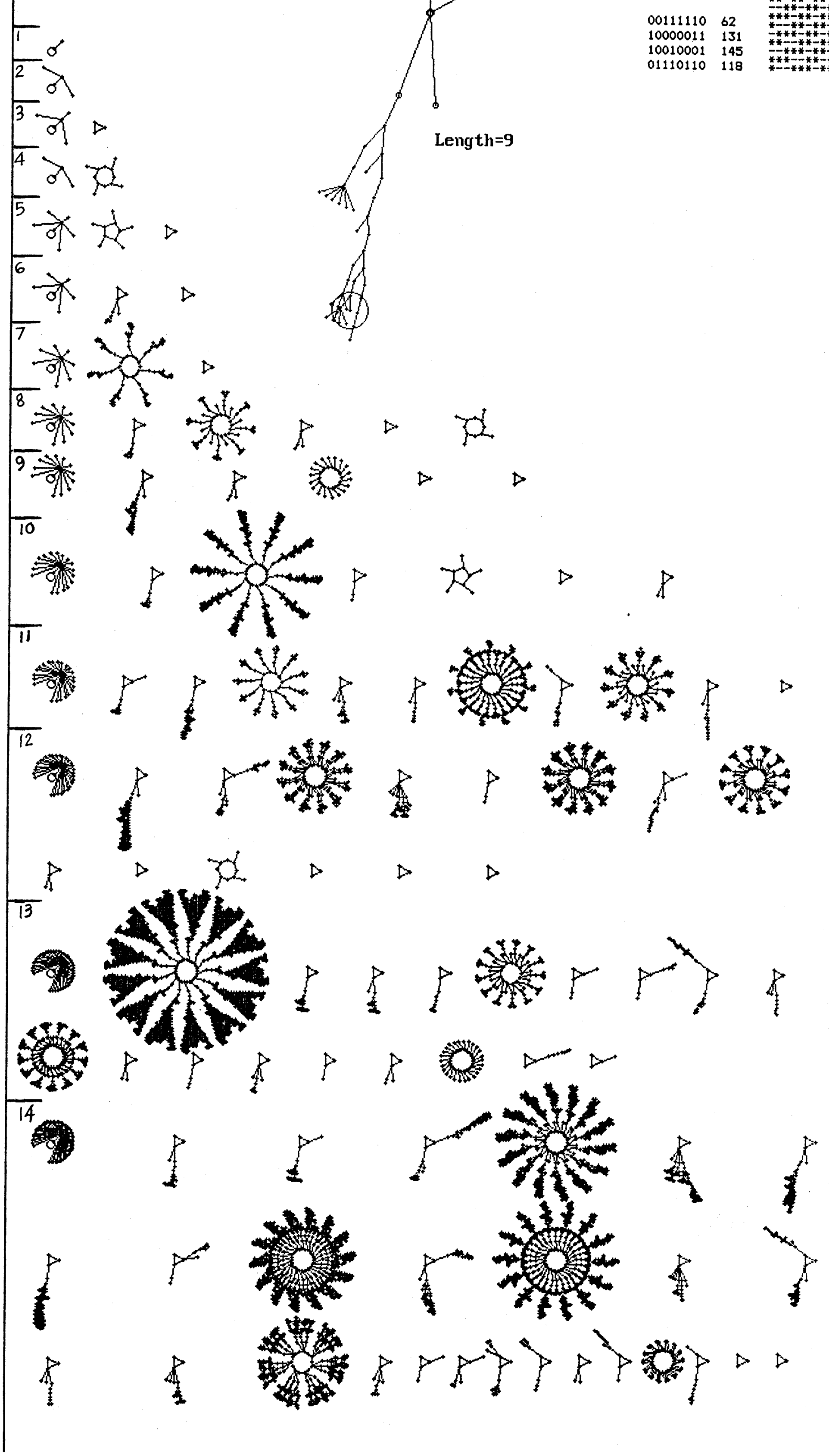
0000111111111100-0000111111111100-rule 268177404
=3-rule 62 -00111110 Length=1 -14



00111110 62
10000011 131
10010001 145
01110110 118

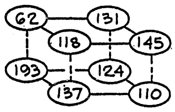




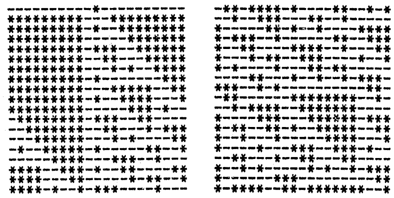


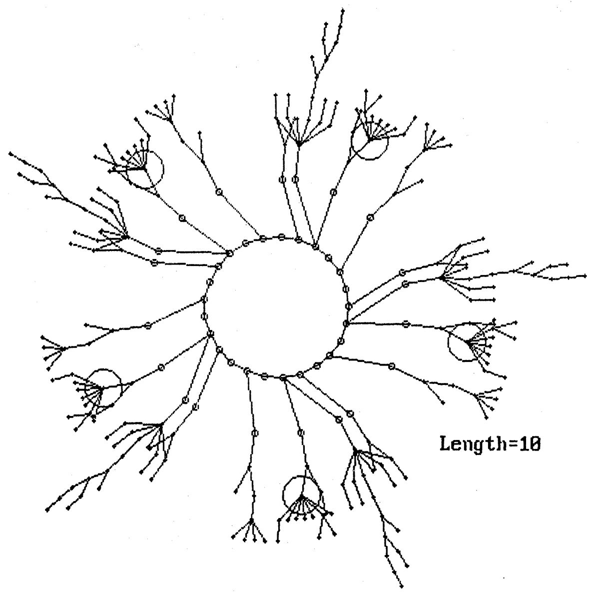

```
3-rule 62 =00111110
ty. at no(p)s g ml mp
L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.000 1(1)5 3 2 3
 2.011 1(3)3 0 0 1
 total NATs = 2
L= 4
 1.0000 1(1)4 2 2 2
 2.1011 1(8)12 4 1 2
 total NATs = 2
L= 5
 1.00000 1(1)7 5 2 5
 2.10011 1(5)10 5 1 2
 3.00011 5(3)3 0 0 1
 total NATs = 7
L= 6
 1.000000 1(1)7 5 2 5
 2.111101 6(3)9 3 4 3
 3.011011 1(3)3 0 0 1
 total NATs = 8
L= 7
 1.0000000 1(1)9 7 2 7
 2.1101101 1(14)98 42 7 3
 3.0001011 7(3)3 0 0 1
 total NATs = 9
L= 8
 1.00000000 1(1)12 10 2 10
 2.11101101 8(3)10 4 4 3
 3.11001101 1(16)80 40 4 4
 4.10001111 8(3)6 2 2 3
 5.00001111 8(3)3 0 0 1
 6.10111011 1(8)12 4 1 2
 total NATs = 27
L= 9
 1.000000000 1(1)14 12 2 12
 2.111011011 9(3)44 18 14 6
 3.100001111 9(3)6 2 2 3
 4.100111011 1(18)36 18 1 2
 5.000101111 3(3)3 0 0 1
 6.011011011 1(3)3 0 0 1
 total NATs = 24
L= 10
 1.0000000000 1(1)19 17 2 17
 2.1110001101 10(3)13 7 4 6
 3.1110110110 1(20)760 330 17 7
 4.1101100011 10(3)5 1 2 2
 5.1001110011 1(5)10 5 1 2
 6.0000101111 10(3)3 0 0 1
 7.1011110111 5(3)5 2 1 3
 total NATs = 38
L= 11
 1.00000000000 1(1)24 22 2 22
 2.11100001101 11(3)15 9 4 7
 3.11110110110 11(3)58 31 14 10
 4.10011011101 1(11)88 33 5 3
 5.11111011110 11(3)20 11 6 6
 6.11111011000 11(3)11 3 6 3
 7.11011011001 1(22)253 132 7 5
 8.01101111101 11(3)15 4 7 2
 9.11101110110 1(22)143 66 5 4
 10.11011111000 11(3)18 4 12 3
 11.00001001111 11(3)3 0 0 1
 total NATs = 81
L= 12
 1.000000000000 1(1)31 29 2 29
 2.111011011011 12(3)193 95 23 13
 3.011111011000 12(3)31 13 7 4
 4.100111011101 1(24)216 108 5 4
 5.111110111110 6(3)45 27 6 7
 6.110001100011 12(3)5 1 2 2
 7.111001110110 1(24)264 144 5 6
 8.100001101111 12(3)21 7 12 3
 9.110011011101 1(24)180 108 4 5
 10.100011110111 12(3)5 2 1 3
 11.000010011111 4(3)3 0 0 1
 12.101110111011 1(8)12 4 1 2
 13.000101101111 12(3)3 0 0 1
 14.000111101011 4(3)3 0 0 1
 15.011011011011 1(3)3 0 0 1
 total NATs = 92
L= 13
 1.0000000000000 1(1)41 39 2 39
 2.0110110110111 1(26)5460 2847 26 13
 3.1111000110110 13(3)33 24 6 17
 4.1111011011110 13(3)25 16 6 10
 5.1111011011000 13(3)12 4 6 4
 6.1001110111001 1(13)117 65 4 4
 7.1100001100011 13(3)6 2 2 2
 8.1100011000011 13(3)10 3 4 2
 9.1110110101101 13(3)33 12 12 4
 10.1111101111000 13(3)12 4 6 4
 11.1110011100110 1(26)221 143 4 6
 12.1000011110111 13(3)5 2 1 3
 13.1111010110001 13(3)6 1 3 2
 14.1000111101111 13(3)11 5 4 4
 15.1000110101111 13(3)4 1 1 2
 16.1000110001111 13(3)6 2 2 3
 17.1001110111011 1(26)52 26 1 2
 18.0000101111011 13(3)10 1 7 2
 19.0000110101111 13(3)4 1 1 2
 total NATs = 187
L= 14
 1.00000000000000 1(1)53 51 2 51
 2.00011111011011 14(3)34 23 8 17
 3.11110000110110 14(3)42 33 6 23
 4.01111011011000 14(3)77 33 17 5
 5.01110110110111 1(28)1036 532 14 7
 6.11110110111110 14(3)103 65 15 13
 7.11100011011011 14(3)92 38 21 8
 8.11101101101101 14(3)269 158 20 12
 9.11000011000011 14(3)12 5 4 3
 10.11101101100110 1(28)1078 574 11 8
 11.00011011111011 14(3)64 29 12 4
 12.11101101101100 1(28)1190 630 15 9
 13.11111011111000 14(3)22 10 6 7
 14.11101101011011 14(3)58 28 12 9
 15.11111011110000 14(3)12 4 6 4
 16.11111011011110 14(3)22 13 6 7
 17.10111011011101 1(14)504 280 8 9
 18.10000111101111 14(3)11 5 4 4
 19.10000110101111 14(3)5 2 1 2
 20.10000110001111 14(3)8 4 2 3
 21.11110111110100 14(3)15 7 4 4
 22.11110100110001 14(3)11 3 3 2
 23.10001111000111 7(3)5 2 1 3
 24.10001101001111 14(3)12 3 7 2
 25.10011100111011 1(28)70 42 1 3
 26.10011011111000 14(3)12 3 7 2
 27.00010110001111 14(3)3 0 0 1
 28.00010110101111 14(3)3 0 0 1
total NATs = 307
```

λ ratio = .75 $Z = .75$ 1111000000000011-1111000000000011-rule 4026789891
=3-rule 193 -11000001 Length=1 -15

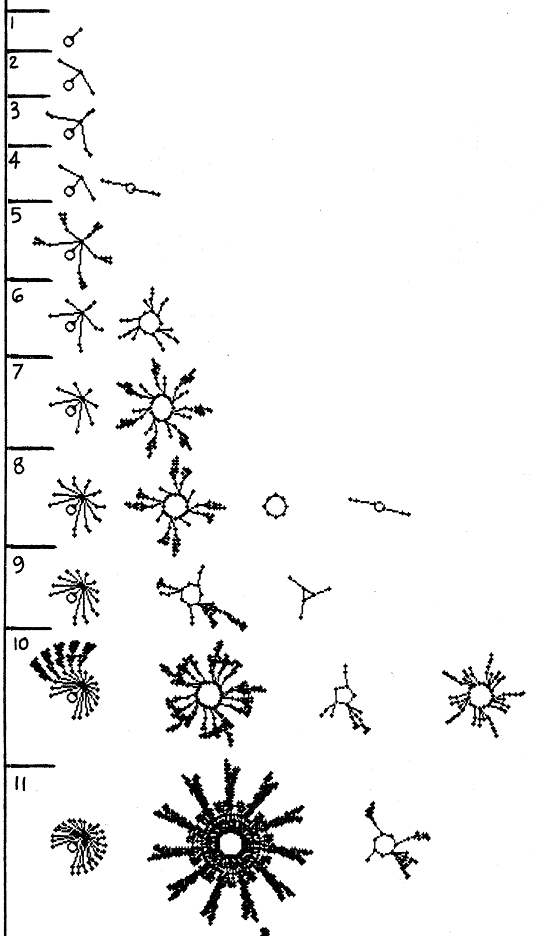


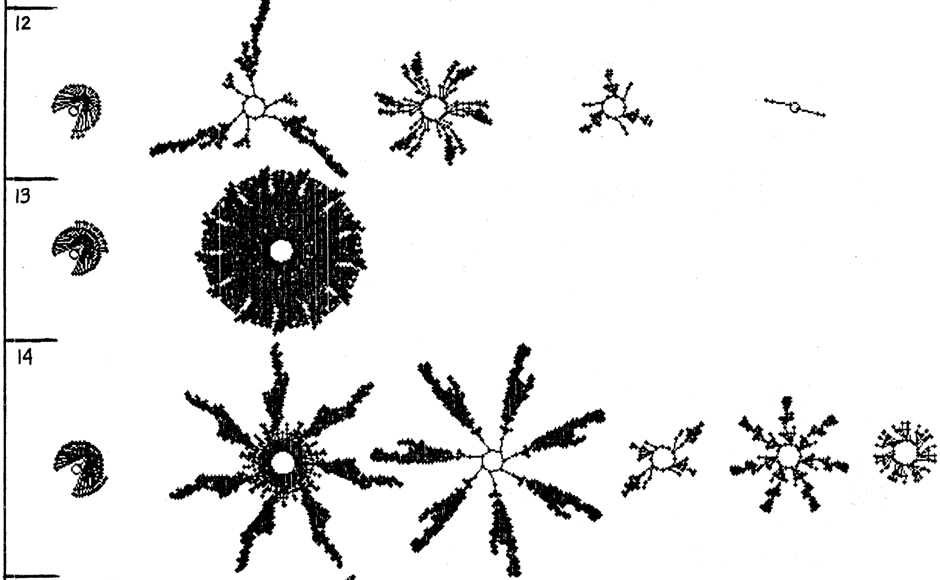
11000001 193
01111100 124
01101110 110
10001001 137

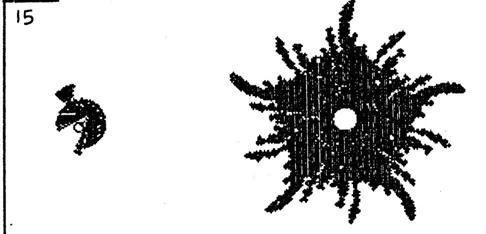




Length=10

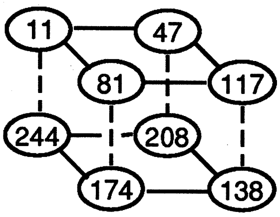


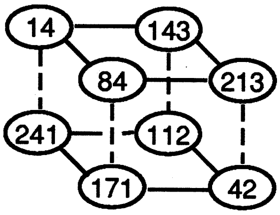


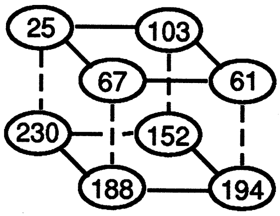


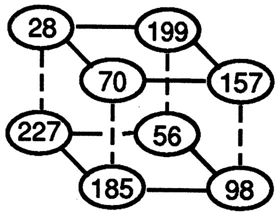

```
3-rule 193 =11000001
ty. at no(p)s g ml mp
L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.111 1(1)8 3 3 3
 total NATs = 1
L= 4
 1.1111 1(1)4 2 2 2
 2.0001 2(2)6 2 2 2
 total NATs = 3
L= 5
 1.11111 1(1)32 10 5 5
 total NATs = 1
L= 6
 1.111111 1(1)10 5 3 5
 2.000001 2(9)27 9 3 3
 total NATs = 3
L= 7
 1.1111111 1(1)9 7 2 7
 2.0001100 1(14)119 42 7 3
 total NATs = 2
L= 8
 1.11111111 1(1)20 10 3 10
 2.00011100 2(16)108 44 7 4
 3.00001011 1(8)8 0 0 1
 4.00010001 2(2)6 2 2 2
 total NATs = 6
L= 9
 1.111111111 1(1)17 12 3 12
 2.000000001 9(7)53 21 12 6
 3.000011001 3(3)6 3 1 2
 total NATs = 13
L= 10
 1.1111111111 1(1)134 72 9 17
 2.0001111100 2(25)270 120 9 7
 3.0000001001 10(5)22 9 5 4
 4.1000100000 2(15)65 20 5 4
 total NATs = 15
L= 11
 1.11111111111 1(1)35 22 3 22
 2.11000100000 1(110)1518 682 20 10
 3.00000000111 11(7)45 21 5 6
 total NATs = 13
L= 12
 1.111111111111 1(1)34 29 3 29
 2.110001111100 4(9)831 402 35 13
 3.100011100000 2(18)312 138 10 6
 4.000001000001 2(9)51 27 4 7
 5.000100010001 2(2)6 2 2 2
 total NATs = 11
L= 13
 1.1111111111111 1(1)54 39 3 39
 2.0111001000100 1(351)8138 4095 21 17
 total NATs = 2
L= 14
 1.11111111111111 1(1)67 51 3 51
 2.00000000100110 2(91)3346 1813 38 23
 3.11000001001101 2(7)4221 2177 38 13
 4.00000000111001 7(12)92 46 8 7
 5.00011000001100 1(14)371 224 10 9
 6.00000100010001 2(21)84 35 2 5
 total NATs = 15
L= 15
 1.111111111111111 1(1)113 88 5 68
 2.110011000001101 3(295)10885 5985 39 30
 total NATs = 4
```

A2.3.4 Fully Asymmetric Rule Clusters (see section 3.3.8)

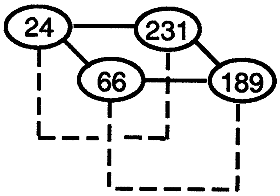


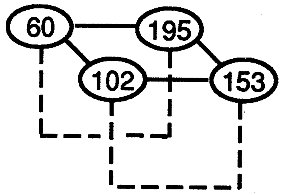




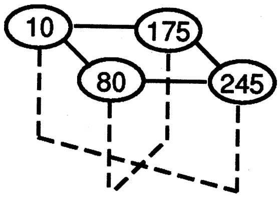


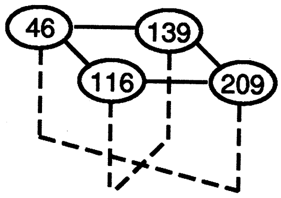
The cluster will collapse if, for a given rule R, $R_c = R_n$,



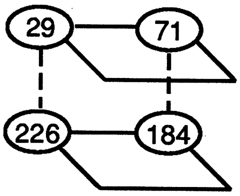


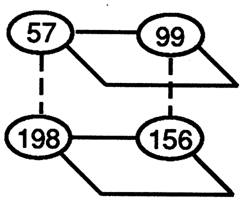
And if, for a given rule R, $R_c = R_{nr}$,



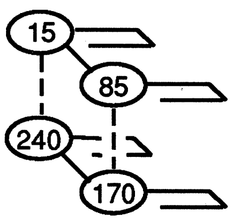


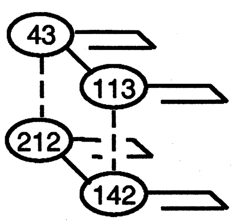
And if, for a given rule R, $R_n = R_r$,

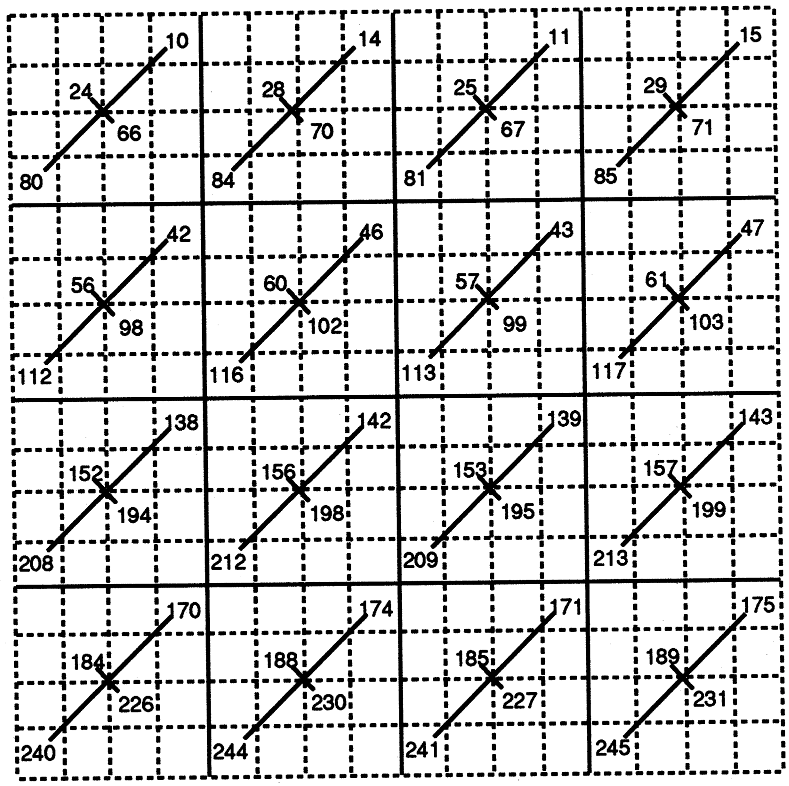




And also if $R = R_n$, then





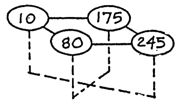


The rule-space matrix (see appendix 4).

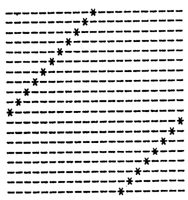
160

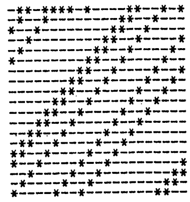
Intentionally blank

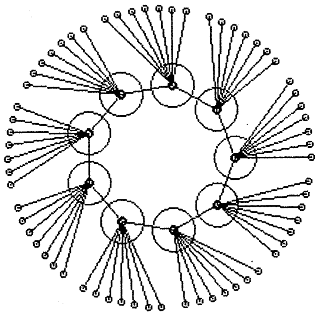
λ ratio = .5   Z = .5   0000000011001100-0000000011001100-rule 13369548
=3-rule 10 -00001010   Length=1 -15



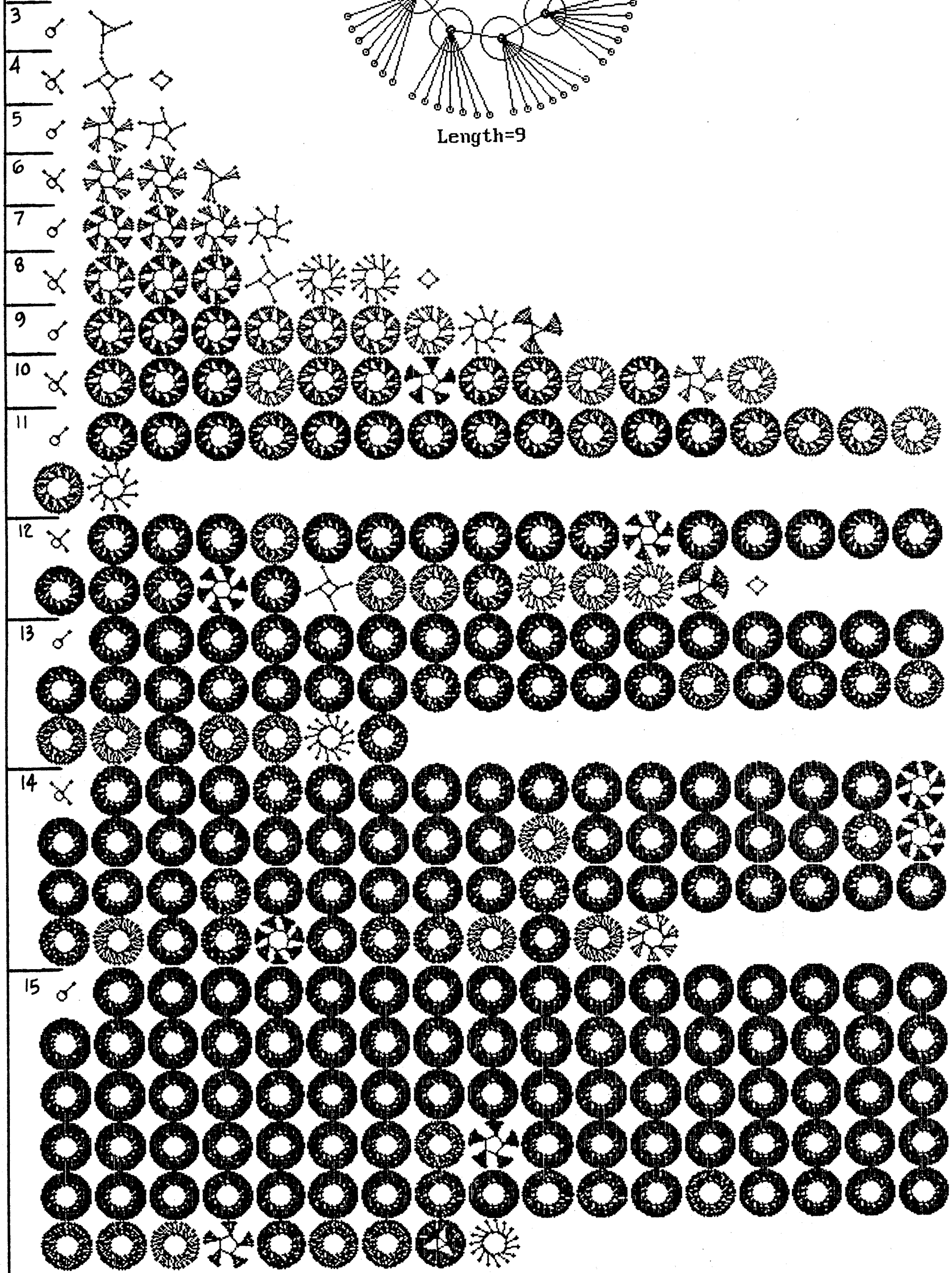

```
00001010  10
10101111  175
11110101  245
01010000  80
```





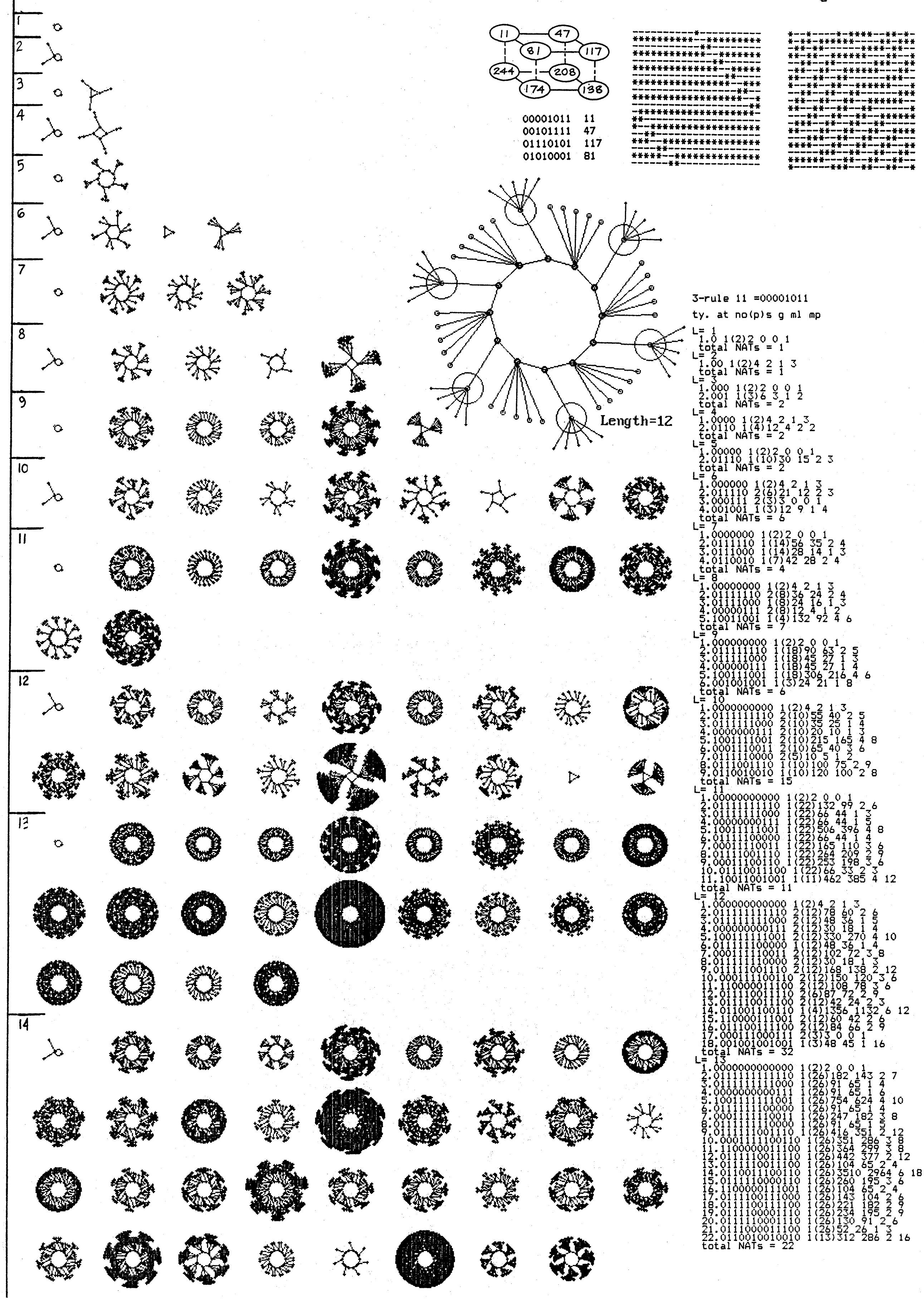


Length=9



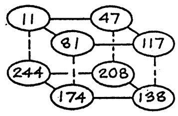

```
3-rule 10 =00001010

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 3 1 4
 total NATs = 1
L= 3
 1.000 1(1)2 1 1 2
 2.001 1(3)6 3 1 2
 total NATs = 2
L= 4
 1.0000 1(1)4 3 1 4
 2.0001 1(4)8 4 1 2
 3.0011 1(4)4 0 0 1
 total NATs = 3
L= 5
 1.00000 1(1)2 1 1 2
 2.00001 1(5)20 15 1 4
 3.00011 1(5)10 5 1 2
 total NATs = 3
L= 6
 1.000000 1(1)4 3 1 4
 2.000001 1(6)24 18 1 4
 3.000011 1(6)24 18 1 4
 4.001001 1(3)12 9 1 4
 total NATs = 4
L= 7
 1.0000000 1(1)2 1 1 2
 2.0000001 1(7)42 35 1 6
 3.0000011 1(7)42 35 1 6
 4.0001001 1(7)28 21 1 4
 5.0010011 1(7)14 7 1 2
 total NATs = 5
L= 8
 1.00000000 1(1)4 3 1 4
 2.00000001 1(8)48 40 1 6
 3.00000011 1(8)72 64 1 9
 4.00001001 1(8)72 64 1 9
 5.00010001 1(4)8 4 1 2
 6.00010011 1(8)24 16 1 3
 7.00011001 1(8)24 16 1 3
 8.00110011 1(4)4 0 0 1
 total NATs = 8
L= 9
 1.000000000 1(1)2 1 1 2
 2.000000001 1(9)72 63 1 8
 3.000000011 1(9)108 99 1 12
 4.000001001 1(9)90 81 1 10
 5.000010001 1(9)54 45 1 6
 6.000010011 1(9)54 45 1 6
 7.000011001 1(9)54 45 1 6
 8.000100011 1(9)36 27 1 4
 9.000110011 1(9)18 9 1 2
 10.001001001 1(3)24 21 1 8
 total NATs = 10
L= 10
 1.0000000000 1(1)4 3 1 4
 2.0000000001 1(10)80 70 1 8
 3.0000000011 1(10)160 150 1 16
 4.0000001001 1(10)160 150 1 16
 5.0000010001 1(10)40 30 1 4
 6.0000010011 1(10)80 70 1 8
 7.0000011001 1(10)80 70 1 8
 8.0000100001 1(5)80 75 1 16
 9.0000100011 1(10)80 70 1 8
 10.0000110001 1(10)80 70 1 8
 11.0000110011 1(10)40 30 1 4
 12.0001001001 1(10)80 70 1 8
 13.0001100011 1(5)20 15 1 4
 14.0010010011 1(10)40 30 1 4
 total NATs = 14
L= 11
 1.00000000000 1(1)2 1 1 2
 2.00000000001 1(11)110 99 1 10
 3.00000000011 1(11)220 209 1 20
 4.00000001001 1(11)198 187 1 18
 5.00000010001 1(11)88 77 1 8
 6.00000010011 1(11)132 121 1 12
 7.00000011001 1(11)132 121 1 12
 8.00000100001 1(11)154 143 1 14
 9.00000100011 1(11)110 99 1 10
 10.00000110001 1(11)110 99 1 10
 11.00000110011 1(11)66 55 1 6
 12.00001000011 1(11)176 165 1 16
 13.00001001001 1(11)198 187 1 18
 14.00001100011 1(11)88 77 1 8
 15.00010001001 1(11)66 55 1 6
 16.00010010011 1(11)66 55 1 6
 17.00010011001 1(11)44 33 1 4
 18.00011001001 1(11)66 55 1 6
 19.00100110011 1(11)22 11 1 2
 total NATs = 19
L= 12
 1.000000000000 1(1)4 3 1 4
 2.000000000001 1(12)120 108 1 10
 3.000000000011 1(12)300 288 1 25
 4.000000001001 1(12)300 288 1 25
 5.000000010001 1(12)72 60 1 6
 6.000000010011 1(12)180 168 1 15
 7.000000011001 1(12)180 168 1 15
 8.000000100001 1(12)300 288 1 25
 9.000000100011 1(12)180 168 1 15
 10.000000110001 1(12)180 168 1 15
 11.000000110011 1(12)108 96 1 9
 12.000001000001 1(6)48 42 1 8
 13.000001000011 1(12)240 228 1 20
 14.000001001001 1(12)240 228 1 20
 15.000001100001 1(12)240 228 1 20
 16.000001100011 1(12)144 132 1 12
 17.000010001001 1(12)180 168 1 15
 18.000010010001 1(12)180 168 1 15
 19.000010010011 1(12)144 132 1 12
 20.000010011001 1(12)108 96 1 9
 21.000011000011 1(6)96 90 1 16
 22.000011001001 1(12)144 132 1 12
 23.000100010001 1(4)8 4 1 2
 24.000100010011 1(12)60 48 1 5
 25.000100011001 1(12)60 48 1 5
 26.000100100011 1(12)108 96 1 9
 27.000100110011 1(12)36 24 1 3
 28.000110010011 1(12)48 36 1 4
 29.000110011001 1(12)36 24 1 3
 30.001001001001 1(3)48 45 1 16
 31.001100110011 1(4)4 0 0 1
 total NATs = 31
```

λ ratio = .75 Z = .75   0000000011001111-0000000011001111-rule 13566159 =3-rule 11 -00001011   Length=1 -14

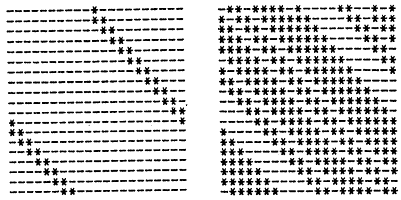


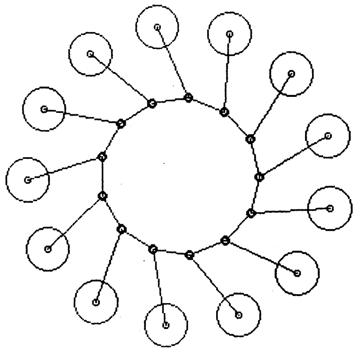

```
3-rule 11 =00001011
ty. at no(p)s g ml mp
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)4 2 1 3
 total NATs = 1
L= 3
 1.000 1(2)2 0 0 1
 2.001 1(3)6 3 1 2
 total NATs = 2
L= 4
 1.0000 1(2)4 2 1 3
 2.0110 1(4)12 4 2 2
 total NATs = 2
L= 5
 1.00000 1(2)2 0 0 1
 2.01110 1(10)30 15 2 3
 total NATs = 2
L= 6
 1.000000 1(2)4 2 1 3
 2.011110 2(6)21 12 2 3
 3.000111 2(3)3 0 0 1
 4.001001 1(3)12 9 1 4
 total NATs = 6
L= 7
 1.0000000 1(2)2 0 0 1
 2.0111110 1(14)56 35 2 4
 3.0111000 1(14)28 14 1 3
 4.0110010 1(7)42 28 2 4
 total NATs = 4
L= 8
 1.00000000 1(2)4 2 1 3
 2.01111110 2(8)36 24 2 4
 3.01111000 1(8)24 16 1 3
 4.00000111 2(8)12 4 1 2
 5.10011001 1(4)132 92 4 6
 total NATs = 7
L= 9
 1.000000000 1(2)2 0 0 1
 2.011111110 1(18)90 63 2 5
 3.011111000 1(18)45 27 1 3
 4.000000111 1(18)45 27 1 4
 5.100111001 1(18)306 216 4 6
 6.001001001 1(3)24 21 1 8
 total NATs = 6
L= 10
 1.0000000000 1(2)4 2 1 3
 2.0111111110 2(10)55 40 2 5
 3.0111111000 2(10)35 25 1 4
 4.0000000111 2(10)20 10 1 3
 5.1001111001 2(10)215 165 4 8
 6.0001110011 2(10)65 40 3 6
 7.0111110000 2(5)10 5 1 2
 8.0111001110 1(10)100 75 2 9
 9.0110010010 1(10)120 100 2 8
 total NATs = 15
L= 11
 1.00000000000 1(2)2 0 0 1
 2.01111111110 1(22)132 99 2 6
 3.01111111000 1(22)66 44 1 3
 4.00000000111 1(22)66 44 1 5
 5.10011111001 1(22)506 396 4 8
 6.01111100000 1(22)66 44 1 4
 7.00011110011 1(22)165 110 3 6
 8.01111001110 1(22)264 209 2 9
 9.00011100110 1(22)253 198 3 6
 10.01110011100 1(22)66 33 2 3
 11.10011001001 1(11)462 385 4 12
 total NATs = 11
L= 12
 1.000000000000 1(2)4 2 1 3
 2.011111111110 2(12)78 60 2 6
 3.011111111000 2(12)48 36 1 5
 4.000000000111 2(12)30 18 1 4
 5.100111111001 2(12)330 270 4 10
 6.011111100000 1(12)48 36 1 4
 7.000111110011 2(12)102 72 3 8
 8.011111110000 2(12)30 18 1 3
 9.011111001110 2(12)168 138 2 12
 10.000111100110 2(12)150 120 3 6
 11.110000011100 2(12)108 78 3 6
 12.011110011110 2(6)87 72 2 9
 13.011110011100 2(12)42 24 2 3
 14.011001100110 1(4)1356 1132 6 12
 15.110000111001 2(12)60 42 2 6
 16.011100111100 2(12)84 66 2 9
 17.000111000111 2(3)3 0 0 1
 18.001001001001 1(3)48 45 1 16
 total NATs = 32
L= 13
 1.0000000000000 1(2)2 0 0 1
 2.0111111111110 1(26)182 143 2 7
 3.0111111111000 1(26)91 65 1 4
 4.0000000000111 1(26)91 65 1 6
 5.1001111111001 1(26)754 624 4 10
 6.0111111100000 1(26)91 65 1 4
 7.0001111110011 1(26)247 182 3 8
 8.0111111110000 1(26)91 65 1 5
 9.0111111001110 1(26)416 351 2 12
 10.0001111100110 1(26)351 286 3 8
 11.1100000011100 1(26)364 299 3 8
 12.0111110011110 1(26)442 377 2 12
 13.0111110011100 1(26)104 65 2 4
 14.0110011100110 1(26)3510 2964 6 18
 15.0111110000110 1(26)260 195 3 6
 16.1100000111001 1(26)104 65 2 4
 17.0111100111000 1(26)143 104 2 6
 18.0111100111100 1(26)221 182 2 9
 19.0111110001110 1(26)234 195 2 9
 20.0111110001110 1(26)130 91 2 6
 21.0111000011100 1(26)52 26 1 3
 22.0110010010010 1(13)312 286 2 16
 total NATs = 22
```

λ ratio = .75 Z = .75 1111111100110000-1111111100110000-rule 4281401136
=3-rule 244 -11110100 Length=1 -13



11110100 244
11010000 208
10001010 138
10101110 174





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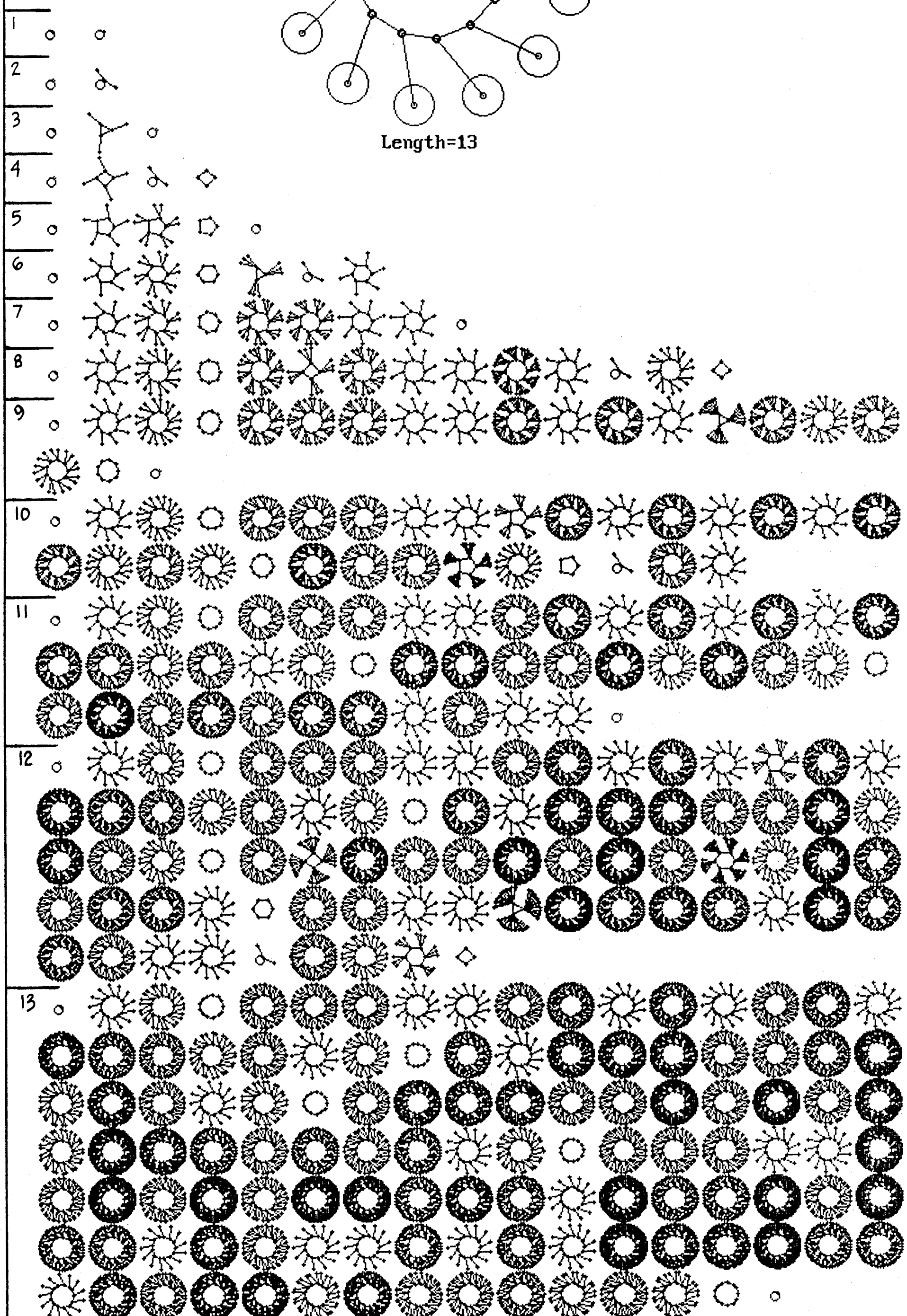
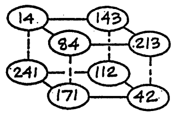
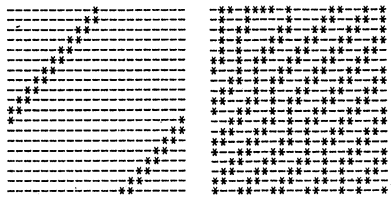


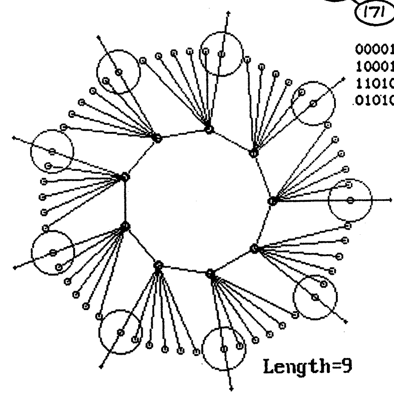

```
3-rule 244 =11110100
ty. at no(p)s g ml mp
L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.11 1(1)3 2 1 3
 total NATs = 2
L= 3
 1.000 1(1)1 0 0 1
 2.101 1(3)6 3 1 2
 3.111 1(1)1 0 0 1
 total NATs = 3
L= 4
 1.0000 1(1)1 0 0 1
 2.1001 1(4)8 4 1 2
 3.1111 1(1)3 2 1 3
 4.0111 1(4)4 0 0 1
 total NATs = 4
L= 5
 1.00000 1(1)1 0 0 1
 2.10001 1(5)10 5 1 2
 3.10111 1(5)15 10 1 3
 4.00111 1(5)5 0 0 1
 5.11111 1(1)1 0 0 1
 total NATs = 5
L= 6
 1.000000 1(1)1 0 0 1
 2.100001 1(6)12 6 1 2
 3.100111 1(6)18 12 1 3
 4.000111 1(6)6 0 0 1
 5.101101 1(3)12 9 1 4
 6.111111 1(1)3 2 1 3
 7.111011 1(6)12 6 1 2
 total NATs = 7
L= 7
 1.0000000 1(1)1 0 0 1
 2.1000001 1(7)14 7 1 2
 3.1000111 1(7)21 14 1 3
 4.0000111 1(7)7 0 0 1
 5.1001101 1(7)28 21 1 4
 6.1011111 1(7)28 21 1 4
 7.1011011 1(7)14 7 1 2
 8.1001111 1(7)14 7 1 2
 9.1111111 1(1)1 0 0 1
 total NATs = 9
L= 8
 1.00000000 1(1)1 0 0 1
 2.10000001 1(8)16 8 1 2
 3.10000111 1(8)24 16 1 3
 4.00000111 1(8)8 0 0 1
 5.10001101 1(8)32 24 1 4
 6.10011001 1(4)16 12 1 4
 7.10011111 1(8)32 24 1 4
 8.10011011 1(8)16 8 1 2
 9.10001111 1(8)16 8 1 2
 10.10110111 1(8)48 40 1 6
 11.10110011 1(8)16 8 1 2
 12.11111111 1(1)3 2 1 3
 13.11111011 1(8)24 16 1 3
 14.01110111 1(4)4 0 0 1
 total NATs = 14
L= 9
 1.000000000 1(1)1 0 0 1
 2.100000001 1(9)18 9 1 2
 3.100000111 1(9)27 18 1 3
 4.000000111 1(9)9 0 0 1
 5.100001101 1(9)36 27 1 4
 6.100011001 1(9)36 27 1 4
 7.100011111 1(9)36 27 1 4
 8.100011011 1(9)18 9 1 2
 9.100001111 1(9)18 9 1 2
 10.100110111 1(9)54 45 1 6
 11.100110011 1(9)18 9 1 2
 12.100111101 1(9)54 45 1 6
 13.100011101 1(9)18 9 1 2
 14.101101101 1(3)24 21 1 8
 15.101111111 1(9)45 36 1 5
 16.101111011 1(9)27 18 1 3
 17.101101111 1(9)36 27 1 4
 18.100111111 1(9)27 18 1 3
 19.001110111 1(9)9 0 0 1
 20.111111111 1(1)1 0 0 1
 total NATs = 20
L= 10
 1.0000000000 1(1)1 0 0 1
 2.1000000001 1(10)20 10 1 2
 3.1000000111 1(10)30 20 1 3
 4.0000000111 1(10)10 0 0 1
 5.1000001101 1(10)40 30 1 4
 6.1000011001 1(10)40 30 1 4
 7.1000011111 1(10)40 30 1 4
 8.1000011011 1(10)20 10 1 2
 9.1000001111 1(10)20 10 1 2
 10.1000110001 1(5)20 15 1 4
 11.1000110111 1(10)60 50 1 6
 12.1000110011 1(10)20 10 1 2
 13.1000111101 1(10)60 50 1 6
 14.1000011101 1(10)20 10 1 2
 15.1001100111 1(10)60 50 1 6
 16.1001100011 1(10)20 10 1 2
 17.1001101101 1(10)80 70 1 8
 18.1001111111 1(10)50 40 1 5
 19.1001111011 1(10)30 20 1 3
 20.1001101111 1(10)40 30 1 4
 21.1000111111 1(10)30 20 1 3
 22.0001110111 1(10)10 0 0 1
 23.1011011111 1(10)80 70 1 8
 24.1011011011 1(10)40 30 1 4
 25.1011001111 1(10)40 30 1 4
 26.1011110111 1(5)45 40 1 9
 27.1011110011 1(10)30 20 1 3
 28.0011100111 1(5)5 0 0 1
 29.1111111111 1(1)3 2 1 3
 30.1111111011 1(10)40 30 1 4
 31.1110111011 1(10)20 10 1 2
 total NATs = 31
```

λ ratio = .75 Z = .75 0000000011111100-0000000011111100-rule 16515324
=3-rule 14 -00001110 Length=1 -14

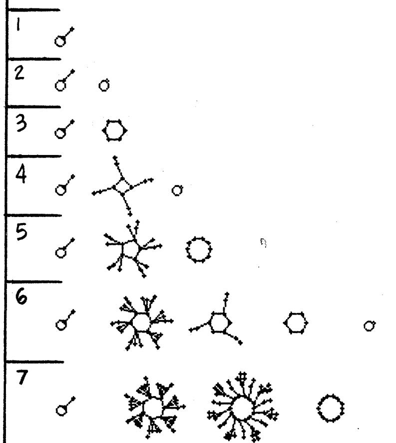


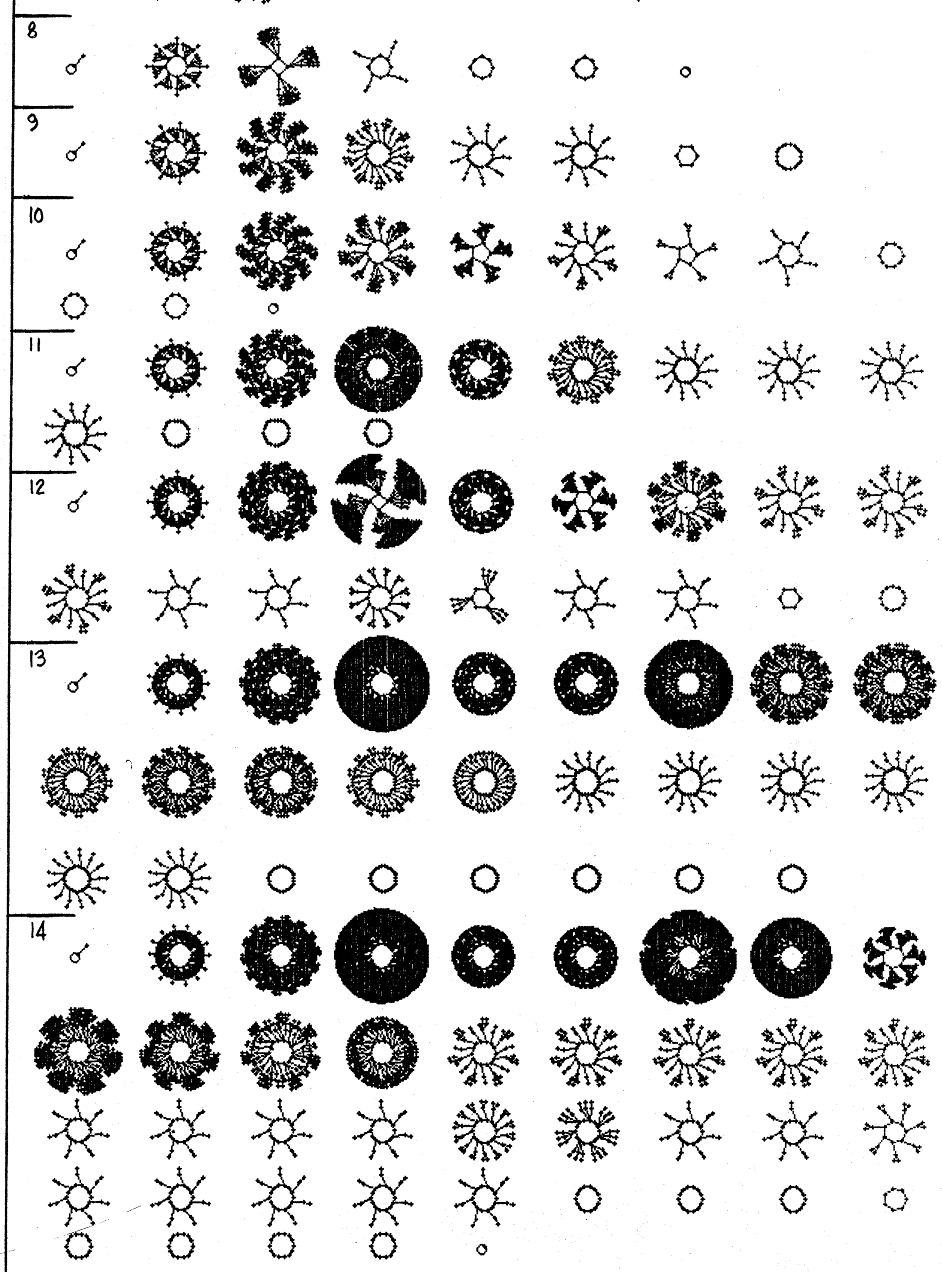
00001110	14
10001111	143
11010101	213
01010100	84





Length=9

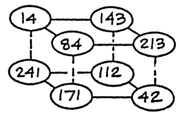




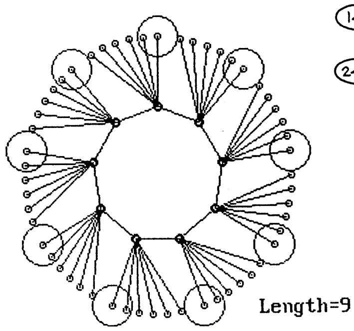

```
3-rule 14 =00001110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)2 1 1 2
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)2 1 1 2
 2.001 1(6)6 0 0 1
 total NATs = 2
L= 4
 1.0000 1(1)2 1 1 2
 2.0011 1(4)12 4 2 2
 3.0101 2(1)1 0 0 1
 total NATs = 4
L= 5
 1.00000 1(1)2 1 1 2
 2.00011 1(5)20 10 2 3
 3.00101 1(10)10 0 0 1
 total NATs = 3
L= 6
 1.000000 1(1)2 1 1 2
 2.000011 1(6)30 18 2 4
 3.001101 2(6)12 3 2 2
 4.001001 1(6)6 0 0 1
 5.010101 2(1)1 0 0 1
 total NATs = 7
L= 7
 1.0000000 1(1)2 1 1 2
 2.0000011 1(7)42 28 2 5
 3.0011001 1(14)70 28 3 3
 4.0010101 1(14)14 0 0 1
 total NATs = 4
L= 8
 1.00000000 1(1)2 1 1 2
 2.00000011 1(8)56 40 2 6
 3.00110011 1(4)140 76 4 4
 4.00110101 2(8)16 4 2 2
 5.00100101 2(8)8 0 0 1
 6.00101101 1(8)8 0 0 1
 7.01010101 2(1)1 0 0 1
 total NATs = 10
L= 9
 1.000000000 1(1)2 1 1 2
 2.000000011 1(9)72 54 2 7
 3.000110011 1(9)252 162 4 6
 4.001100101 1(18)90 36 3 3
 5.001101101 1(18)36 9 2 2
 6.001101001 1(18)36 9 2 2
 7.001001001 1(6)6 0 0 1
 8.001010101 1(18)18 0 0 1
 total NATs = 8
L= 10
 1.0000000000 1(1)2 1 1 2
 2.0000000011 1(10)90 70 2 8
 3.0000110011 1(10)330 230 4 8
 4.0011001101 2(10)140 70 4 4
 5.0001100011 1(5)90 70 2 9
 6.0011001001 2(10)50 20 3 3
 7.0011011001 2(5)20 10 2 2
 8.0011010101 2(10)20 5 2 2
 9.0010010101 2(10)10 0 0 1
 10.0010100101 1(10)10 0 0 1
 11.0010101101 2(10)10 0 0 1
 12.0101010101 2(1)1 0 0 1
 total NATs = 19
L= 11
 1.00000000000 1(1)2 1 1 2
 2.00000000011 1(11)110 88 2 9
 3.00000110011 1(11)418 308 4 10
 4.00110011001 1(22)924 550 5 6
 5.00001100011 1(11)242 198 2 12
 6.00110010101 1(22)110 44 3 3
 7.00110110101 1(22)44 11 2 2
 8.00110101101 1(22)44 11 2 2
 9.00110101001 1(22)44 11 2 2
 10.00110100101 1(22)44 11 2 2
 11.00100100101 1(22)22 0 0 1
 12.00100101101 1(22)22 0 0 1
 13.00101010101 1(22)22 0 0 1
 total NATs = 13
L= 12
 1.000000000000 1(1)2 1 1 2
 2.000000000011 1(12)132 108 2 10
 3.000000110011 1(12)516 396 4 12
 4.001100110011 1(4)1848 1264 6 9
 5.000001100011 1(12)312 264 2 15
 6.000011000011 1(6)162 138 2 16
 7.001100110101 2(12)168 84 4 4
 8.001100100101 2(12)60 24 3 3
 9.001100101001 2(12)60 24 3 3
 10.001100101101 2(12)60 24 3 3
 11.001101101101 2(12)24 6 2 2
 12.001101101001 2(12)24 6 2 2
 13.001101100101 2(12)48 24 2 2
 14.001101001101 2(6)24 9 2 4
 15.001101010101 2(12)24 6 2 2
 16.001101001001 2(12)24 6 2 2
 17.001001001001 1(6)6 0 0 1
 18.001001010101 2(12)12 0 0 1
 19.001010010101 2(12)12 0 0 1
 20.001010101101 2(12)12 0 0 1
 21.001010110101 1(12)12 0 0 1
 22.010101010101 2(1)1 0 0 1
 total NATs = 36
L= 13
 1.0000000000000 1(1)2 1 1 2
 2.0000000000011 1(13)156 130 2 11
 3.0000000110011 1(13)624 494 4 14
 4.0001100110011 1(13)3432 2574 6 12
 5.0000001100011 1(13)390 338 2 18
 6.0000011000011 1(13)416 364 2 20
 7.0011001100101 1(26)1092 650 5 6
 8.0011001101101 1(26)364 182 4 4
 9.0011001101001 1(26)364 182 4 4
 10.0011001001001 1(26)130 52 3 3
 11.0011001001101 1(26)260 156 3 4
 12.0011001011001 1(26)260 130 3 6
 13.0011001010101 1(26)130 52 3 3
 14.0011011011001 1(26)104 52 2 2
 15.0011011010101 1(26)52 13 2 2
 16.0011010110101 1(26)52 13 2 2
 17.0011010101101 1(26)52 13 2 2
 18.0011010101001 1(26)52 13 2 2
 19.0011010100101 1(26)52 13 2 2
 20.0011010010101 1(26)52 13 2 2
 21.0010010010101 1(26)26 0 0 1
 22.0010010100101 1(26)26 0 0 1
 23.0010010101101 1(26)26 0 0 1
 24.0010010110101 1(26)26 0 0 1
 25.0010100101101 1(26)26 0 0 1
 26.0010101010101 1(26)26 0 0 1
 total NATs = 26
```

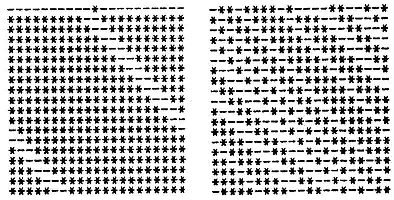
λ ratio = .75 Z = .75

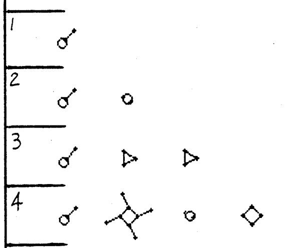
1111111100000011-1111111100000011-rule 4278451971
=3-rule 241 -11110001
Length=1 -12

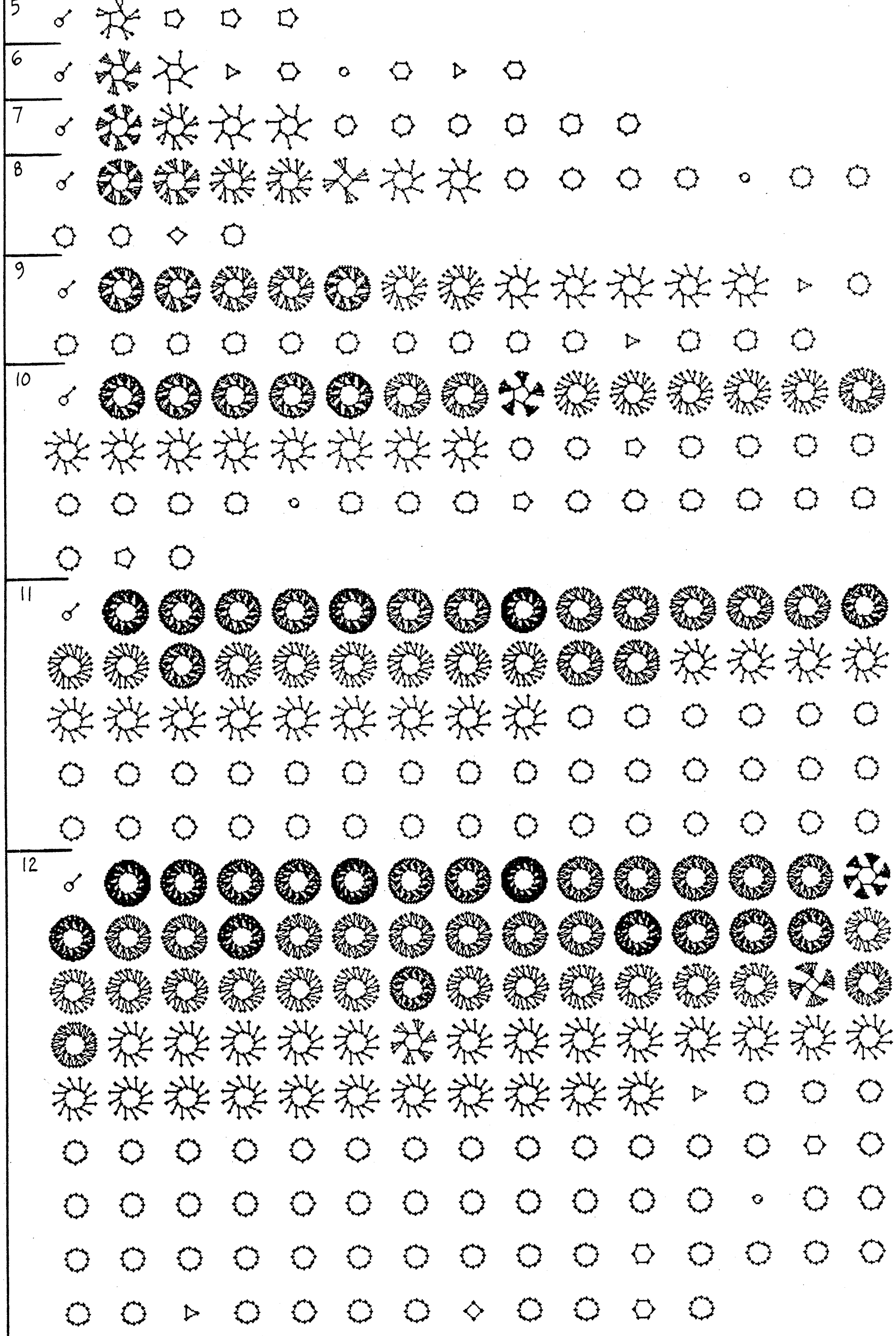


11110001	241
01110000	112
00101010	42
10101011	171







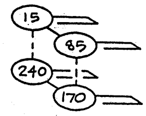



```
3-rule 241 =11110001

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.111 1(1)2 1 1 2
 2.001 1(3)3 0 0 1
 3.011 1(3)3 0 0 1
 total NATs = 3
L= 4
 1.1111 1(1)2 1 1 2
 2.1100 1(4)8 4 1 2
 3.0101 1(2)2 0 0 1
 4.0111 1(4)4 0 0 1
 total NATs = 4
L= 5
 1.11111 1(1)2 1 1 2
 2.11100 1(5)15 10 1 3
 3.00101 1(5)5 0 0 1
 4.01011 1(5)5 0 0 1
 5.01111 1(5)5 0 0 1
 total NATs = 5
L= 6
 1.111111 1(1)2 1 1 2
 2.111100 1(6)24 18 1 4
 3.110010 1(6)12 6 1 2
 4.001001 1(3)3 0 0 1
 5.001101 1(6)6 0 0 1
 6.010101 1(2)2 0 0 1
 7.010111 1(6)6 0 0 1
 8.011011 1(3)3 0 0 1
 9.011111 1(6)6 0 0 1
 total NATs = 9
L= 7
 1.1111111 1(1)2 1 1 2
 2.1111100 1(7)35 28 1 5
 3.1110010 1(7)21 14 1 3
 4.1100100 1(7)14 7 1 2
 5.1100110 1(7)14 7 1 2
 6.0010101 1(7)7 0 0 1
 7.0011101 1(7)7 0 0 1
 8.0101011 1(7)7 0 0 1
 9.0101111 1(7)7 0 0 1
 10.0110111 1(7)7 0 0 1
 11.0111111 1(7)7 0 0 1
 total NATs = 11
L= 8
 1.11111111 1(1)2 1 1 2
 2.11111100 1(8)48 40 1 6
 3.11110010 1(8)32 24 1 4
 4.11100100 1(8)24 16 1 3
 5.11100110 1(8)24 16 1 3
 6.11001100 1(4)16 12 1 4
 7.11001010 1(8)16 8 1 2
 8.11001110 1(8)16 8 1 2
 9.00100101 1(8)8 0 0 1
 10.00101101 1(8)8 0 0 1
 11.00110101 1(8)8 0 0 1
 12.00111101 1(8)8 0 0 1
 13.01010101 1(2)2 0 0 1
 14.01010111 1(8)8 0 0 1
 15.01011011 1(8)8 0 0 1
 16.01011111 1(8)8 0 0 1
 17.01101111 1(8)8 0 0 1
 18.01110111 1(4)4 0 0 1
 19.01111111 1(8)8 0 0 1
 total NATs = 19
L= 9
 1.111111111 1(1)2 1 1 2
 2.111111100 1(9)63 54 1 7
 3.111110010 1(9)45 36 1 5
 4.111100100 1(9)36 27 1 4
 5.111100110 1(9)36 27 1 4
 6.111001100 1(9)54 45 1 6
 7.111001010 1(9)27 18 1 3
 8.111001110 1(9)27 18 1 3
 9.110010010 1(9)18 9 1 2
 10.110010100 1(9)18 9 1 2
 11.110010110 1(9)18 9 1 2
 12.110011010 1(9)18 9 1 2
 13.110011110 1(9)18 9 1 2
 14.001001001 1(3)3 0 0 1
 15.001001101 1(9)9 0 0 1
 16.001010101 1(9)9 0 0 1
 17.001011101 1(9)9 0 0 1
 18.001101101 1(9)9 0 0 1
 19.001110101 1(9)9 0 0 1
 20.001111101 1(9)9 0 0 1
 21.010101011 1(9)9 0 0 1
 22.010101111 1(9)9 0 0 1
 23.010110111 1(9)9 0 0 1
 24.010111011 1(9)9 0 0 1
 25.010111111 1(9)9 0 0 1
 26.011011011 1(3)3 0 0 1
 27.011011111 1(9)9 0 0 1
 28.011101111 1(9)9 0 0 1
 29.011111111 1(9)9 0 0 1
 total NATs = 29
L= 10
 1.1111111111 1(1)2 1 1 2
 2.1111111100 1(10)80 70 1 8
 3.1111110010 1(10)60 50 1 6
 4.1111100100 1(10)50 40 1 5
 5.1111100110 1(10)50 40 1 5
 6.1111001100 1(10)80 70 1 8
 7.1111001010 1(10)40 30 1 4
 8.1111001110 1(10)40 30 1 4
 9.1110011100 1(5)45 40 1 9
 10.1110010010 1(10)30 20 1 3

 35.0101010101 1(2)2 0 0 1
 36.0101010111 1(10)10 0 0 1
 37.0101011011 1(10)10 0 0 1
 38.0101011111 1(10)10 0 0 1
 39.0101101011 1(5)5 0 0 1
 40.0101101111 1(10)10 0 0 1
 41.0101110111 1(10)10 0 0 1
 42.0101111011 1(10)10 0 0 1
 43.0101111111 1(10)10 0 0 1
 44.0110110111 1(10)10 0 0 1
 45.0110111111 1(10)10 0 0 1
 46.0111011111 1(10)10 0 0 1
 47.0111101111 1(5)5 0 0 1
 48.0111111111 1(10)10 0 0 1
 total NATs = 48
```

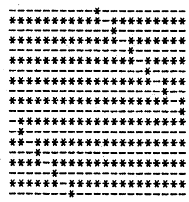
λ ratio = 1 $Z = 1$

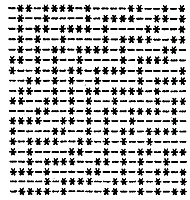
0000000011111111-0000000011111111-rule 16711935
=3-rule 15 -00001111

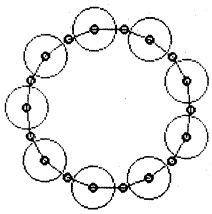
Length=1 -13



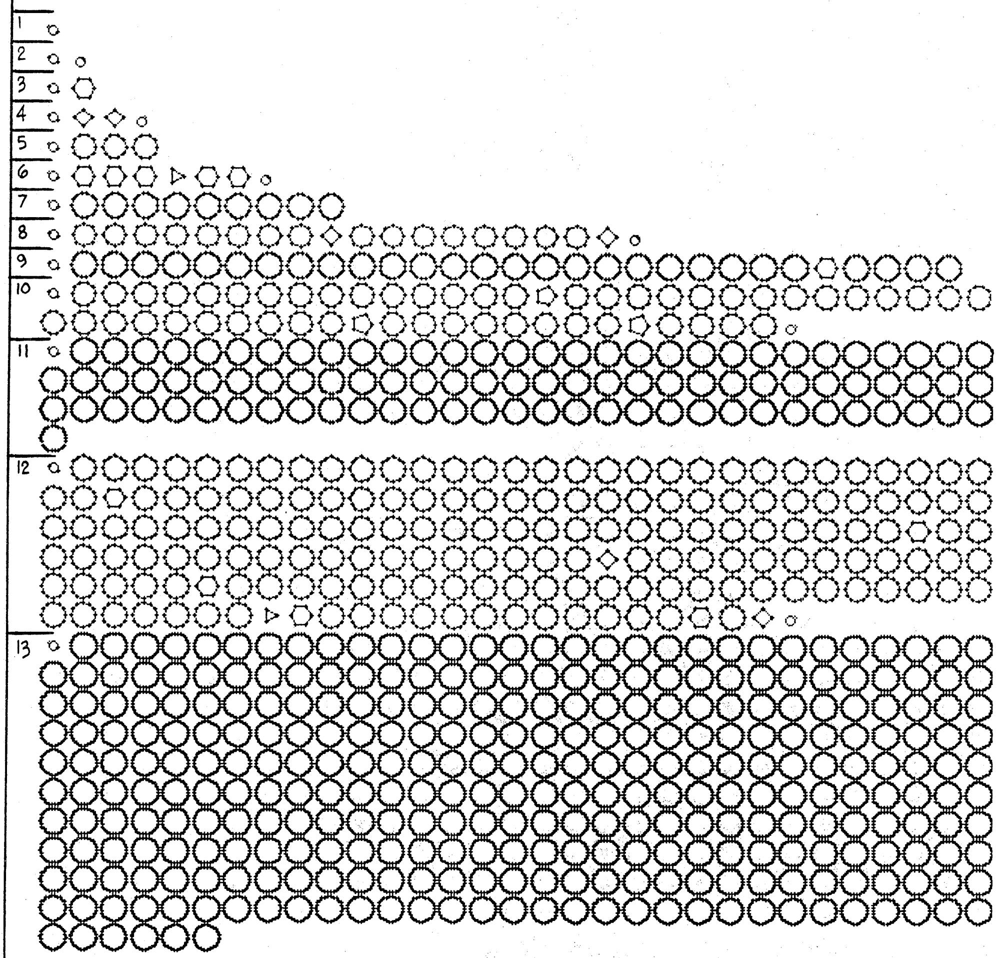
00001111 15
01010101 85







Length=9



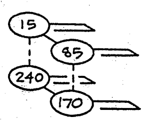
3-rule 15 =00001111

ty. at no(p)s g ml mp

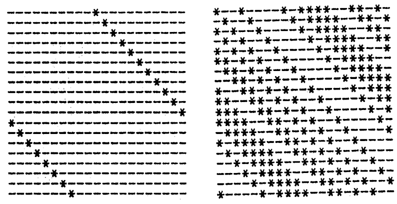
L= 1
1.0 1(2)2 0 0 1
total NATs = 1
L= 2
1.00 1(2)2 0 0 1
2.01 2(1)1 0 0 1
total NATs = 3
L= 3
1.000 1(2)2 0 0 1
2.001 1(6)6 0 0 1
total NATs = 2
L= 4
1.0000 1(2)2 0 0 1
2.0001 2(4)4 0 0 1
3.0011 1(4)4 0 0 1
4.0101 2(1)1 0 0 1
total NATs = 6
L= 5
1.00000 1(2)2 0 0 1
2.00001 1(10)10 0 0 1
3.00011 1(10)10 0 0 1
4.00101 1(10)10 0 0 1
total NATs = 4
L= 6
1.000000 1(2)2 0 0 1
2.000001 2(6)6 0 0 1
3.000011 2(6)6 0 0 1
4.000101 2(6)6 0 0 1
5.000111 2(3)3 0 0 1
6.001001 1(6)6 0 0 1
7.001011 2(6)6 0 0 1
8.010101 2(1)1 0 0 1
total NATs = 14
L= 7
1.0000000 1(2)2 0 0 1
2.0000001 1(14)14 0 0 1
3.0000011 1(14)14 0 0 1
4.0000101 1(14)14 0 0 1
5.0000111 1(14)14 0 0 1
6.0001001 1(14)14 0 0 1
7.0001011 1(14)14 0 0 1
8.0001101 1(14)14 0 0 1
9.0010011 1(14)14 0 0 1
10.0010101 1(14)14 0 0 1
total NATs = 10
L= 8
1.00000000 1(2)2 0 0 1
2.00000001 2(8)8 0 0 1
3.00000011 2(8)8 0 0 1
4.00000101 2(8)8 0 0 1
5.00000111 2(8)8 0 0 1
6.00001001 2(8)8 0 0 1
7.00001011 2(8)8 0 0 1
8.00001101 2(8)8 0 0 1
9.00001111 1(8)8 0 0 1
10.00010001 2(4)4 0 0 1
11.00010011 2(8)8 0 0 1
12.00010101 2(8)8 0 0 1
13.00010111 2(8)8 0 0 1
14.00011001 2(8)8 0 0 1
15.00011011 2(8)8 0 0 1
16.00100101 2(8)8 0 0 1
17.00101011 2(8)8 0 0 1
18.00101101 1(8)8 0 0 1
19.00110011 1(4)4 0 0 1
20.01010101 2(1)1 0 0 1
total NATs = 36
L= 9
1.000000000 1(2)2 0 0 1
2.000000001 1(18)18 0 0 1
3.000000011 1(18)18 0 0 1
4.000000101 1(18)18 0 0 1
5.000000111 1(18)18 0 0 1
6.000001001 1(18)18 0 0 1
7.000001011 1(18)18 0 0 1
8.000001101 1(18)18 0 0 1
9.000001111 1(18)18 0 0 1
10.000010001 1(18)18 0 0 1
11.000010011 1(18)18 0 0 1
12.000010101 1(18)18 0 0 1
13.000010111 1(18)18 0 0 1
14.000011001 1(18)18 0 0 1
15.000011011 1(18)18 0 0 1
16.000011101 1(18)18 0 0 1
17.000100011 1(18)18 0 0 1
18.000100101 1(18)18 0 0 1
19.000100111 1(18)18 0 0 1
20.000101001 1(18)18 0 0 1
21.000101011 1(18)18 0 0 1
22.000101101 1(18)18 0 0 1
23.000110011 1(18)18 0 0 1
24.000110101 1(18)18 0 0 1
25.000110111 1(18)18 0 0 1
26.001001001 1(6)6 0 0 1
27.001001011 1(18)18 0 0 1
28.001001101 1(18)18 0 0 1
29.001010011 1(18)18 0 0 1
30.001010101 1(18)18 0 0 1
total NATs = 30

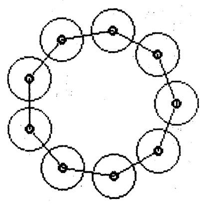
λ ratio = 1 $Z = 1$

1111111100000000-1111111100000000-rule 4278255360
=3-rule 240 -11110000
Length=1 -12

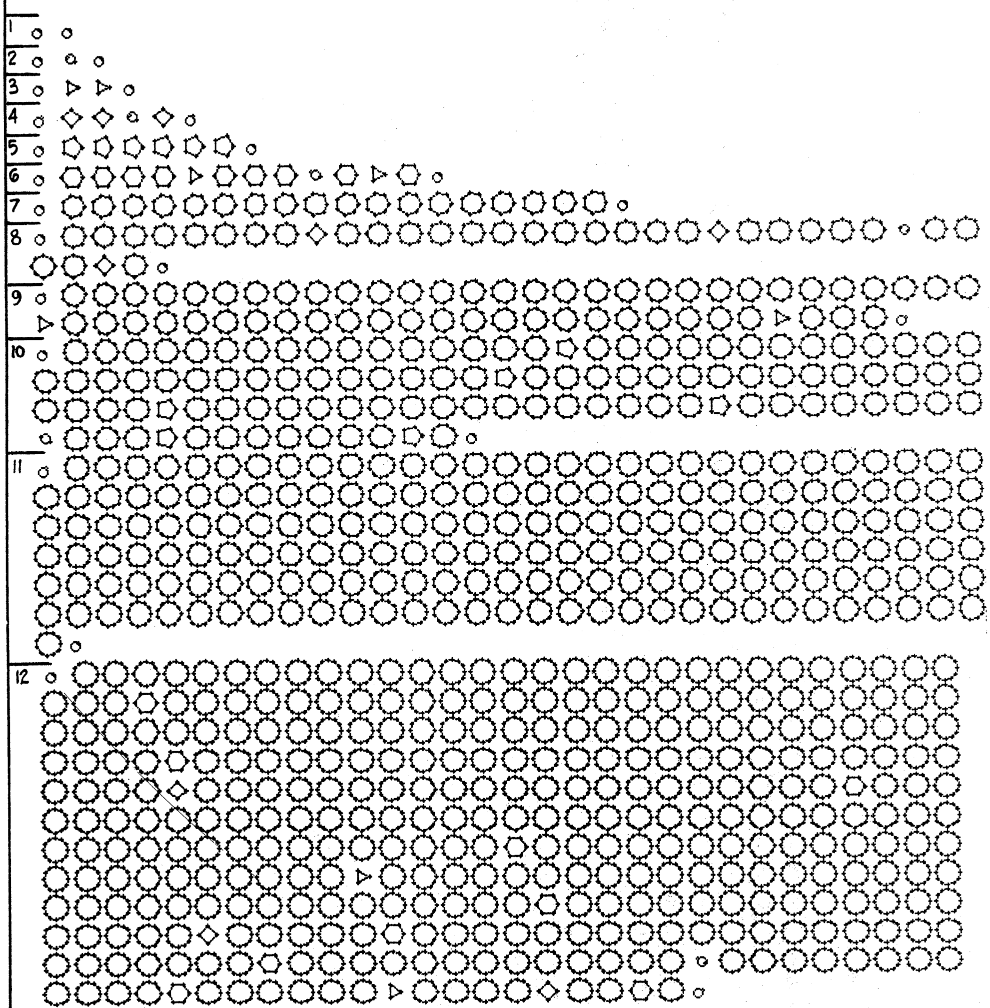


11110000 240
10101010 170





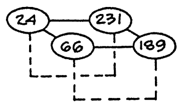
Length=9



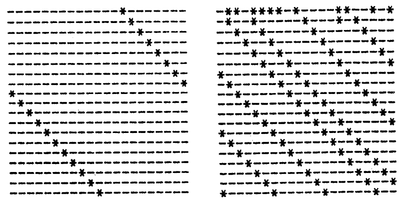

```
3-rule 240 =11110000

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 1(2)2 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)1 0 0 1
 2.001 1(3)3 0 0 1
 3.011 1(3)3 0 0 1
 4.111 1(1)1 0 0 1
 total NATs = 4
L= 4
 1.0000 1(1)1 0 0 1
 2.0001 1(4)4 0 0 1
 3.0011 1(4)4 0 0 1
 4.0101 1(2)2 0 0 1
 5.0111 1(4)4 0 0 1
 6.1111 1(1)1 0 0 1
 total NATs = 6
L= 5
 1.00000 1(1)1 0 0 1
 2.00001 1(5)5 0 0 1
 3.00011 1(5)5 0 0 1
 4.00101 1(5)5 0 0 1
 5.00111 1(5)5 0 0 1
 6.01011 1(5)5 0 0 1
 7.01111 1(5)5 0 0 1
 8.11111 1(1)1 0 0 1
 total NATs = 8
L= 6
 1.000000 1(1)1 0 0 1
 2.000001 1(6)6 0 0 1
 3.000011 1(6)6 0 0 1
 4.000101 1(6)6 0 0 1
 5.000111 1(6)6 0 0 1
 6.001001 1(3)3 0 0 1
 7.001011 1(6)6 0 0 1
 8.001101 1(6)6 0 0 1
 9.001111 1(6)6 0 0 1
 10.010101 1(2)2 0 0 1
 11.010111 1(6)6 0 0 1
 12.011011 1(3)3 0 0 1
 13.011111 1(6)6 0 0 1
 14.111111 1(1)1 0 0 1
 total NATs = 14
L= 7
 1.0000000 1(1)1 0 0 1
 2.0000001 1(7)7 0 0 1
 3.0000011 1(7)7 0 0 1
 4.0000101 1(7)7 0 0 1
 5.0000111 1(7)7 0 0 1
 6.0001001 1(7)7 0 0 1
 7.0001011 1(7)7 0 0 1
 8.0001101 1(7)7 0 0 1
 9.0001111 1(7)7 0 0 1
 10.0010011 1(7)7 0 0 1
 11.0010101 1(7)7 0 0 1
 12.0010111 1(7)7 0 0 1
 13.0011011 1(7)7 0 0 1
 14.0011101 1(7)7 0 0 1
 15.0011111 1(7)7 0 0 1
 16.0101011 1(7)7 0 0 1
 17.0101111 1(7)7 0 0 1
 18.0110111 1(7)7 0 0 1
 19.0111111 1(7)7 0 0 1
 20.1111111 1(1)1 0 0 1
 total NATs = 20
L= 8
 1.00000000 1(1)1 0 0 1
 2.00000001 1(8)8 0 0 1
 3.00000011 1(8)8 0 0 1
 4.00000101 1(8)8 0 0 1
 5.00000111 1(8)8 0 0 1
 6.00001001 1(8)8 0 0 1
 7.00001011 1(8)8 0 0 1
 8.00001101 1(8)8 0 0 1
 9.00001111 1(8)8 0 0 1
 10.00010001 1(4)4 0 0 1
 11.00010011 1(8)8 0 0 1
 12.00010101 1(8)8 0 0 1
 13.00010111 1(8)8 0 0 1
 14.00011001 1(8)8 0 0 1
 15.00011011 1(8)8 0 0 1
 16.00011101 1(8)8 0 0 1
 17.00011111 1(8)8 0 0 1
 18.00100101 1(8)8 0 0 1
 19.00100111 1(8)8 0 0 1
 20.00101011 1(8)8 0 0 1
 21.00101101 1(8)8 0 0 1
 22.00101111 1(8)8 0 0 1
 23.00110011 1(4)4 0 0 1
 24.00110101 1(8)8 0 0 1
 25.00110111 1(8)8 0 0 1
 26.00111011 1(8)8 0 0 1
 27.00111101 1(8)8 0 0 1
 28.00111111 1(8)8 0 0 1
 29.01010101 1(2)2 0 0 1
 30.01010111 1(8)8 0 0 1
 31.01011011 1(8)8 0 0 1
 32.01011111 1(8)8 0 0 1
 33.01101111 1(8)8 0 0 1
 34.01110111 1(4)4 0 0 1
 35.01111111 1(8)8 0 0 1
 36.11111111 1(1)1 0 0 1
 total NATs = 36
```

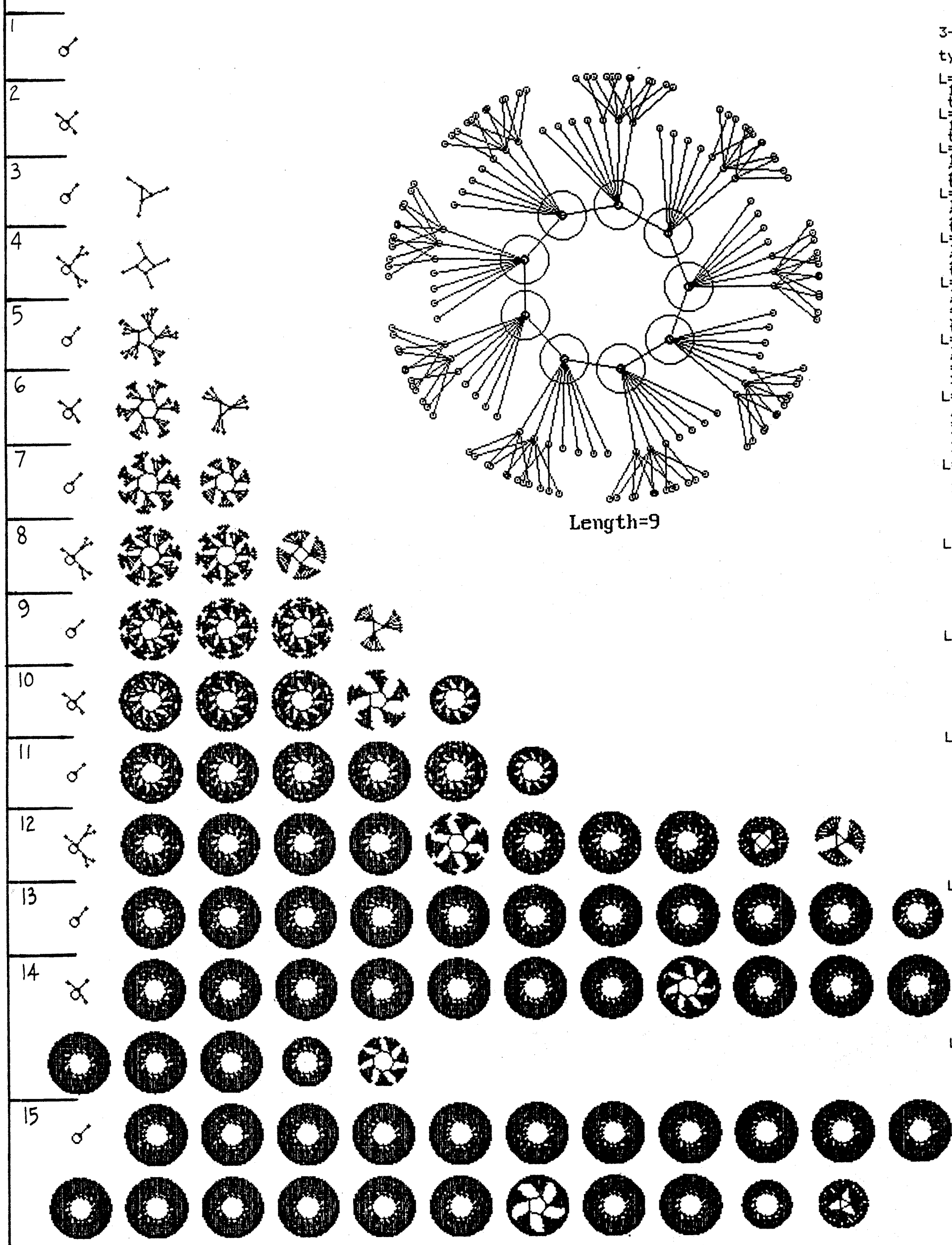
λ ratio = .5 Z = .5

00000001111000000-00000001111000000-rule 62915520
=3-rule 24 -00011000   Length=1 -15



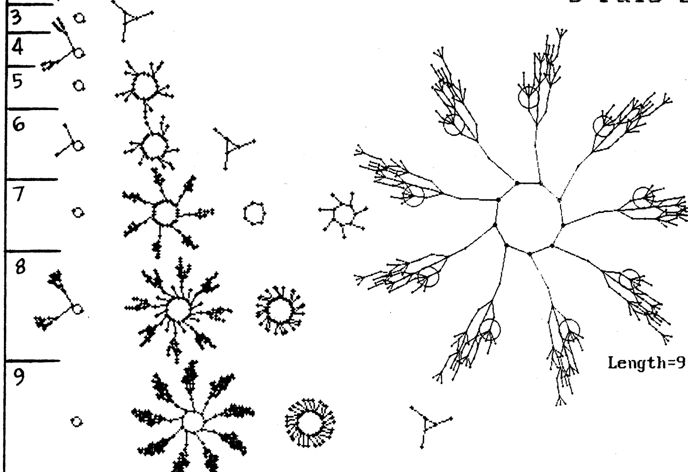
00011000 24
11100111 231
10111101 189
01000010 66

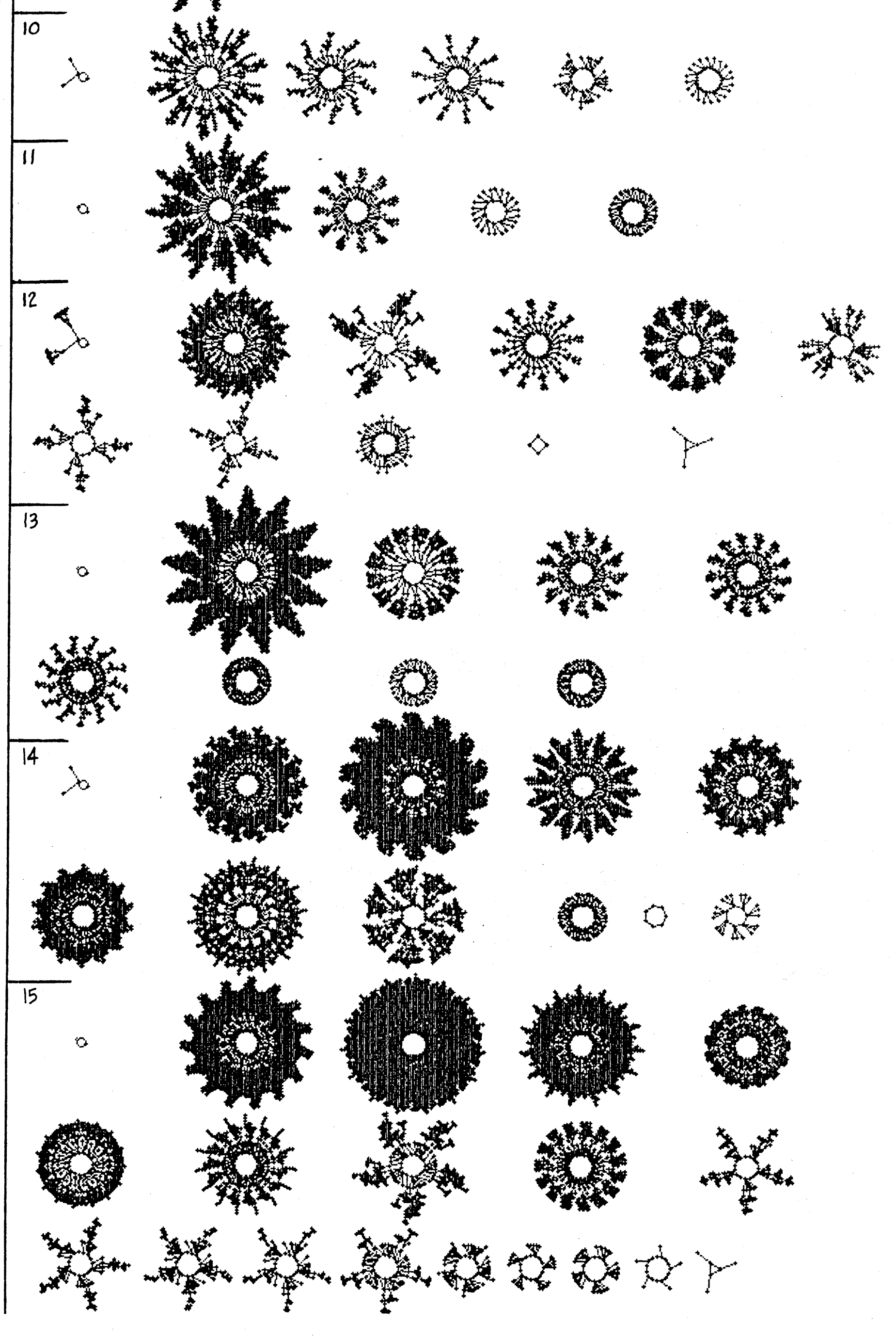


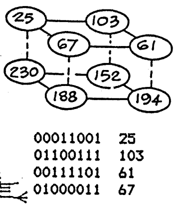


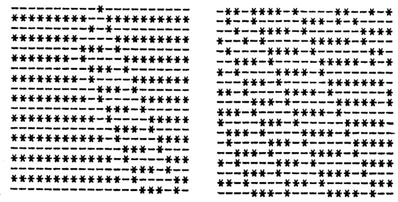

```
3-rule 24 =00011000

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 3 1 4
 total NATs = 1
L= 3
 1.000 1(1)2 1 1 2
 2.001 1(3)6 3 1 2
 total NATs = 2
L= 4
 1.0000 1(1)8 5 2 4
 2.0001 1(4)8 4 1 2
 total NATs = 2
L= 5
 1.00000 1(1)2 1 1 2
 2.00001 1(5)30 20 2 4
 total NATs = 2
L= 6
 1.000000 1(1)4 3 1 4
 2.000001 1(6)48 36 2 4
 3.100100 1(3)12 9 1 4
 total NATs = 3
L= 7
 1.0000000 1(1)2 1 1 2
 2.0000001 1(7)84 63 2 6
 3.1000100 1(7)42 35 1 6
 total NATs = 3
L= 8
 1.00000000 1(1)8 5 2 4
 2.00000001 1(8)112 88 2 6
 3.10000100 1(8)96 80 2 8
 4.10001000 1(4)40 36 1 10
 total NATs = 4
L= 9
 1.000000000 1(1)2 1 1 2
 2.000000001 1(9)180 144 2 8
 3.100000100 1(9)144 126 2.10
 4.100001000 1(9)162 144 2 12
 5.100100100 1(3)24 21 1 8
 total NATs = 5
L= 10
 1.0000000000 1(1)4 3 1 4
 2.0000000001 1(10)240 200 2 8
 3.1000000100 1(10)240 210 2 12
 4.1000001000 1(10)240 220 2 16
 5.0100001000 1(5)180 160 2 16
 6.1000100100 1(10)120 110 1 12
 total NATs = 6
L= 11
 1.00000000000 1(1)2 1 1 2
 2.00000000001 1(11)330 275 2 10
 3.10000000100 1(11)330 297 2 14
 4.10000001000 1(11)396 363 2 18
 5.01000001000 1(11)528 484 2 20
 6.10000100100 1(11)264 242 2 16
 7.10001000100 1(11)198 187 1 18
 total NATs = 7
L= 12
 1.000000000000 1(1)8 5 2 4
 2.000000000001 1(12)408 348 2 10
 3.100000000100 1(12)480 432 2 16
 4.100000001000 1(12)552 516 2 22
 5.010000001000 1(12)864 792 2 24
 6.100000100000 1(6)384 360 2 26
 7.100000100100 1(12)384 360 2 20
 8.100001000100 1(12)432 408 2 24
 9.100001001000 1(12)432 408 2 24
 10.100010001000 1(4)104 100 1 26
 11.100100100100 1(3)48 45 1 16
 total NATs = 11
L= 13
 1.0000000000000 1(1)2 1 1 2
 2.0000000000001 1(13)546 468 2 12
 3.1000000000100 1(13)624 572 2 18
 4.1000000001000 1(13)780 728 2 24
 5.0100000001000 1(13)1170 1092 2 28
 6.1000000100000 1(13)1248 1170 2 30
 7.1000000100100 1(13)624 585 2 24
 8.1000001000100 1(13)624 598 2 30
 9.1000001001000 1(13)624 598 2 30
 10.1000010001000 1(13)702 676 2 36
 11.1000010000100 1(13)936 884 2 32
 12.1000100100100 1(13)312 299 1 24
 total NATs = 12
L= 14
 1.00000000000000 1(1)4 3 1 4
 2.00000000000001 1(14)672 588 2 12
 3.10000000000100 1(14)840 770 2 20
 4.10000000001000 1(14)1008 952 2 28
 5.01000000001000 1(14)1680 1568 2 32
 6.10000000100000 1(14)1680 1596 2 36
 7.10000000100100 1(14)840 798 2 28
 8.10000001000100 1(14)1008 966 2 36
 9.01000000100000 1(7)1008 945 2 36
 10.10000001001000 1(14)1008 966 2 36
 11.10000010001000 1(14)1008 980 2 44
 12.10000010000100 1(14)1344 1288 2 40
 13.01000001001000 1(14)1344 1288 2 40
 14.10000100010000 1(14)1512 1456 2 48
 15.10000100100100 1(14)672 644 2 32
 16.10001000100100 1(14)504 490 1 36
 17.10001001000100 1(7)252 245 1 36
 total NATs = 17
L= 15
 1.000000000000000 1(1)2 1 1 2
 2.000000000000001 1(15)840 735 2 14
 3.100000000000100 1(15)1050 975 2 22
 4.100000000001000 1(15)1350 1275 2 30
 5.010000000001000 1(15)2160 2040 2 36
 6.100000000100000 1(15)2400 2280 2 40
 7.100000000100100 1(15)1200 1140 2 32
 8.100000001000100 1(15)1350 1305 2 42
 9.010000000100000 1(15)2700 2565 2 42
 10.100000001001000 1(15)1350 1305 2 42
 11.100000010001000 1(15)1620 1575 2 54
 12.100000010000100 1(15)2160 2070 2 48
 13.010000001001000 1(15)2160 2070 2 48
 14.100000100010000 1(15)2160 2100 2 60
 15.010000010000010 1(15)1920 1860 2 50
 16.100000100001000 1(15)2160 2100 2 60
 17.100000100100100 1(15)960 930 2 40
 18.100001000100100 1(15)1080 1050 2 48
 19.010000100001000 1(5)1080 1040 2 64
 20.100001001000100 1(15)1080 1050 2 48
 21.100001001001000 1(15)1080 1050 2 48
 22.100010001000100 1(15)810 795 1 54
 23.100100100100100 1(3)96 93 1 32
 total NATs = 23
```

λ ratio = .75 Z = .75 0000001111000011-0000001111000011-rule 63112131
=3-rule 25 -00011001 Length=1 -15





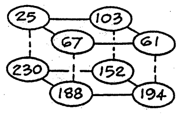




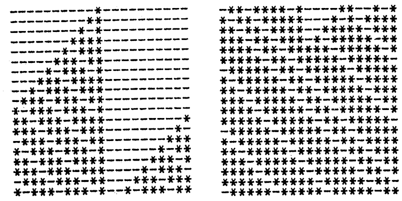
```
3-rule 25 =00011001

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)4 2 1 3
 total NATs = 1
L= 3
 1.000 1(2)2 0 0 1
 2.001 1(3)6 3 1 2
 total NATs = 2
L= 4
 1.0000 1(2)16 4 4 3
 total NATs = 1
L= 5
 1.00000 1(2)2 0 0 1
 2.00001 1(15)30 10 2 3
 total NATs = 2
L= 6
 1.000000 1(2)4 2 1 3
 2.000001 2(15)27 9 2 3
 3.001001 1(3)6 3 1 2
 total NATs = 4
L= 7
 1.0000000 1(2)2 0 0 1
 2.1000010 1(21)105 42 6 4
 3.0001011 1(7)7 0 0 1
 4.0001101 1(7)14 7 1 2
 total NATs = 4
L= 8
 1.00000000 1(2)40 20 5 5
 2.01000111 1(16)168 56 8 4
 3.00001001 1(24)48 24 1 3
 total NATs = 3
L= 9
 1.000000000 1(2)2 0 0 1
 2.010000111 1(9)459 189 11 5
 3.000001101 1(18)45 27 1 3
 4.001001001 1(3)6 3 1 2
 total NATs = 4
L= 10
 1.0000000000 1(2)4 2 1 3
 2.0100000111 1(20)600 260 12 7
 3.1000010010 1(30)190 100 6 4
 4.1010001111 1(20)130 50 6 3
 5.0000100001 1(15)50 30 2 5
 6.0000100111 1(30)50 20 1 3
 total NATs = 6
L= 11
 1.00000000000 1(2)2 0 0 1
 2.01000000111 1(22)1749 814 17 6
 3.00000001101 1(22)187 99 5 4
 4.11110100000 1(11)33 22 1 3
 5.00001001001 1(33)77 44 1 3
 total NATs = 5
L= 12
 1.000000000000 1(2)40 28 4 7
 2.010000000111 1(24)1104 516 10 6
 3.010000100111 3(12)340 184 11 6
 4.101000001111 1(24)216 96 6 3
 5.000000001101 1(24)720 384 7 9
 6.100000100000 2(15)129 51 5 10
 7.100001010010 3(12)112 64 7 7
 8.100001010000 3(12)76 36 6 6
 9.000001000011 2(30)78 36 2 5
 10.000010100111 3(4)4 0 0 1
 11.001001001001 1(3)6 3 1 2
 total NATs = 21
L= 13
 1.0000000000000 1(2)2 0 0 1
 2.0100000000111 1(26)6461 3367 22 10
 3.1010000001111 1(13)533 234 7 5
 4.0000000001101 1(26)299 169 6 5
 5.0011110100000 1(26)286 169 5 4
 6.1000010010010 1(39)299 182 6 6
 7.0000100001001 1(39)130 91 1 5
 8.0000100100111 1(39)78 39 1 4
 9.0000100101001 1(39)104 65 1 5
 total NATs = 9
L= 14
 1.00000000000000 1(2)4 2 1 3
 2.01000000000111 1(28)2114 1204 11 11
 3.10100001000011 1(42)9268 5194 17 13
 4.10100000001111 1(28)896 392 10 5
 5.00000000001101 1(28)854 476 8 7
 6.01000001111111 1(28)1386 770 8 11
 7.01000010100111 1(42)1078 504 11 9
 8.10000101000010 1(21)637 378 8 10
 9.00001001001001 1(42)112 70 1 4
 10.00010110001011 1(7)7 0 0 1
 11.00011010001101 1(7)28 21 1 4
 total NATs = 11
L= 15
 1.000000000000000 1(2)2 0 0 1
 2.010000000000111 1(30)5625 3300 14 14
 3.101000001000011 1(75)17655 9825 16 15
 4.101000000001111 1(30)3195 1695 12 10
 5.000000000001101 1(30)615 390 5 6
 6.001111110100000 1(15)555 330 6 4
 7.111111110100000 1(30)465 270 8 5
 8.010000100100111 3(15)520 325 11 9
 9.010001110100011 1(30)525 345 6 6
 10.100001010010010 3(15)160 105 7 7
 11.100001001010010 3(15)150 100 7 7
 12.100001001010000 3(15)140 85 6 10
 13.100001001000010 3(15)110 65 6 8
 14.100001001110010 3(15)160 110 6 7
 15.000010000100001 1(15)75 55 2 9
 16.000010000100111 3(15)40 25 1 6
 17.000010000101001 3(15)50 35 1 7
 18.000010010100111 3(15)20 5 1 2
 19.001001001001001 1(3)6 3 1 2
 total NATs = 37
```

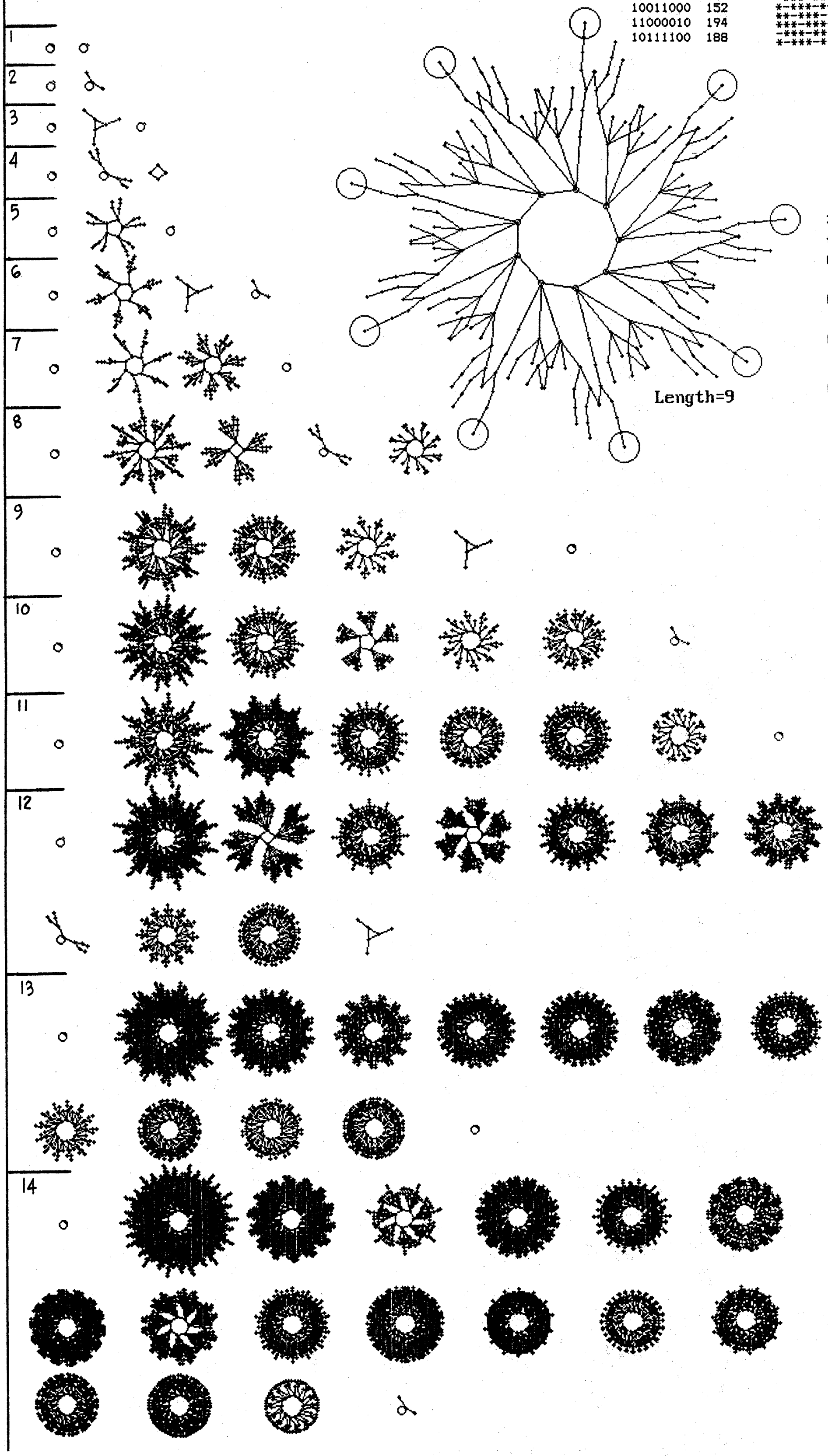
λ ratio = .75   Z = .75

1111110000111100-1111110000111100-rule 4231855164
=3-rule 230 -11100110   Length=1 -14



11100110 230
10011000 152
11000010 194
10111100 188



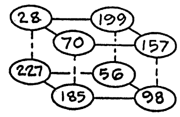



```
3-rule 230 =11100110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.11 1(1)3 2 1 3
 total NATs = 2
L= 3
 1.000 1(1)1 0 0 1
 2.011 1(3)6 3 1 2
 3.111 1(1)1 0 0 1
 total NATs = 3
L= 4
 1.0000 1(1)1 0 0 1
 2.1111 1(1)11 4 3 3
 3.0111 1(4)4 0 0 1
 total NATs = 3
L= 5
 1.00000 1(1)1 0 0 1
 2.01111 1(5)30 10 3 3
 3.11111 1(1)1 0 0 1
 total NATs = 3
L= 6
 1.000000 1(1)1 0 0 1
 2.111011 1(6)54 18 5 3
 3.011011 1(3)6 3 1 2
 4.111111 1(1)3 2 1 3
 total NATs = 4
L= 7
 1.0000000 1(1)1 0 0 1
 2.0111011 1(7)49 14 5 2
 3.1110111 1(7)77 35 4 4
 4.1111111 1(1)1 0 0 1
 total NATs = 4
L= 8
 1.00000000 1(1)1 0 0 1
 2.11101111 1(8)136 48 7 4
 3.01110111 1(4)68 24 4 5
 4.11111111 1(1)11 4 3 3
 5.01101111 1(8)40 24 2 3
 total NATs = 5
L= 9
 1.000000000 1(1)1 0 0 1
 2.011101111 1(9)270 108 7 4
 3.011111111 1(9)171 72 4 5
 4.111011011 1(9)63 36 3 3
 5.011011011 1(3)6 3 1 2
 6.111111111 1(1)1 0 0 1
 total NATs = 6
L= 10
 1.0000000000 1(1)1 0 0 1
 2.1110111011 1(10)500 200 9 7
 3.1110111111 1(10)240 100 5 5
 4.0111101111 1(5)110 70 3 5
 5.0111011011 1(10)60 30 3 3
 6.1110110111 1(10)110 60 3 4
 7.1111111111 1(1)3 2 1 3
 total NATs = 7
L= 11
 1.00000000000 1(1)1 0 0 1
 2.01110111011 1(11)341 143 9 6
 3.11101110111 1(11)792 363 8 6
 4.11101111011 1(11)374 198 5 6
 5.01111111011 1(11)176 88 3 4
 6.11101111111 1(11)297 143 4 6
 7.01101101111 1(11)66 44 2 3
 8.11111111111 1(1)1 0 0 1
 total NATs = 8
L= 12
 1.000000000000 1(1)1 0 0 1
 2.111011101111 1(12)1356 636 11 9
 3.011101110111 1(4)472 192 8 7
 4.011101111011 1(12)300 180 5 6
 5.111011111011 1(6)330 150 5 10
 6.111011111101 1(12)540 324 5 7
 7.111011111111 1(12)396 180 5 6
 8.111011101101 1(12)384 204 5 5
 9.111111111111 1(1)11 4 3 3
 10.111011011011 1(12)96 60 3 4
 11.111011011111 1(12)204 120 3 5
 12.011011011011 1(3)6 3 1 2
 total NATs = 12
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.0111011101111 1(13)2093 962 11 10
 3.1110111011111 1(13)1716 923 7 8
 4.0111011111011 1(13)481 260 5 9
 5.1110111101111 1(13)949 546 5 8
 6.1110111111011 1(13)910 468 5 9
 7.1110111011011 1(13)702 390 5 8
 8.0111111111111 1(13)507 234 4 7
 9.0111011011011 1(13)91 52 3 4
 10.1111111011011 1(13)273 156 3 5
 11.1110110110111 1(13)156 91 3 4
 12.1110110111101 1(13)312 234 3 6
 13.1111111111111 1(1)1 0 0 1
 total NATs = 13
L= 14
 1.00000000000000 1(1)1 0 0 1
 2.11101110111011 1(14)3906 1862 13 11
 3.11101110111111 1(14)2478 1344 7 11
 4.01110110111011 1(7)203 126 5 8
 5.01110111101111 1(14)1848 1078 6 10
 6.01110111111011 1(14)700 420 5 9
 7.01110111011011 1(14)476 252 5 7
 8.11101111101111 1(14)1652 826 5 13
 9.11101111110111 1(7)665 357 5 10
 10.11101111111111 1(14)644 294 5 7
 11.11101110110111 1(14)1092 644 5 7
 12.01111011111111 1(14)1050 728 4 9
 13.01111111011011 1(14)252 140 3 5
 14.11101111101101 1(14)476 322 4 8
 15.11111110110111 1(14)378 210 3 6
 16.11101101111101 1(14)462 336 3 8
 17.01101101101111 1(14)98 70 2 4
 18.11111111111111 1(1)3 2 1 3
 total NATs = 18
```

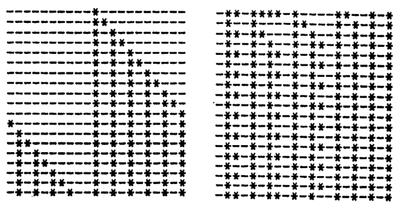
λ ratio = X $Z = .75$
0·75

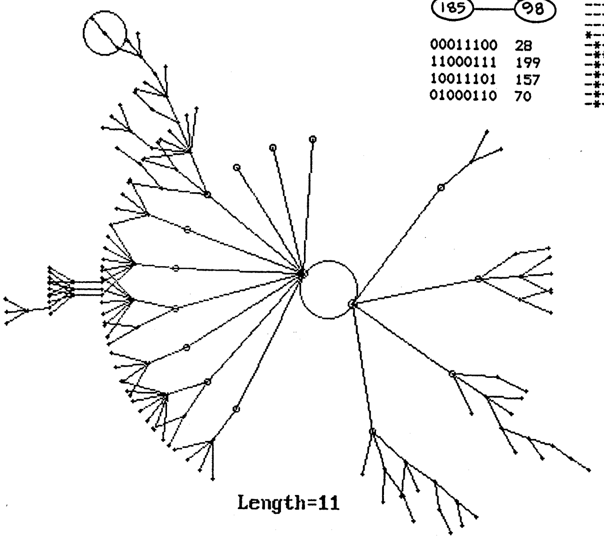
0000001111110000-0000001111110000-rule 66061296
=3-rule 28 -00011100

Length=1 -15

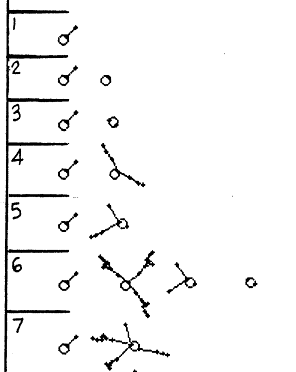


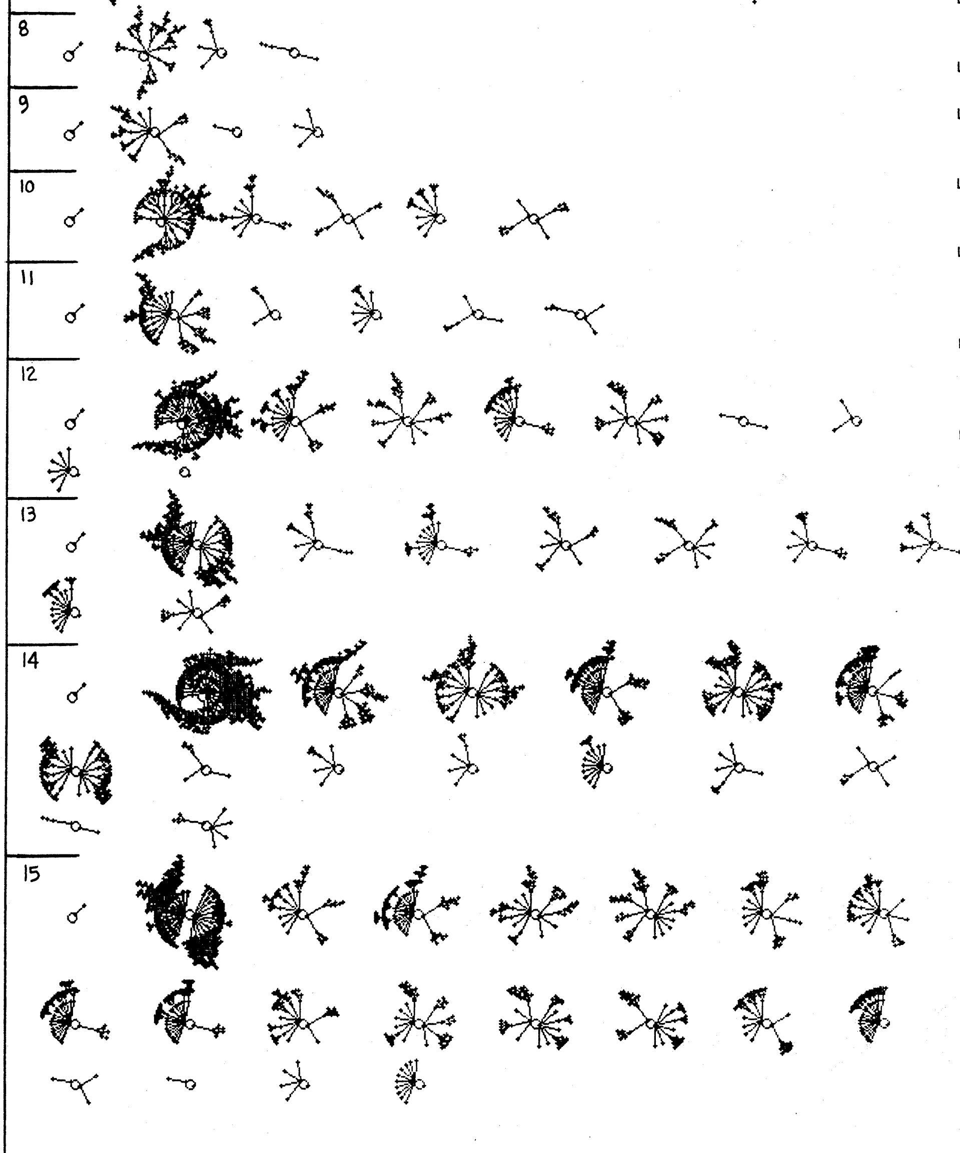
00011100	28
11000111	199
10011101	157
01000110	70





Length=11





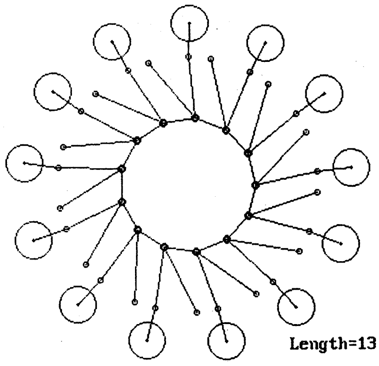
3-rule 28 =00011100

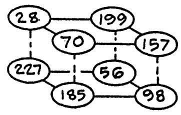
ty. at no(p)s g ml mp

```
L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)2 1 1 2
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)2 1 1 2
 2.001 3(2)2 0 0 1
 total NATs = 4
L= 4
 1.0000 1(1)2 1 1 2
 2.0101 2(1)7 2 3 3
 total NATs = 3
L= 5
 1.00000 1(1)2 1 1 2
 2.01001 5(2)6 2 3 3
 total NATs = 6
L= 6
 1.000000 1(1)2 1 1 2
 2.010101 2(1)22 9 5 4
 3.100100 3(2)4 2 1 3
 4.001101 3(2)2 0 0 1
 total NATs = 9
L= 7
 1.0000000 1(1)2 1 1 2
 2.0101001 7(2)18 8 5 4
 total NATs = 8
L= 8
 1.00000000 1(1)2 1 1 2
 2.01010101 2(1)67 34 7 7
 3.10100100 8(2)9 5 3 4
 4.01001101 8(2)6 2 3 2
 total NATs = 19
L= 9
 1.000000000 1(1)2 1 1 2
 2.010101001 9(2)52 28 7 7
 3.100110100 9(2)3 1 1 2
 4.100100100 3(2)5 3 1 4
 total NATs = 22
L= 10
 1.0000000000 1(1)2 1 1 2
 2.0101010101 2(1)196 115 9 11
 3.1010100100 10(2)26 16 5 7
 4.0101001101 10(2)18 8 5 3
 5.1010010100 5(2)22 14 3 7
 6.0100101101 5(2)16 8 3 3
 total NATs = 33
L= 11
 1.00000000000 1(1)2 1 1 2
 2.01010101001 11(2)150 92 9 11
 3.10100110100 11(2)7 3 3 3
 4.10100100100 11(2)13 9 3 7
 5.01001001101 11(2)8 4 3 3
 6.01001101101 11(2)8 4 3 3
 total NATs = 56
L= 12
 1.000000000000 1(1)2 1 1 2
 2.010101010101 2(1)568 371 11 18
 3.101010100100 12(2)74 50 7 11
 4.010101001101 12(2)52 28 7 5
 5.101010010100 12(2)64 44 5 11
 6.010100101101 12(2)48 28 5 5
 7.100110110100 6(2)4 2 1 2
 8.100110100100 12(2)4 2 1 3
 9.100100100100 3(2)8 6 1 7
 10.001101001101 3(2)2 0 0 1
 total NATs = 75
L= 13
 1.0000000000000 1(1)2 1 1 2
 2.0101010101001 13(2)432 292 11 18
 3.1010100110100 13(2)20 10 5 5
 4.1010100100100 13(2)38 28 5 11
 5.0101001001101 13(2)24 14 5 5
 6.0101001101101 13(2)24 14 5 5
 7.1010010110100 13(2)19 11 3 5
 8.1010011010100 13(2)19 11 3 5
 9.1010010100100 13(2)33 25 3 11
 10.0100100101101 13(2)21 13 3 5
 total NATs = 118
L= 14
 1.00000000000000 1(1)2 1 1 2
 2.01010101010101 2(1)1639 1162 13 29
 3.10101010100100 14(2)213 155 9 18
 4.01010101001101 14(2)150 92 9 8
 5.10101010010100 14(2)183 135 7 18
 6.01010100101101 14(2)138 90 7 8
 7.10101001010100 7(2)186 136 5 18
 8.01010010101101 7(2)144 94 5 8
 9.10100110110100 14(2)9 5 3 3
 10.10100110100100 14(2)10 6 3 5
 11.10100100110100 14(2)10 6 3 5
 12.10100100100100 14(2)20 16 3 11
 13.01001001001101 14(2)11 7 3 5
 14.01001001101101 14(2)10 6 3 3
 15.01001101001101 14(2)6 2 3 2
 16.01001101101101 14(2)11 7 3 5
 total NATs = 185
L= 15
 1.000000000000000 1(1)2 1 1 2
 2.010101010101001 15(2)1244 906 13 29
 3.101010100110100 15(2)57 33 7 8
 4.101010100100100 15(2)109 85 7 18
 5.010101001001101 15(2)69 45 7 8
 6.010101001101101 15(2)69 45 7 8
 7.101010010110100 15(2)55 35 5 8
 8.101010011010100 15(2)55 35 5 8
 9.101010010100100 15(2)96 76 5 18
 10.101010010010100 15(2)96 76 5 18
 11.010100101001101 15(2)57 37 5 8
 12.010100100101101 15(2)63 43 5 8
 13.010100101101101 15(2)63 43 5 8
 14.010100110101101 15(2)57 37 5 8
 15.101001011010100 15(2)50 34 3 8
 16.101001010010100 5(2)84 68 3 18
 17.100110110110100 15(2)5 3 1 3
 18.100110100110100 15(2)3 1 1 2
 19.100110100100100 15(2)6 4 1 5
 20.100100100100100 3(2)12 10 1 11
 total NATs = 264
```

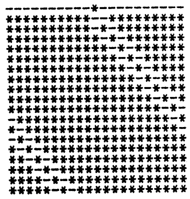
λ ratio = ~~1~~ 0·75 $Z = .75$

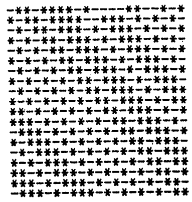
1111110000001111-1111110000001111-rule 4228905999
=3-rule 227 -11100011
Length=1 -13

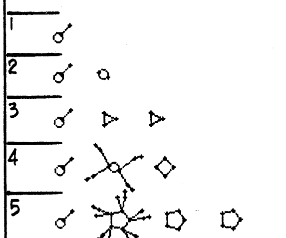


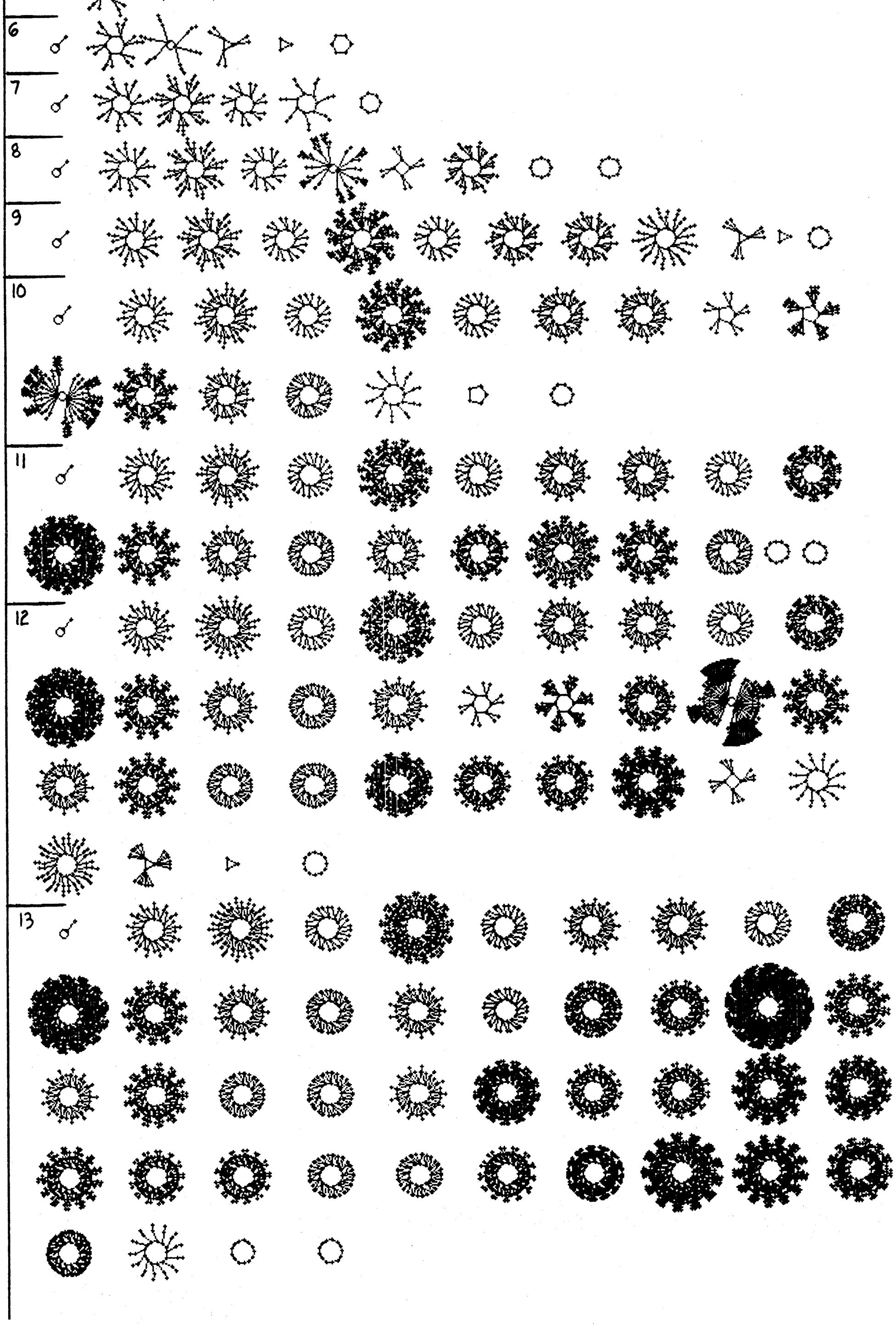


11100011	227
00111000	56
01100010	98
10111001	185







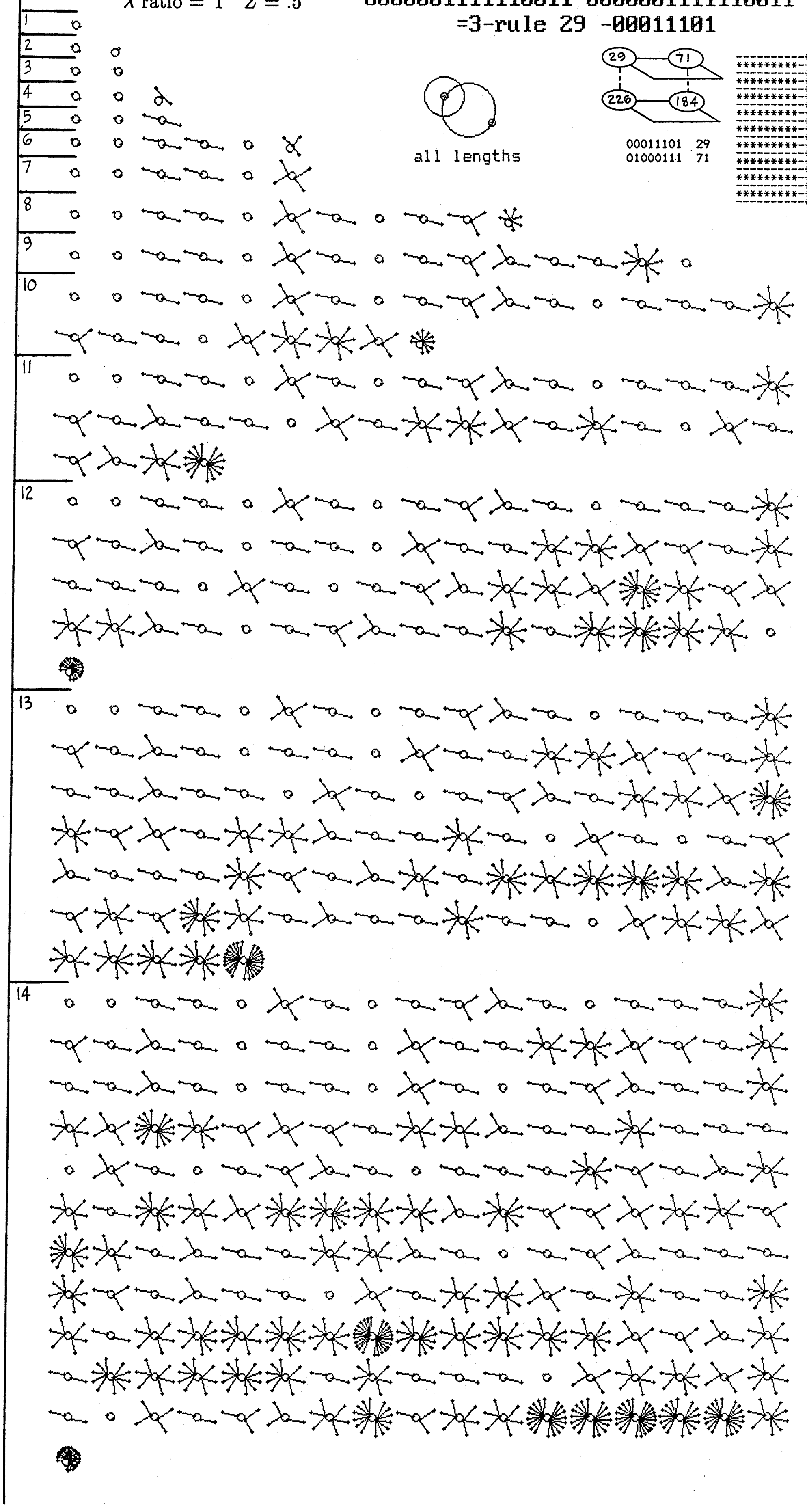



```
3-rule 227 =11100011

ty. at no(p)s g ml mp

L= 1
 1.1 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.11 1(1)2 1 1 2
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.111 1(1)2 1 1 2
 2.001 1(3)3 0 0 1
 3.011 1(3)3 0 0 1
 total NATs = 3
L= 4
 1.1111 1(1)2 1 1 2
 2.1010 1(2)10 4 2 3
 3.0111 1(4)4 0 0 1
 total NATs = 3
L= 5
 1.11111 1(1)2 1 1 2
 2.10110 1(5)20 10 2 3
 3.00101 1(5)5 0 0 1
 4.01111 1(5)5 0 0 1
 total NATs = 4
L= 6
 1.111111 1(1)2 1 1 2
 2.101110 1(6)24 12 2 3
 3.010101 1(2)20 6 3 4
 4.011011 1(3)9 6 1 3
 5.001001 1(3)3 0 0 1
 6.011111 1(6)6 0 0 1
 total NATs = 6
L= 7
 1.1111111 1(1)2 1 1 2
 2.1011110 1(7)28 14 2 3
 3.0101101 1(7)49 21 3 4
 4.0111011 1(7)21 14 1 3
 5.1010100 1(7)21 7 2 2
 6.0111111 1(7)7 0 0 1
 total NATs = 6
L= 8
 1.11111111 1(1)2 1 1 2
 2.10111110 1(8)32 16 2 3
 3.01011101 1(8)56 24 3 4
 4.01111011 1(8)24 16 1 3
 5.10101010 1(2)74 36 4 7
 6.01110111 1(4)12 8 1 3
 7.01101011 1(8)40 24 2 4
 8.00100101 1(8)8 0 0 1
 9.01111111 1(8)8 0 0 1
 total NATs = 9
L= 9
 1.111111111 1(1)2 1 1 2
 2.101111110 1(9)36 18 2 3
 3.010111101 1(9)63 27 3 4
 4.011111011 1(9)27 18 1 3
 5.101011010 1(9)189 108 4 7
 6.011110111 1(9)27 18 1 3
 7.011101011 1(9)45 27 2 4
 8.011101101 1(9)45 27 2 4
 9.101010100 1(9)54 18 3 3
 10.011011011 1(3)12 9 1 4
 11.001001001 1(3)3 0 0 1
 12.011111111 1(9)9 0 0 1
 total NATs = 12
L= 10
 1.1111111111 1(1)2 1 1 2
 2.1011111110 1(10)40 20 2 3
 3.0101111101 1(10)70 30 3 4
 4.0111111011 1(10)30 20 1 3
 5.1010111010 1(10)210 120 4 7
 6.0111110111 1(10)30 20 1 3
 7.0111101011 1(10)50 30 2 4
 8.0111101101 1(10)50 30 2 4
 9.0111101111 1(5)15 10 1 3
 10.1011010110 1(5)60 40 2 7
 11.0101010101 1(2)202 100 5 11
 12.1011010101 1(10)130 90 3 7
 13.0111010111 1(10)50 30 2 4
 14.0111011011 1(10)40 30 1 4
 15.1010100100 1(10)30 10 2 2
 16.0010100101 1(5)5 0 0 1
 17.0111111111 1(10)10 0 0 1
 total NATs = 17
L= 11
 1.11111111111 1(1)2 1 1 2
 2.10111111110 1(11)44 22 2 3
 3.01011111101 1(11)77 33 3 4
 4.01111111011 1(11)33 22 1 3
 5.10101111010 1(11)231 132 4 7
 6.01111110111 1(11)33 22 1 3
 7.01111101011 1(11)55 33 2 4
 8.01111101101 1(11)55 33 2 4
 9.01111101111 1(11)33 22 1 3
 10.10111010110 1(11)132 88 2 7
 11.01010110101 1(11)561 330 5 11
 12.10111010101 1(11)143 99 3 7
 13.01111010111 1(11)55 33 2 4
 14.01111011011 1(11)44 33 1 4
 15.01111011101 1(11)55 33 2 4
 16.10110101101 1(11)99 77 2 7
 17.10101010010 1(11)187 88 4 5
 18.10110101011 1(11)143 99 3 7
 19.01110110111 1(11)44 33 1 4
 20.00100100101 1(11)11 0 0 1
 21.01111111111 1(11)11 0 0 1
 total NATs = 21
```

λ ratio = 1   Z = .5

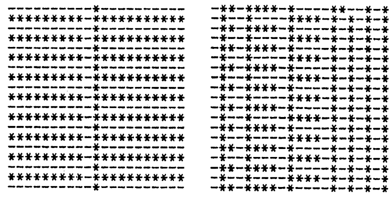
0000001111110011-0000001111110011-rule 66257907
=3-rule 29 -00011101

Length=1 -14



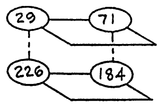
all lengths

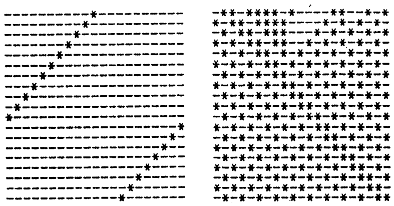
00011101 29
01000111 71

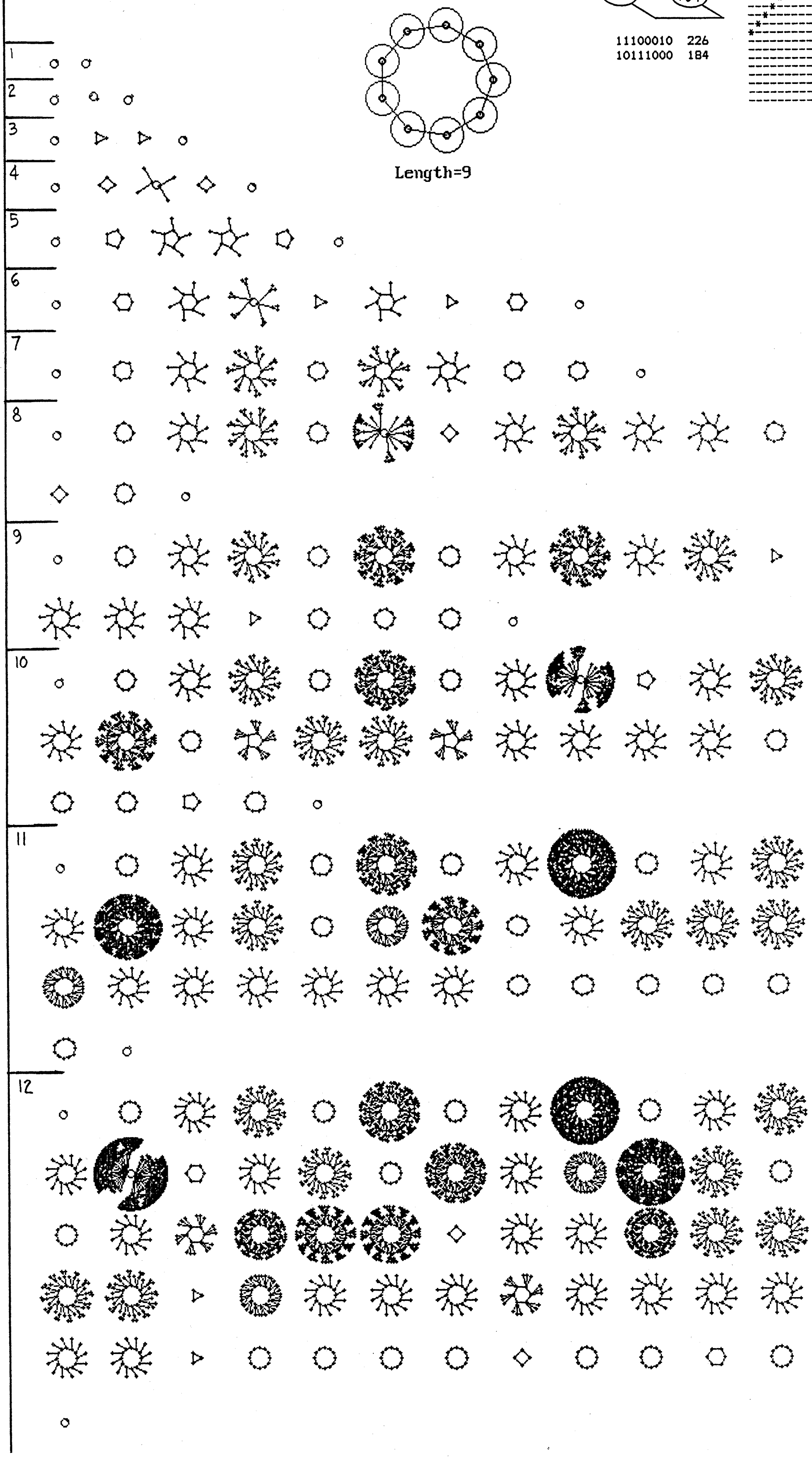


```
3-rule 29 =00011101

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(2)2 0 0 1
 2.001 3(2)2 0 0 1
 total NATs = 4
L= 4
 1.0000 1(2)2 0 0 1
 2.0001 4(2)2 0 0 1
 3.1010 2(1)3 2 1 3
 total NATs = 7
L= 5
 1.00000 1(2)2 0 0 1
 2.00001 5(2)2 0 0 1
 3.11010 5(2)4 2 1 2
 total NATs = 11
L= 6
 1.000000 1(2)2 0 0 1
 2.000001 6(2)2 0 0 1
 3.111010 6(2)4 2 1 2
 4.110100 3(2)4 2 1 2
 5.001001 3(2)2 0 0 1
 6.101010 2(1)4 3 1 4
 total NATs = 21
L= 7
 1.0000000 1(2)2 0 0 1
 2.0000001 7(2)2 0 0 1
 3.1111010 7(2)4 2 1 2
 4.1110100 7(2)4 2 1 2
 5.0001001 7(2)2 0 0 1
 6.1101010 7(2)6 4 1 3
 total NATs = 36
L= 8
 1.00000000 1(2)2 0 0 1
 2.00000001 8(2)2 0 0 1
 3.11111010 8(2)4 2 1 2
 4.11110100 8(2)4 2 1 2
 5.00001001 8(2)2 0 0 1
 6.11101010 8(2)6 4 1 3
 7.11101000 4(2)4 2 1 2
 8.00010001 4(2)2 0 0 1
 9.11011010 8(2)4 2 1 2
 10.11010100 8(2)5 3 1 3
 11.10101010 2(1)7 6 1 7
 total NATs = 67
L= 9
 1.000000000 1(2)2 0 0 1
 2.000000001 9(2)2 0 0 1
 3.111111010 9(2)4 2 1 2
 4.111110100 9(2)4 2 1 2
 5.000001001 9(2)2 0 0 1
 6.111101010 9(2)6 4 1 3
 7.111101000 9(2)4 2 1 2
 8.000010001 9(2)2 0 0 1
 9.111011010 9(2)4 2 1 2
 10.111010100 9(2)5 3 1 3
 11.111010010 9(2)5 3 1 3
 12.110111010 9(2)4 2 1 2
 13.110110100 9(2)4 2 1 2
 14.110101010 9(2)10 8 1 5
 15.001001001 3(2)2 0 0 1
 total NATs = 121
L= 10
 1.0000000000 1(2)2 0 0 1
 2.0000000001 10(2)2 0 0 1
 3.1111111010 10(2)4 2 1 2
 4.1111110100 10(2)4 2 1 2
 5.0000001001 10(2)2 0 0 1
 6.1111101010 10(2)6 4 1 3
 7.1111101000 10(2)4 2 1 2
 8.0000010001 10(2)2 0 0 1
 9.1111011010 10(2)4 2 1 2
 10.1111010100 10(2)5 3 1 3
 11.1111010010 10(2)5 3 1 3
 12.1111010000 5(2)4 2 1 2
 13.0000100001 5(2)2 0 0 1
 14.1110111010 10(2)4 2 1 2
 15.1110110100 10(2)4 2 1 2
 16.0000101001 10(2)4 2 1 2
 17.1110101010 10(2)10 8 1 5
 18.1110101000 10(2)5 3 1 3
 19.1110100100 10(2)4 2 1 2
 20.1101110100 10(2)4 2 1 2
 21.0001001001 10(2)2 0 0 1
 22.1101101010 10(2)6 4 1 3
 23.1101011010 5(2)8 6 1 4
 24.1101010100 10(2)9 7 1 5
 25.1101010010 5(2)6 4 1 3
 26.1010101010 2(1)11 10 1 11
 total NATs = 223
L= 11
 1.00000000000 1(2)2 0 0 1
 2.00000000001 11(2)2 0 0 1
 3.11111111010 11(2)4 2 1 2
 4.11111110100 11(2)4 2 1 2
 5.00000001001 11(2)2 0 0 1
 6.11111101010 11(2)6 4 1 3
 7.11111101000 11(2)4 2 1 2
 8.00000010001 11(2)2 0 0 1
 9.11111011010 11(2)4 2 1 2
 10.11111010100 11(2)5 3 1 3
 11.11111010010 11(2)5 3 1 3
 12.11111010000 11(2)4 2 1 2
 13.00000100001 11(2)2 0 0 1
 14.11110111010 11(2)4 2 1 2
 15.11110110100 11(2)4 2 1 2
 16.00000101001 11(2)4 2 1 2
 17.11110101010 11(2)10 8 1 5
 18.11110101000 11(2)5 3 1 3
 19.11110100100 11(2)4 2 1 2
 20.11110100010 11(2)5 3 1 3
 21.11101111010 11(2)4 2 1 2
 22.11101110100 11(2)4 2 1 2
 23.00001001001 11(2)2 0 0 1
 24.11101101010 11(2)6 4 1 3
 25.11101101000 11(2)4 2 1 2
 26.11101011010 11(2)8 6 1 4
 27.11101010100 11(2)9 7 1 5
 28.11101010010 11(2)6 4 1 3
 29.00001101001 11(2)4 2 1 2
 30.11101001010 11(2)9 7 1 5
 31.11101001000 11(2)4 2 1 2
 32.00010001001 11(2)2 0 0 1
 33.11011101010 11(2)6 4 1 3
 34.11011011010 11(2)4 2 1 2
 35.11011010100 11(2)5 3 1 3
 36.11011010010 11(2)5 3 1 3
 37.11010110100 11(2)8 6 1 4
 38.11010101010 11(2)16 14 1 8
total NATs = 408
```

λ ratio = 1 $Z = .5$ 1111110000001100-1111110000001100-rule 4228709388
=3-rule 226 -11100010   Length=1 -12



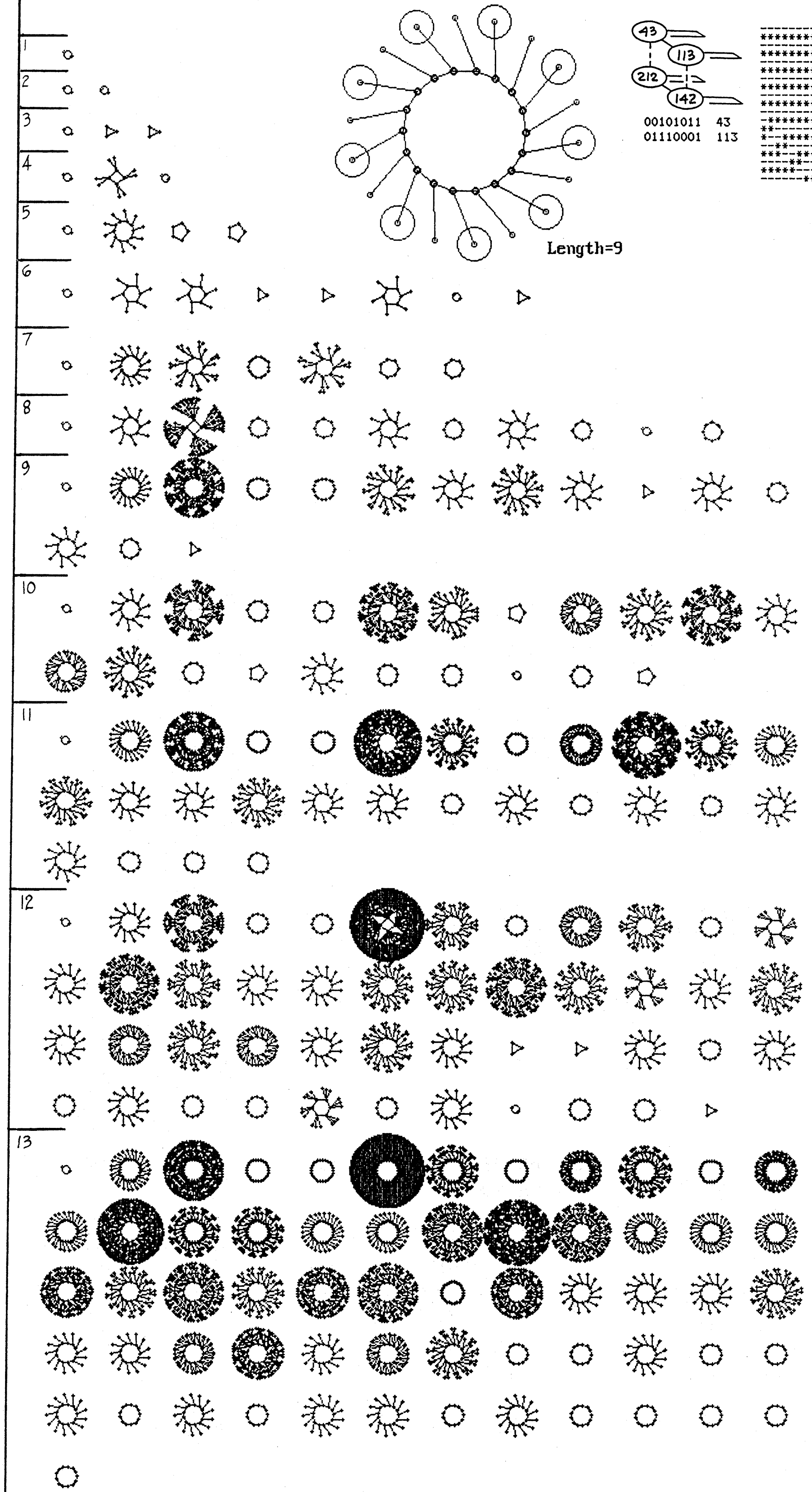


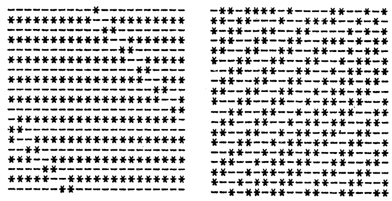



```
3-rule 226 =11100010
ty. at no(p)s g ml mp
L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 1(2)2 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(1)1 0 0 1
 2.001 1(3)3 0 0 1
 3.011 1(3)3 0 0 1
 4.111 1(1)1 0 0 1
 total NATs = 4
L= 4
 1.0000 1(1)1 0 0 1
 2.0001 1(4)4 0 0 1
 3.0101 1(2)6 4 1 3
 4.0111 1(4)4 0 0 1
 5.1111 1(1)1 0 0 1
 total NATs = 5
L= 5
 1.00000 1(1)1 0 0 1
 2.00001 1(5)5 0 0 1
 3.00101 1(5)10 5 1 2
 4.01011 1(5)10 5 1 2
 5.01111 1(5)5 0 0 1
 6.11111 1(1)1 0 0 1
 total NATs = 6
L= 6
 1.000000 1(1)1 0 0 1
 2.000001 1(6)6 0 0 1
 3.000101 1(6)12 6 1 2
 4.010101 1(2)20 12 2 4
 5.001001 1(3)3 0 0 1
 6.010111 1(6)12 6 1 2
 7.011011 1(3)3 0 0 1
 8.011111 1(6)6 0 0 1
 9.111111 1(1)1 0 0 1
 total NATs = 9
L= 7
 1.0000000 1(1)1 0 0 1
 2.0000001 1(7)7 0 0 1
 3.0000101 1(7)14 7 1 2
 4.0010101 1(7)35 21 2 3
 5.0001001 1(7)7 0 0 1
 6.0101011 1(7)35 21 2 3
 7.0101111 1(7)14 7 1 2
 8.0110111 1(7)7 0 0 1
 9.0111111 1(7)7 0 0 1
 10.1111111 1(1)1 0 0 1
 total NATs = 10
L= 8
 1.00000000 1(1)1 0 0 1
 2.00000001 1(8)8 0 0 1
 3.00000101 1(8)16 8 1 2
 4.00010101 1(8)40 24 2 3
 5.00001001 1(8)8 0 0 1
 6.01010101 1(2)70 44 3 7
 7.00010001 1(4)4 0 0 1
 8.00100101 1(8)16 8 1 2
 9.01010111 1(8)40 24 2 3
 10.01011011 1(8)16 8 1 2
 11.01011111 1(8)16 8 1 2
 12.01101111 1(8)8 0 0 1
 13.01110111 1(4)4 0 0 1
 14.01111111 1(8)8 0 0 1
 15.11111111 1(1)1 0 0 1
 total NATs = 15
L= 9
 1.000000000 1(1)1 0 0 1
 2.000000001 1(9)9 0 0 1
 3.000000101 1(9)18 9 1 2
 4.000010101 1(9)45 27 2 3
 5.000001001 1(9)9 0 0 1
 6.001010101 1(9)126 81 3 5
 7.000010001 1(9)9 0 0 1
 8.000100101 1(9)18 9 1 2
 9.010101011 1(9)126 81 3 5
 10.001000101 1(9)18 9 1 2
 11.010101111 1(9)45 27 2 3
 12.001001001 1(3)3 0 0 1
 13.010110111 1(9)18 9 1 2
 14.010111011 1(9)18 9 1 2
 15.010111111 1(9)18 9 1 2
 16.011011011 1(3)3 0 0 1
 17.011011111 1(9)9 0 0 1
 18.011101111 1(9)9 0 0 1
 19.011111111 1(9)9 0 0 1
 20.111111111 1(1)1 0 0 1
 total NATs = 20
L= 10
 1.0000000000 1(1)1 0 0 1
 2.0000000001 1(10)10 0 0 1
 3.0000000101 1(10)20 10 1 2
 4.0000010101 1(10)50 30 2 3
 5.0000001001 1(10)10 0 0 1
 6.0001010101 1(10)140 90 3 5
 7.0000010001 1(10)10 0 0 1
 8.0000100101 1(10)20 10 1 2
 9.0101010101 1(2)252 170 4 11
 10.0000100001 1(5)5 0 0 1
 11.0001000101 1(10)20 10 1 2
 12.0010010101 1(10)50 30 2 3
 13.0000101001 1(10)20 10 1 2
 14.0101010111 1(10)140 90 3 5
 15.0001001001 1(10)10 0 0 1
 16.0010100101 1(5)20 15 1 4
 17.0101011011 1(10)50 30 2 3
 18.0101011111 1(10)50 30 2 3
 19.0101101011 1(5)20 15 1 4
 20.0101101111 1(10)20 10 1 2
 21.0101110111 1(10)20 10 1 2
 22.0101111011 1(10)20 10 1 2
 23.0101111111 1(10)20 10 1 2
 24.0110110111 1(10)10 0 0 1
 25.0110111111 1(10)10 0 0 1
 26.0111011111 1(10)10 0 0 1
 27.0111101111 1(5)5 0 0 1
 28.0111111111 1(10)10 0 0 1
 29.1111111111 1(1)1 0 0 1
 total NATs = 29
```

λ ratio = 1 $Z = .5$

0000110011001111-0000110011001111-rule 214895823
=3-rule 43 -00101011
Length=1 -13





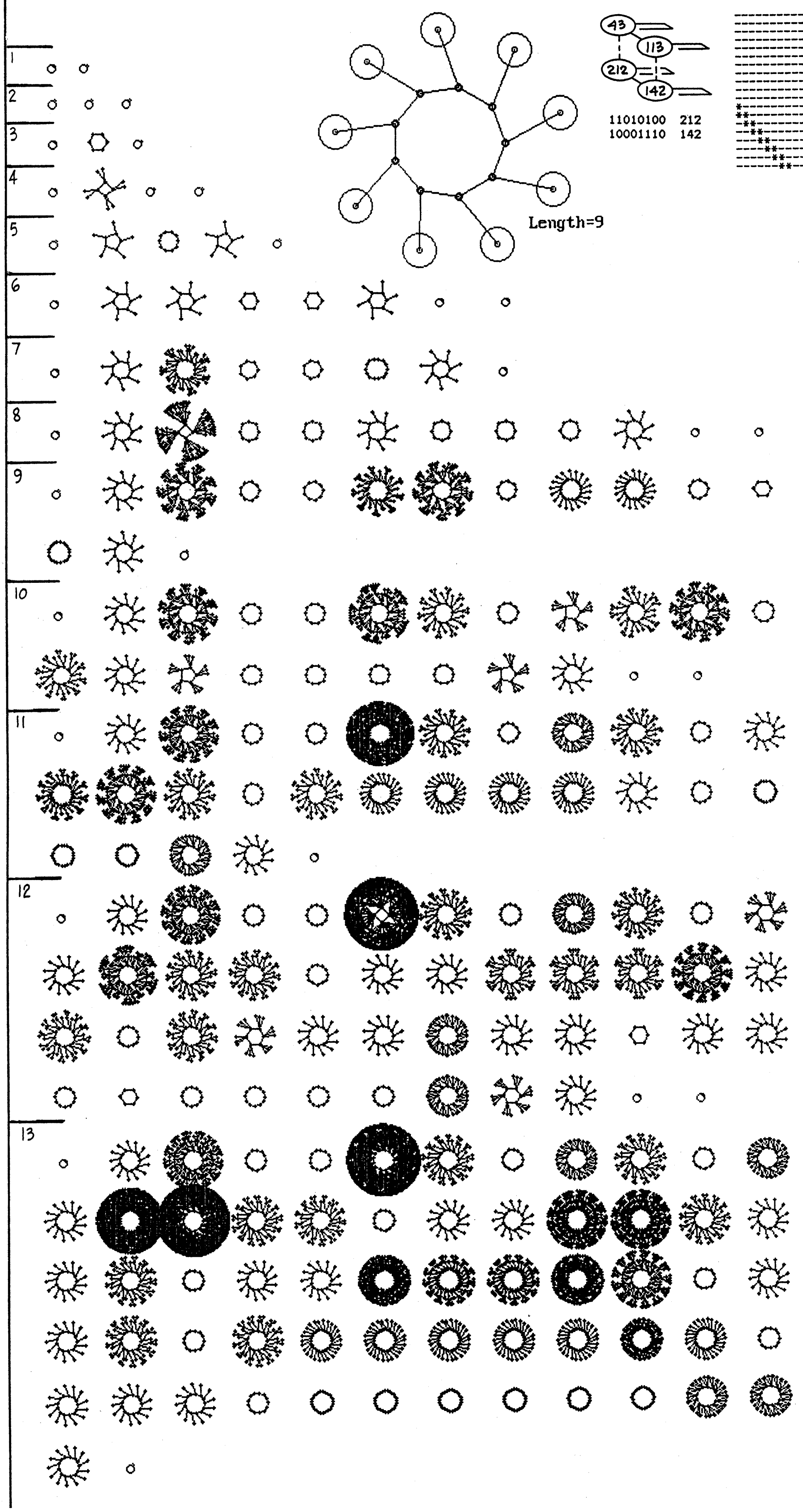
```
3-rule 43 =00101011

ty. at no(p)s g ml mp

L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 1(2)2 0 0 1
 total NATs = 2
L= 3
 1.000 1(2)2 0 0 1
 2.001 1(3)3 0 0 1
 3.011 1(3)3 0 0 1
 total NATs = 3
L= 4
 1.0000 1(2)2 0 0 1
 2.0110 1(4)12 8 1 3
 3.0101 1(2)2 0 0 1
 total NATs = 3
L= 5
 1.00000 1(2)2 0 0 1
 2.01110 1(10)20 10 1 2
 3.00101 1(5)5 0 0 1
 4.01011 1(5)5 0 0 1
 total NATs = 4
L= 6
 1.000000 1(2)2 0 0 1
 2.011110 2(6)12 6 1 2
 3.011010 1(6)12 6 1 2
 4.000111 2(3)3 0 0 1
 5.001001 1(3)3 0 0 1
 6.001011 1(6)12 6 1 2
 7.010101 1(2)2 0 0 1
 8.011011 1(3)3 0 0 1
 total NATs = 10
L= 7
 1.0000000 1(2)2 0 0 1
 2.0111110 1(14)28 14 1 2
 3.1100100 1(7)35 21 2 3
 4.0000111 1(14)14 0 0 1
 5.0110110 1(7)35 21 2 3
 6.0010101 1(7)7 0 0 1
 7.0101011 1(7)7 0 0 1
 total NATs = 7
L= 8
 1.00000000 1(2)2 0 0 1
 2.01111110 2(8)16 8 1 2
 3.10011001 1(4)140 88 3 7
 4.00000111 2(8)8 0 0 1
 5.00001111 1(8)8 0 0 1
 6.01101010 1(8)16 8 1 2
 7.00100101 1(8)8 0 0 1
 8.00101011 1(8)16 8 1 2
 9.00101101 1(8)8 0 0 1
 10.01010101 1(2)2 0 0 1
 11.01011011 1(8)8 0 0 1
 total NATs = 13
L= 9
 1.000000000 1(2)2 0 0 1
 2.011111110 1(18)36 18 1 2
 3.100111001 1(18)252 162 3 5
 4.000000111 1(18)18 0 0 1
 5.000001111 1(18)18 0 0 1
 6.110010100 1(9)45 27 2 3
 7.011010010 1(9)18 9 1 2
 8.011010110 1(9)45 27 2 3
 9.011011010 1(9)18 9 1 2
 10.001001001 1(3)3 0 0 1
 11.001001011 1(9)18 9 1 2
 12.001010101 1(9)9 0 0 1
 13.001011011 1(9)18 9 1 2
 14.010101011 1(9)9 0 0 1
 15.011011011 1(3)3 0 0 1
 total NATs = 15
L= 10
 1.0000000000 1(2)2 0 0 1
 2.0111111110 2(10)20 10 1 2
 3.1001111001 2(10)140 90 3 5
 4.0000000111 2(10)10 0 0 1
 5.0000001111 2(10)10 0 0 1
 6.1001101001 1(10)140 90 3 5
 7.1100011001 2(10)50 30 2 3
 8.0000011111 2(5)5 0 0 1
 9.0111001110 1(10)40 30 1 4
 10.1100100100 1(10)50 30 2 3
 11.1100101100 1(10)140 90 3 5
 12.0110101010 1(10)20 10 1 2
 13.0110110010 1(10)40 30 1 4
 14.0110110110 1(10)50 30 2 3
 15.0010010101 1(10)10 0 0 1
 16.0010100101 1(5)5 0 0 1
 17.0010101011 1(10)20 10 1 2
 18.0010101101 1(10)10 0 0 1
 19.0010110101 1(10)10 0 0 1
 20.0101010101 1(2)2 0 0 1
 21.0101011011 1(10)10 0 0 1
 22.0101101011 1(5)5 0 0 1
 total NATs = 28
L= 11
 1.00000000000 1(2)2 0 0 1
 2.01111111110 1(22)44 22 1 2
 3.10011111001 1(22)308 198 3 5
 4.00000000111 1(22)22 0 0 1
 5.00000001111 1(22)22 0 0 1
 6.00110010011 1(11)462 319 4 8
 7.11000011001 1(22)110 66 2 3
 8.00000011111 1(22)22 0 0 1
 9.01111001110 1(22)88 66 1 4
 10.10011011001 1(11)462 319 4 8
 11.11000110011 1(22)110 66 2 3
 12.01110011100 1(22)44 22 1 2
 13.11001010100 1(11)55 33 2 3
 14.01101001010 1(11)22 11 1 2
 15.01101010010 1(11)22 11 1 2
 16.01101010110 1(11)55 33 2 3
 17.01101011010 1(11)22 11 1 2
 18.01101101010 1(11)22 11 1 2
 19.00100100101 1(11)11 0 0 1
 20.00100101011 1(11)22 11 1 2
 21.00100101101 1(11)11 0 0 1
 22.00101001011 1(11)22 11 1 2
 23.00101010101 1(11)11 0 0 1
 24.00101011011 1(11)22 11 1 2
 25.00101101011 1(11)22 11 1 2
 26.00101101101 1(11)11 0 0 1
 27.01010101011 1(11)11 0 0 1
 28.01011011011 1(11)11 0 0 1
 total NATs = 28
```

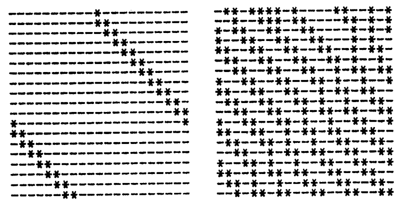
λ ratio = 1 $Z = .5$

11110011100110000-11110011100110000-rule 4080071472

=3-rule 212 -11010100

Length=1 -13





3-rule 212 =11010100

ty. at no(p)s g ml mp

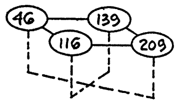
L= 1
1.0 1(1)1 0 0 1
2.1 1(1)1 0 0 1
total NATs = 2
L= 2
1.00 1(1)1 0 0 1
2.01 2(1)1 0 0 1
3.11 1(1)1 0 0 1
total NATs = 4
L= 3
1.000 1(1)1 0 0 1
2.001 1(6)6 0 0 1
3.111 1(1)1 0 0 1
total NATs = 3
L= 4
1.0000 1(1)1 0 0 1
2.1001 1(4)12 8 1 3
3.0101 2(1)1 0 0 1
4.1111 1(1)1 0 0 1
total NATs = 5
L= 5
1.00000 1(1)1 0 0 1
2.10001 1(5)10 5 1 2
3.00101 1(10)10 0 0 1
4.00111 1(5)10 5 1 2
5.11111 1(1)1 0 0 1
total NATs = 5
L= 6
1.000000 1(1)1 0 0 1
2.100001 1(6)12 6 1 2
3.100101 2(6)12 6 1 2
4.000111 1(6)6 0 0 1
5.001001 1(6)6 0 0 1
6.001111 1(6)12 6 1 2
7.010101 2(1)1 0 0 1
8.111111 1(1)1 0 0 1
total NATs = 10
L= 7
1.0000000 1(1)1 0 0 1
2.1000001 1(7)14 7 1 2
3.1100100 1(14)70 42 2 3
4.0000111 1(7)7 0 0 1
5.0001111 1(7)7 0 0 1
6.0010101 1(14)14 0 0 1
7.0011111 1(7)14 7 1 2
8.1111111 1(1)1 0 0 1
total NATs = 8
L= 8
1.00000000 1(1)1 0 0 1
2.10000001 1(8)16 8 1 2
3.01100110 1(4)140 88 3 7
4.00000111 1(8)8 0 0 1
5.00001111 1(8)8 0 0 1
6.10010101 2(8)16 8 1 2
7.00011111 1(8)8 0 0 1
8.00100101 2(8)8 0 0 1
9.00101101 1(8)8 0 0 1
10.00111111 1(8)16 8 1 2
11.01010101 2(1)1 0 0 1
12.11111111 1(1)1 0 0 1
total NATs = 15
L= 9
1.000000000 1(1)1 0 0 1
2.100000001 1(9)18 9 1 2
3.011000110 1(9)126 81 3 5
4.000000111 1(9)9 0 0 1
5.000001111 1(9)9 0 0 1
6.110010100 1(18)90 54 2 3
7.110011001 1(9)126 81 3 5
8.000011111 1(9)9 0 0 1
9.100101101 1(18)36 18 1 2
10.100100101 1(18)36 18 1 2
11.000111111 1(9)9 0 0 1
12.001001001 1(6)6 0 0 1
13.001010101 1(18)18 0 0 1
14.001111111 1(9)18 9 1 2
15.111111111 1(1)1 0 0 1
total NATs = 15
L= 10
1.0000000000 1(1)1 0 0 1
2.1000000001 1(10)20 10 1 2
3.0110000110 1(10)140 90 3 5
4.0000000111 1(10)10 0 0 1
5.0000001111 1(10)10 0 0 1
6.0110010110 2(10)140 90 3 5
7.1100011001 1(10)50 30 2 3
8.0000011111 1(10)10 0 0 1
9.1000110001 1(5)20 15 1 4
10.1100100100 2(10)50 30 2 3
11.1100110011 1(10)140 90 3 5
12.0000111111 1(10)10 0 0 1
13.1001100011 1(10)50 30 2 3
14.1001010101 2(10)20 10 1 2
15.1001001101 2(5)20 15 1 4
16.0001111111 1(10)10 0 0 1
17.0010010101 2(10)10 0 0 1
18.0010100101 1(10)10 0 0 1
19.0010101101 2(10)10 0 0 1
20.0011100111 1(5)20 15 1 4
21.0011111111 1(10)20 10 1 2
22.0101010101 2(1)1 0 0 1
23.1111111111 1(1)1 0 0 1
total NATs = 30
L= 11
1.00000000000 1(1)1 0 0 1
2.10000000001 1(11)22 11 1 2
3.01100000110 1(11)154 99 3 5
4.00000000111 1(11)11 0 0 1
5.00000001111 1(11)11 0 0 1
6.00110010011 1(22)924 638 4 8
7.11000011001 1(11)55 33 2 3
8.00000011111 1(11)11 0 0 1
9.10000110001 1(11)44 33 1 4
10.11000110011 1(11)55 33 2 3
11.00000111111 1(11)11 0 0 1
12.10001100011 1(11)22 11 1 2
13.11001010100 1(22)110 66 2 3
14.11001100111 1(11)154 99 3 5
15.10000111001 1(11)55 33 2 3

25.00100101101 1(22)22 0 0 1
26.00101010101 1(22)22 0 0 1
27.00111001111 1(11)44 33 1 4
28.00111111111 1(11)22 11 1 2
29.11111111111 1(1)1 0 0 1
total NATs = 29

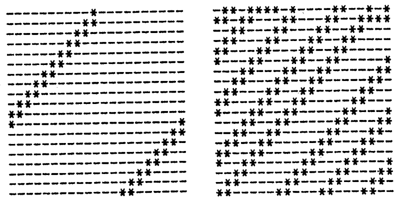
178

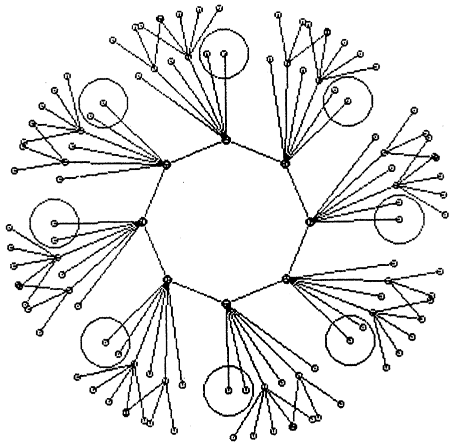
intentionally blank

λ ratio = 1 $Z = .5$ 00001100011111100-00001100011111100-rule 217844988
=3-rule 46 -00101110   Length=1 -15

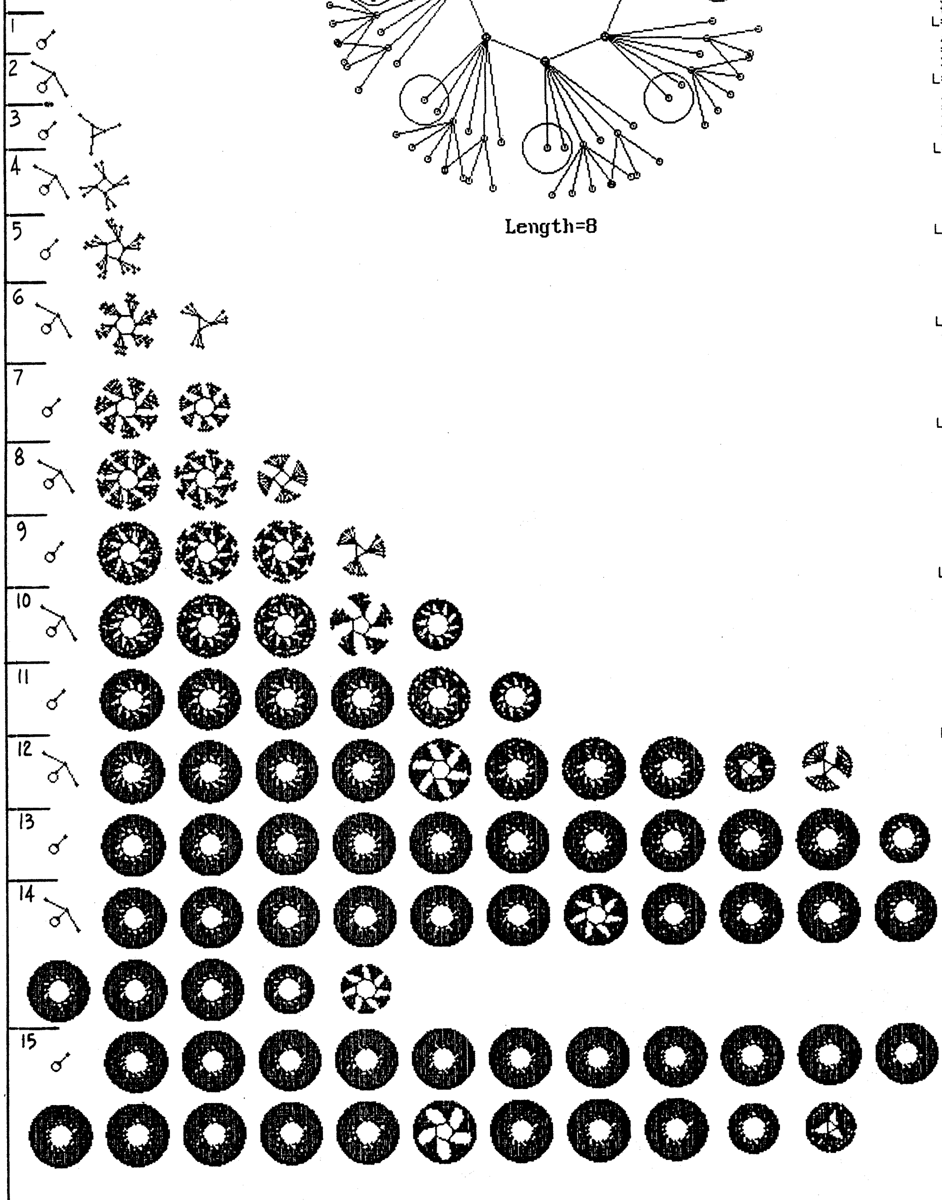


00101110	46
10001011	139
11010001	209
01110100	116

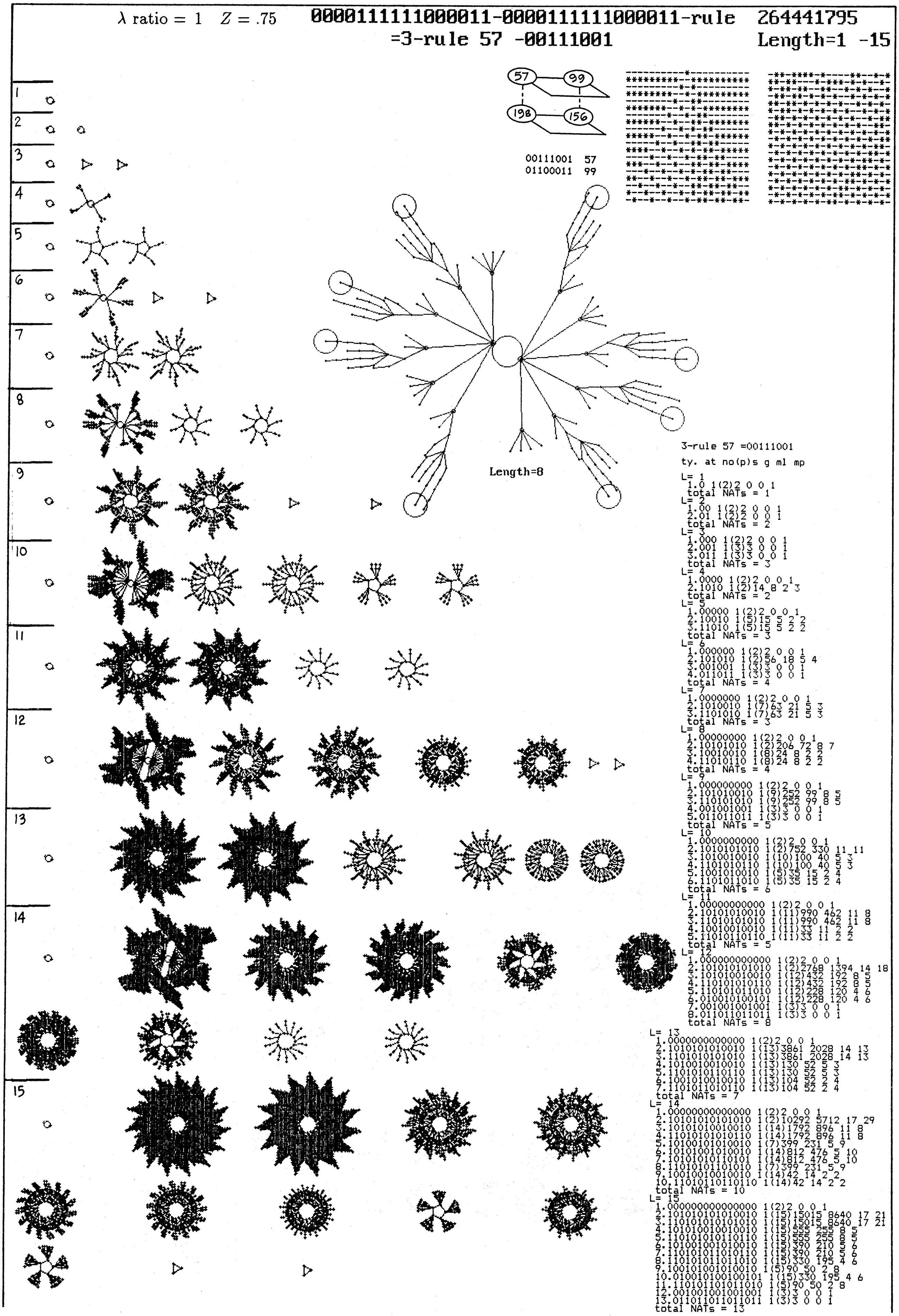




Length=8

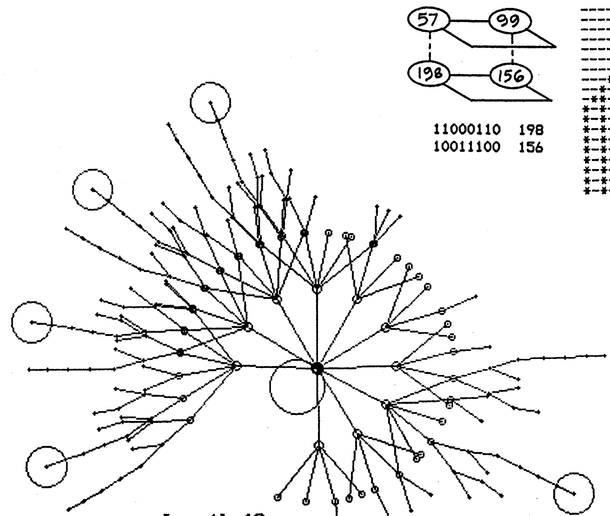


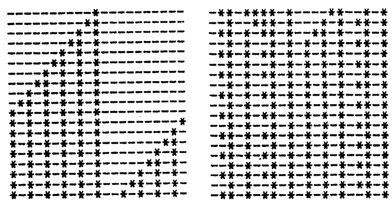

```
3-rule 46 =00101110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.000 1(1)2 1 1 2
 2.011 1(3)6 3 1 2
 total NATs = 2
L= 4
 1.0000 1(1)4 2 2 2
 2.0011 1(4)12 8 1 3
 total NATs = 2
L= 5
 1.00000 1(1)2 1 1 2
 2.00011 1(5)30 20 2 4
 total NATs = 2
L= 6
 1.000000 1(1)4 2 2 2
 2.000011 1(6)48 36 2 5
 3.011011 1(3)12 9 1 4
 total NATs = 3
L= 7
 1.0000000 1(1)2 1 1 2
 2.0000011 1(7)84 63 2 6
 3.0011011 1(7)42 35 1 6
 total NATs = 3
L= 8
 1.00000000 1(1)4 2 2 2
 2.00000011 1(8)120 96 2 7
 3.00011011 1(8)96 80 2 8
 4.00110011 1(4)36 32 1 9
 total NATs = 4
L= 9
 1.000000000 1(1)2 1 1 2
 2.000000011 1(9)180 144 2 8
 3.000011011 1(9)144 126 2 10
 4.000110011 1(9)162 144 2 12
 5.011011011 1(3)24 21 1 8
 total NATs = 5
L= 10
 1.0000000000 1(1)4 2 2 2
 2.0000000011 1(10)240 200 2 9
 3.0000011011 1(10)240 210 2 12
 4.0000110011 1(10)240 220 2 15
 5.0001100011 1(5)180 160 2 16
 6.0011011011 1(10)120 110 1 12
 total NATs = 6
L= 11
 1.00000000000 1(1)2 1 1 2
 2.00000000011 1(11)330 275 2 10
 3.00000011011 1(11)330 297 2 14
 4.00000110011 1(11)396 363 2 18
 5.00001100011 1(11)528 484 2 20
 6.00011011011 1(11)264 242 2 16
 7.00110011011 1(11)198 187 1 18
 total NATs = 7
L= 12
 1.000000000000 1(1)4 2 2 2
 2.000000000011 1(12)420 360 2 11
 3.000000011011 1(12)480 432 2 16
 4.000000110011 1(12)540 504 2 21
 5.000001100011 1(12)864 792 2 24
 6.000011000011 1(6)384 360 2 25
 7.000011011011 1(12)384 360 2 20
 8.000110011011 1(12)432 408 2 24
 9.000110110011 1(12)432 408 2 24
 10.001100110011 1(4)108 104 1 27
 11.011011011011 1(3)48 45 1 16
 total NATs = 11
L= 13
 1.0000000000000 1(1)2 1 1 2
 2.0000000000011 1(13)546 468 2 12
 3.0000000011011 1(13)624 572 2 18
 4.0000000110011 1(13)780 728 2 24
 5.0000001100011 1(13)1170 1092 2 28
 6.0000011000011 1(13)1248 1170 2 30
 7.0000011011011 1(13)624 585 2 24
 8.0000110011011 1(13)624 598 2 30
 9.0000110110011 1(13)624 598 2 30
 10.0001100011011 1(13)936 884 2 32
 11.0001100110011 1(13)702 676 2 36
 12.0011011011011 1(13)312 299 1 24
 total NATs = 12
L= 14
 1.00000000000000 1(1)4 2 2 2
 2.00000000000011 1(14)672 588 2 13
 3.00000000011011 1(14)840 770 2 20
 4.00000000110011 1(14)1008 952 2 27
 5.00000001100011 1(14)1680 1568 2 32
 6.00000011000011 1(14)1680 1596 2 35
 7.00000011011011 1(14)840 798 2 28
 8.00000110000011 1(7)1008 945 2 36
 9.00000110011011 1(14)1008 966 2 36
 10.00000110110011 1(14)1008 966 2 36
 11.00001100011011 1(14)1344 1288 2 40
 12.00001100110011 1(14)1008 980 2 45
 13.00001101100011 1(14)1344 1288 2 40
 14.00011000110011 1(14)1512 1456 2 48
 15.00011011011011 1(14)672 644 2 32
 16.00110011011011 1(14)504 490 1 36
 17.00110110011011 1(7)252 245 1 36
 total NATs = 17
L= 15
 1.000000000000000 1(1)2 1 1 2
 2.000000000000011 1(15)840 735 2 14
 3.000000000011011 1(15)1050 975 2 22
 4.000000000110011 1(15)1350 1275 2 30
 5.000000001100011 1(15)2160 2040 2 36
 6.000000011000011 1(15)2400 2280 2 40
 7.000000011011011 1(15)1200 1140 2 32
 8.000000110000011 1(15)2700 2565 2 42
 9.000000110011011 1(15)1350 1305 2 42
 10.000000110110011 1(15)1350 1305 2 42
 11.000001100011011 1(15)2160 2070 2 48
 12.000001100110011 1(15)1620 1575 2 54
 13.000001101100011 1(15)2160 2070 2 48
 14.000011000011011 1(15)1920 1860 2 50
 15.000011000110011 1(15)2160 2100 2 60
 16.000011001100011 1(15)2160 2100 2 60
 17.000011011011011 1(15)960 930 2 40
 18.000110001100011 1(5)1080 1040 2 64
 19.000110011011011 1(15)1080 1050 2 48
 20.000110110011011 1(15)1080 1050 2 48
 21.000110110110011 1(15)1080 1050 2 48
 22.001100110011011 1(15)810 795 1 54
 23.011011011011011 1(3)96 93 1 32
 total NATs = 23
```


λ ratio = 1 Z = .75
0000111111000011-0000111111000011-rule 264441795
=3-rule 57 -00111001
Length=1 -15
57
99
198
156
00111001 57
01100011 99
Length=8
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
3-rule 57 =00111001
ty. at no(p)s g ml mp
L= 1
1.0 1(2)2 0 0 1
total NATs = 1
L= 2
1.00 1(2)2 0 0 1
2.01 1(2)2 0 0 1
total NATs = 2
L= 3
1.000 1(2)2 0 0 1
2.001 1(3)3 0 0 1
3.011 1(3)3 0 0 1
total NATs = 3
L= 4
1.0000 1(2)2 0 0 1
2.1010 1(2)14 8 2 3
total NATs = 2
L= 5
1.00000 1(2)2 0 0 1
2.10010 1(5)15 5 2 2
3.11010 1(5)15 5 2 2
total NATs = 3
L= 6
1.000000 1(2)2 0 0 1
2.101010 1(2)56 18 5 4
3.001001 1(3)3 0 0 1
4.011011 1(3)3 0 0 1
total NATs = 4
L= 7
1.0000000 1(2)2 0 0 1
2.1010010 1(7)63 21 5 3
3.1101010 1(7)63 21 5 3
total NATs = 3
L= 8
1.00000000 1(2)2 0 0 1
2.10101010 1(2)206 72 8 7
3.10010010 1(8)24 8 2 2
4.11010110 1(8)24 8 2 2
total NATs = 4
L= 9
1.000000000 1(2)2 0 0 1
2.101010010 1(9)252 99 8 5
3.110101010 1(9)252 99 8 5
4.001001001 1(3)3 0 0 1
5.011011011 1(3)3 0 0 1
total NATs = 5
L= 10
1.0000000000 1(2)2 0 0 1
2.1010101010 1(2)752 330 11 11
3.1010010010 1(10)100 40 5 3
4.1101010110 1(10)100 40 5 3
5.1001010010 1(5)35 15 2 4
6.1101011010 1(5)35 15 2 4
total NATs = 6
L= 11
1.00000000000 1(2)2 0 0 1
2.10101010010 1(11)990 462 11 8
3.11010101010 1(11)990 462 11 8
4.10010010010 1(11)33 11 2 2
5.11010110110 1(11)33 11 2 2
total NATs = 5
L= 12
1.000000000000 1(2)2 0 0 1
2.101010101010 1(2)2768 1394 14 18
3.101010010010 1(12)432 192 8 5
4.110101010110 1(12)432 192 8 5
5.110101011010 1(12)228 120 4 6
6.010010100101 1(12)228 120 4 6
7.001001001001 1(3)3 0 0 1
8.011011011011 1(3)3 0 0 1
total NATs = 8
L= 13
1.0000000000000 1(2)2 0 0 1
2.1010101010010 1(13)3861 2028 14 13
3.1101010101010 1(13)3861 2028 14 13
4.1010010010010 1(13)130 52 5 3
5.1101010110110 1(13)130 52 5 3
6.1001010010010 1(13)104 52 2 4
7.1101011010110 1(13)104 52 2 4
total NATs = 7
L= 14
1.00000000000000 1(2)2 0 0 1
2.10101010101010 1(2)10292 5712 17 29
3.10101010010010 1(14)1792 896 11 8
4.11010101010110 1(14)1792 896 11 8
5.10100101010010 1(7)399 231 5 9
6.10101001010010 1(14)812 476 5 10
7.10101010110101 1(14)812 476 5 10
8.11010101101010 1(7)399 231 5 9
9.10010010010010 1(14)42 14 2 2
10.11010110110110 1(14)42 14 2 2
total NATs = 10
L= 15
1.000000000000000 1(2)2 0 0 1
2.101010101010010 1(15)15015 8640 17 21
3.110101010101010 1(15)15015 8640 17 21
4.101010010010010 1(15)555 255 8 5
5.110101010110110 1(15)555 255 8 5
6.101001001010010 1(15)390 210 5 6
7.110101011010110 1(15)390 210 5 6
8.110101011011010 1(15)330 195 4 6
9.100101001010010 1(5)90 50 2 8
10.010010100100101 1(15)330 195 4 6
11.110101101011010 1(5)90 50 2 8
12.001001001001001 1(3)3 0 0 1
13.011011011011011 1(3)3 0 0 1
total NATs = 13

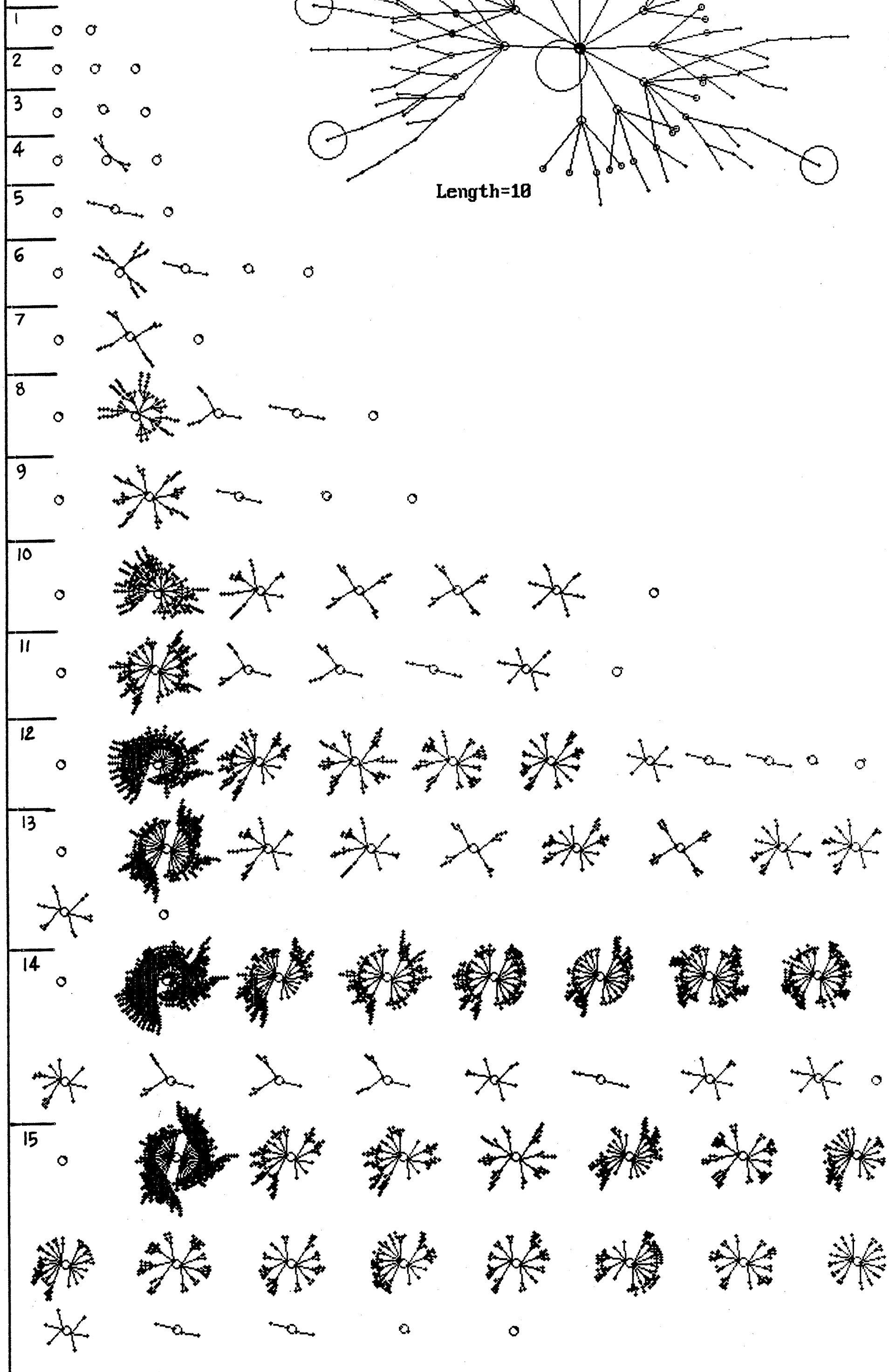
λ ratio = 1   Z = .75

1111000000111100-1111000000111100-rule 4030525500
=3-rule 198 -11000110
Length=1 -15





Length=10



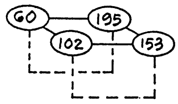

```
3-rule 198 =11000110

ty. at no(p)s g ml mp

L= 1
 1.0 1(1)1 0 0 1
 2.1 1(1)1 0 0 1
 total NATs = 2
L= 2
 1.00 1(1)1 0 0 1
 2.01 2(1)1 0 0 1
 3.11 1(1)1 0 0 1
 total NATs = 4
L= 3
 1.000 1(1)1 0 0 1
 2.001 3(2)2 0 0 1
 3.111 1(1)1 0 0 1
 total NATs = 5
L= 4
 1.0000 1(1)1 0 0 1
 2.0101 2(1)7 4 2 3
 3.1111 1(1)1 0 0 1
 total NATs = 4
L= 5
 1.00000 1(1)1 0 0 1
 2.00101 5(2)6 2 2 2
 3.11111 1(1)1 0 0 1
 total NATs = 7
L= 6
 1.000000 1(1)1 0 0 1
 2.010101 2(1)22 6 4 4
 3.001011 3(2)4 2 1 2
 4.001001 3(2)2 0 0 1
 5.111111 1(1)1 0 0 1
 total NATs = 10
L= 7
 1.0000000 1(1)1 0 0 1
 2.0010101 7(2)18 6 4 3
 3.1111111 1(1)1 0 0 1
 total NATs = 9
L= 8
 1.00000000 1(1)1 0 0 1
 2.01010101 2(1)67 24 6 7
 3.00101011 8(2)9 3 3 3
 4.00101001 8(2)6 2 2 2
 5.11111111 1(1)1 0 0 1
 total NATs = 20
L= 9
 1.000000000 1(1)1 0 0 1
 2.001010101 9(2)52 20 6 5
 3.001011011 9(2)4 2 1 2
 4.001001001 3(2)2 0 0 1
 5.111111111 1(1)1 0 0 1
 total NATs = 23
L= 10
 1.0000000000 1(1)1 0 0 1
 2.0101010101 2(1)196 90 8 11
 3.0010101011 10(2)27 11 5 5
 4.0010101001 10(2)20 8 4 3
 5.0010101101 5(2)18 8 3 3
 6.0010100101 5(2)14 6 2 4
 7.1111111111 1(1)1 0 0 1
 total NATs = 34
L= 11
 1.00000000000 1(1)1 0 0 1
 2.00101010101 11(2)150 70 8 8
 3.00101011011 11(2)10 4 3 3
 4.00101011001 11(2)10 4 3 3
 5.00101001001 11(2)6 2 2 2
 6.00101001011 11(2)10 6 2 4
 7.11111111111 1(1)1 0 0 1
 total NATs = 57
L= 12
 1.000000000000 1(1)1 0 0 1
 2.010101010101 2(1)568 292 10 18
 3.001010101011 12(2)75 36 7 8
 4.001010101001 12(2)60 24 6 5
 5.001010101101 12(2)57 29 5 6
 6.001010100101 12(2)46 26 4 6
 7.001011001011 3(2)8 6 1 4
 8.001011011011 12(2)4 2 1 2
 9.001011011001 6(2)4 2 1 2
 10.001001001001 3(2)2 0 0 1
 11.111111111111 1(1)1 0 0 1
 total NATs = 76
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.0010101010101 13(2)432 228 10 13
 3.0010101011011 13(2)29 13 5 5
 4.0010101011001 13(2)29 13 5 5
 5.0010101001001 13(2)20 8 4 3
 6.0010101001011 13(2)34 22 4 6
 7.0010101101101 13(2)24 10 3 6
 8.0010101100101 13(2)23 13 3 6
 9.0010101101011 13(2)23 13 3 6
 10.0010100101001 13(2)16 8 2 4
 11.1111111111111 1(1)1 0 0 1
 total NATs = 119
L= 14
 1.00000000000000 1(1)1 0 0 1
 2.01010101010101 2(1)1639 924 12 29
 3.00101010101011 14(2)214 118 9 13
 4.00101010101001 14(2)176 82 8 8
 5.00101010101101 14(2)160 83 7 9
 6.00101010100101 14(2)134 76 6 10
 7.00101010110101 7(2)176 104 5 10
 8.00101010010101 7(2)154 98 4 9
 9.00101011001011 14(2)18 12 3 6
 10.00101011011011 14(2)10 4 3 3
 11.00101011011001 14(2)11 5 3 3
 12.00101011001001 14(2)10 4 3 3
 13.00101001011001 14(2)11 7 2 4
 14.00101001001001 14(2)6 2 2 2
 15.00101001001011 14(2)11 7 2 4
 16.00101001011011 14(2)10 6 2 4
 17.11111111111111 1(1)1 0 0 1
 total NATs = 186
```

182
intentionally blank

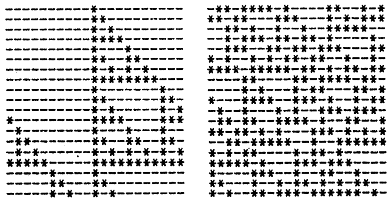
λ ratio = 1 Z = 1

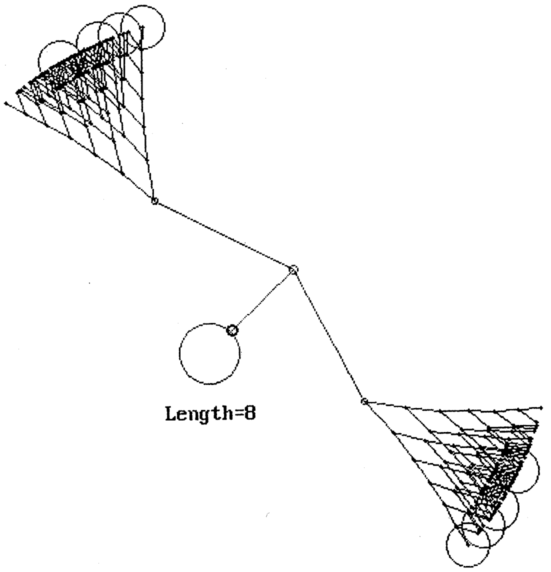
0000111111110000-0000111111110000-rule 267390960
=3-rule 60 -00111100

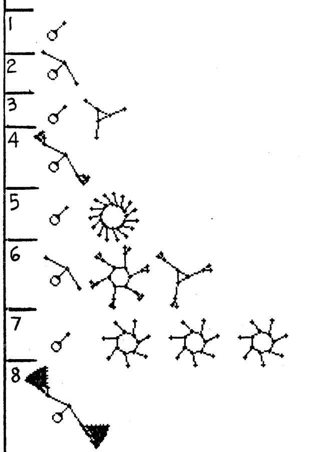
Length=1 -15

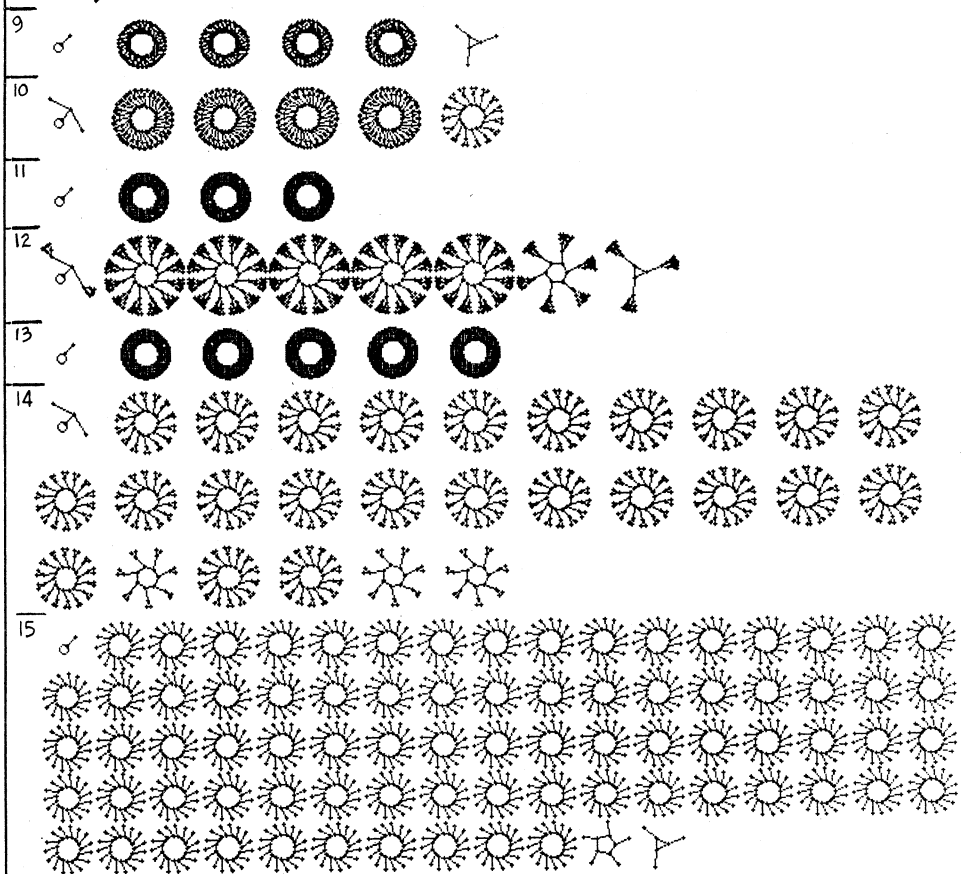


00111100	60
11000011	195
10011001	153
01100110	102









3-rule 60 =00111100

ty. at no(p)s g ml mp

L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)4 2 2 2
total NATs = 1
L= 3
1.000 1(1)2 1 1 2
2.101 1(3)6 3 1 2
total NATs = 2
L= 4
1.0000 1(1)16 8 4 2
total NATs = 1
L= 5
1.00000 1(1)2 1 1 2
2.10001 1(15)30 15 1 2
total NATs = 2
L= 6
1.000000 1(1)4 2 2 2
2.010001 2(6)24 12 2 2
3.110110 1(3)12 6 2 2
total NATs = 4
L= 7
1.0000000 1(1)2 1 1 2
2.1000001 7(7)14 7 1 2
3.1001110 1(7)14 7 1 2
4.1001011 1(7)14 7 1 2
total NATs = 10
L= 8
1.00000000 1(1)256 128 8 2
total NATs = 1
L= 9
1.000000000 1(1)2 1 1 2
2.100000001 1(63)126 63 1 2
3.100000100 1(63)126 63 1 2
4.100001110 1(63)126 63 1 2
5.100001011 1(63)126 63 1 2
6.101101101 1(3)6 3 1 2
total NATs = 6
L= 10
1.0000000000 1(1)4 2 2 2
2.0100000001 2(30)120 60 2 2
3.1100000110 2(30)120 60 2 2
4.1100001001 2(30)120 60 2 2
5.0100001110 2(30)120 60 2 2
6.1100011000 1(15)60 30 2 2
total NATs = 10
L= 11
1.00000000000 1(1)2 1 1 2
2.10000000001 1(341)682 341 1 2
3.10000001110 1(341)682 341 1 2
4.10000001011 1(341)682 341 1 2
total NATs = 4
L= 12
1.000000000000 1(1)16 8 4 2
2.000100000001 4(12)192 96 4 2
3.011100000111 4(12)192 96 4 2
4.101100001011 4(12)192 96 4 2
5.110100001101 4(12)192 96 4 2
6.001100010010 4(12)192 96 4 2
7.010100010100 2(6)96 48 4 2
8.101101101101 1(3)48 24 4 2
total NATs = 24
L= 13
1.0000000000000 1(1)2 1 1 2
2.1000000000001 1(819)1638 819 1 2
3.1000000001110 1(819)1638 819 1 2
4.1000000001011 1(819)1638 819 1 2
5.1000000011010 1(819)1638 819 1 2
6.1000000010101 1(819)1638 819 1 2
total NATs = 6
L= 14
1.00000000000000 1(1)4 2 2 2
2.01000000000001 14(14)56 28 2 2
3.11000000000110 14(14)56 28 2 2
4.11000000001001 14(14)56 28 2 2
5.01000000001110 14(14)56 28 2 2
6.11000000010111 14(14)56 28 2 2
7.01000000011111 14(14)56 28 2 2
8.11000000011000 14(14)56 28 2 2
9.11000000101011 14(14)56 28 2 2
10.01000000101100 14(14)56 28 2 2
11.01000000100011 14(14)56 28 2 2
12.11000000100100 14(14)56 28 2 2
13.01000000111101 14(14)56 28 2 2
14.11000000111010 14(14)56 28 2 2
15.11000000110101 14(14)56 28 2 2
16.01000000110010 14(14)56 28 2 2
17.11000001010011 14(14)56 28 2 2
18.01000001010100 2(14)56 28 2 2
19.01000001011011 14(14)56 28 2 2
20.11000001011100 14(14)56 28 2 2
21.01000001000101 2(14)56 28 2 2
22.11000001000010 14(14)56 28 2 2
23.01000001110110 14(14)56 28 2 2
24.11000001100000 7(7)28 14 2 2
25.11000111011010 2(14)56 28 2 2
26.11000110101101 2(14)56 28 2 2
27.11001011100101 1(7)28 14 2 2
28.01001110100111 1(7)28 14 2 2
total NATs = 298

184
intentionally blank

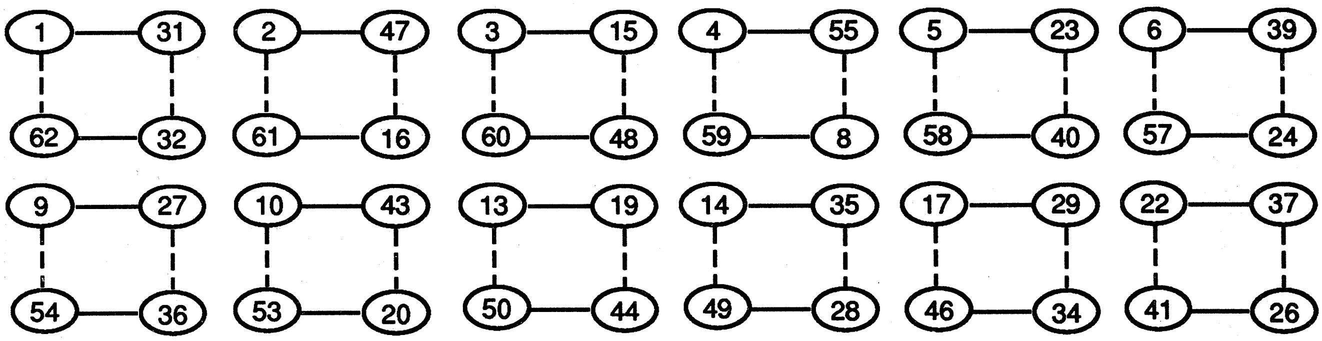
A2.4 n=5 Rules, Totalistic Code

A2.4.1 Index

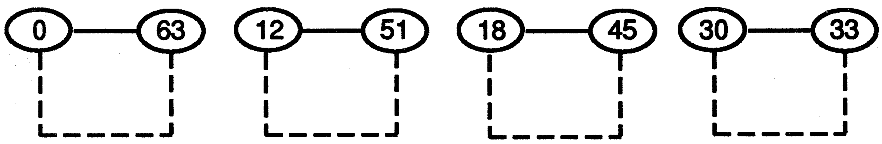
Key: [decimal ~~rule~~ code number],*[hex ~~rule~~ code number]*-[page number]

A2.4.2 Totalistic Code Clusters (see section 3.3.10)

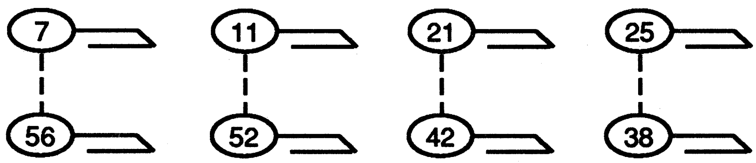
$C = C_r$, so the reflection links collapse.



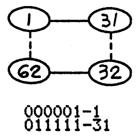
The clusters collapse further where for a given rule C, $C_c = C_n$

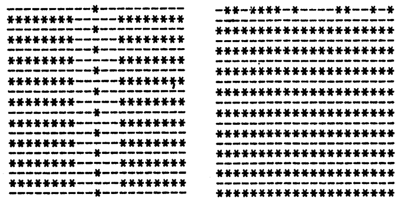


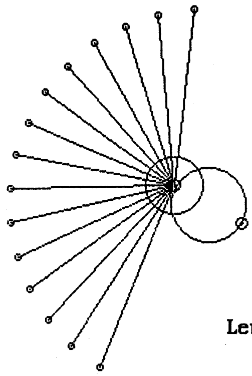
and also if $C = C_n$



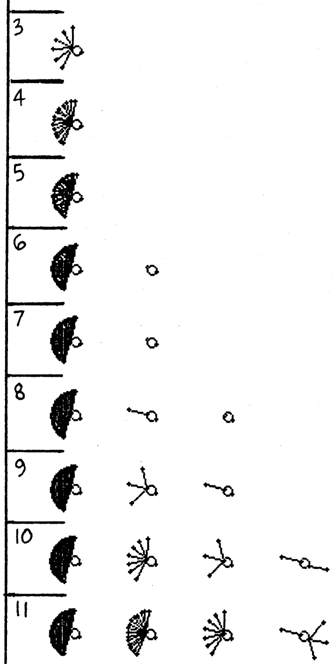
λ ratio = .0625 Z = .0625 0000000000000000-0000000000000001-rule 1
= 5-code 1 -000001 Length=3 -11







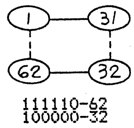
Length=11

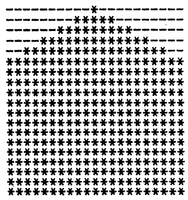


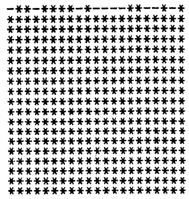

```
5-code  1 =000001

ty. at no(p)s g ml mp

L= 3
 1.000 1(2)8 6 1 7
 total NATs = 1
L= 4
 1.0000 1(2)16 14 1 15
 total NATs = 1
L= 5
 1.00000 1(2)32 30 1 31
 total NATs = 1
L= 6
 1.000000 1(2)58 56 1 57
 2.000001 3(2)2 0 0 1
 total NATs = 4
L= 7
 1.0000000 1(2)114 112 1 113
 2.0000001 7(2)2 0 0 1
 total NATs = 8
L= 8
 1.00000000 1(2)224 222 1 223
 2.00000001 8(2)3 1 1 2
 3.00000011 4(2)2 0 0 1
 total NATs = 13
L= 9
 1.000000000 1(2)440 438 1 439
 2.000000001 9(2)5 3 1 4
 3.000000011 9(2)3 1 1 2
 total NATs = 19
L= 10
 1.0000000000 1(2)864 862 1 863
 2.0000000001 10(2)9 7 1 8
 3.0000000011 10(2)5 3 1 4
 4.0011100000 5(2)4 2 1 2
 total NATs = 26
L= 11
 1.00000000000 1(2)1696 1694 1 1695
 2.00000000001 11(2)17 15 1 16
 3.00000000011 11(2)9 7 1 8
 4.00111100000 11(2)6 4 1 4
 total NATs = 34
```

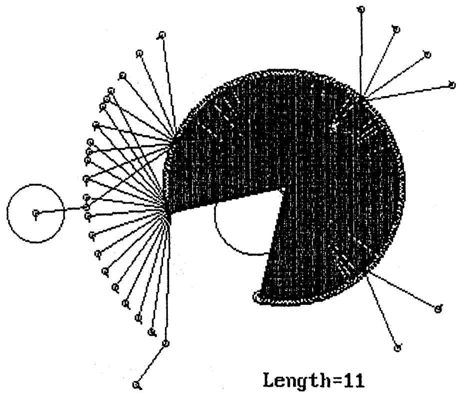
λ ratio = .0625 Z = .0625

1111111111111111-1111111111111110-rule 4294967294
= 5-code 62 -111110   Length=3 -11

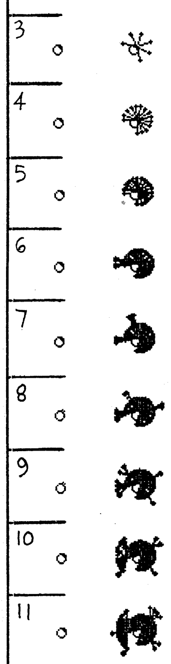








Length=11



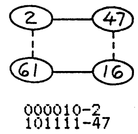
5-code 62 =111110

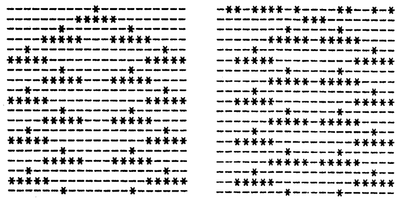
ty. at no(p)s g ml mp

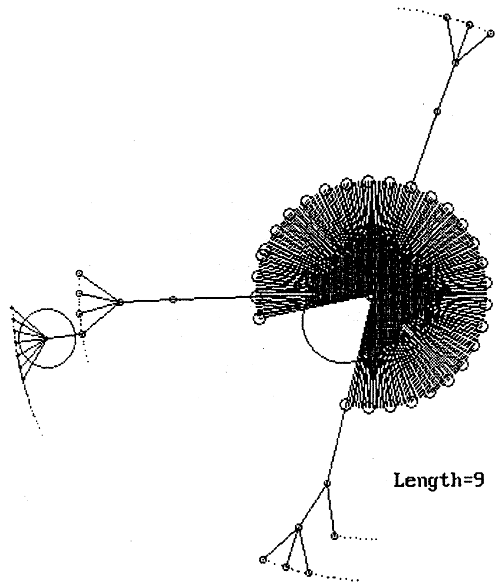
```
L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)7 6 1 7
 total NATs = 2
L= 4
 1.0000 1(1)1 0 0 1
 2.1111 1(1)15 14 1 15
 total NATs = 2
L= 5
 1.00000 1(1)1 0 0 1
 2.11111 1(1)31 30 1 31
 total NATs = 2
L= 6
 1.000000 1(1)1 0 0 1
 2.111111 1(1)63 56 2 57
 total NATs = 2
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111111 1(1)127 112 2 113
 total NATs = 2
L= 8
 1.00000000 1(1)1 0 0 1
 2.11111111 1(1)255 230 2 223
 total NATs = 2
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111111 1(1)511 474 2 439
 total NATs = 2
L= 10
 1.0000000000 1(1)1 0 0 1
 2.1111111111 1(1)1023 972 3 863
 total NATs = 2
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111111 1(1)2047 1980 3 1695
 total NATs = 2
L= 12
```

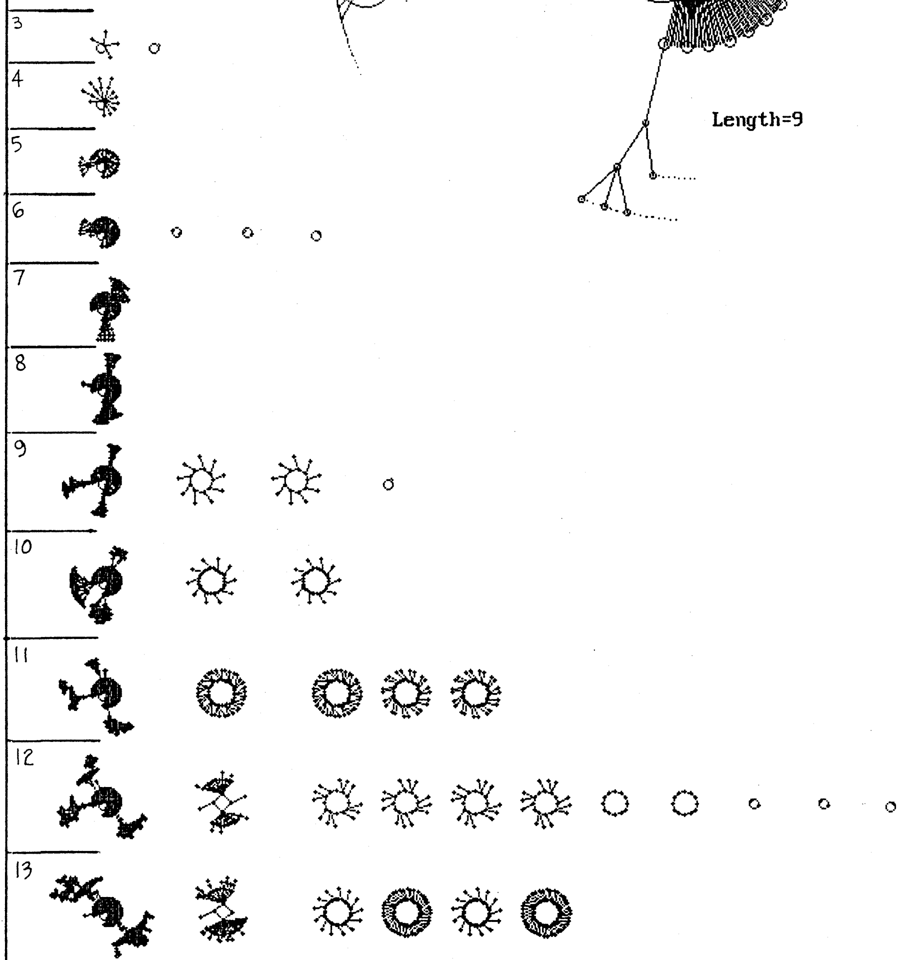
λ ratio = .3135 Z = .3125

0000000000000001-0000000100010110-rule 65814 = 5-code 2 -000010 Length=3 -13





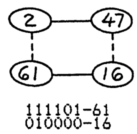


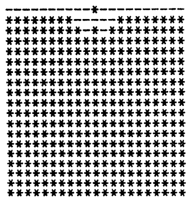


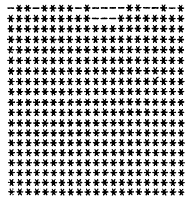

```
5-code  2 =000010

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)5 4 1 5
 2.001 3(1)1 0 0 1
 total NATs = 4
L= 4
 1.0000 1(1)16 11 2 12
 total NATs = 1
L= 5
 1.00000 1(1)32 30 2 27
 total NATs = 1
L= 6
 1.000000 1(1)49 42 2 43
 2.000011 3(2)2 0 0 1
 3.000101 3(2)2 0 0 1
 4.001001 3(1)1 0 0 1
 total NATs = 10
L= 7
 1.0000000 1(1)128 85 4 72
 total NATs = 1
L= 8
 1.00000000 1(1)256 187 4 132
 total NATs = 1
L= 9
 1.000000000 1(1)473 373 6 248
 2.000001011 1(9)18 9 1 2
 3.000001101 1(9)18 9 1 2
 4.001001001 3(1)1 0 0 1
 total NATs = 6
L= 10
 1.0000000000 1(1)944 812 6 459
 2.0000001011 1(30)40 10 1 2
 3.0000001101 1(30)40 10 1 2
 total NATs = 3
L= 11
 1.00000000000 1(1)1806 1563 7 849
 2.00000001011 1(33)66 33 1 4
 3.00000001101 1(33)66 33 1 4
 4.00000010011 1(33)55 22 1 2
 5.00000011001 1(33)55 22 1 2
 total NATs = 5
L= 12
 1.000000000000 1(1)3529 3116 8 1551
 2.000000000101 6(4)40 30 2 15
 3.000000010011 3(12)24 12 1 3
 4.000000010111 3(12)24 12 1 3
 5.000000011001 3(12)24 12 1 3
 6.000000011101 3(12)24 12 1 3
 7.000000100111 1(12)12 0 0 1
 8.000000111001 1(12)12 0 0 1
 9.000011000011 3(2)2 0 0 1
 10.000101000101 3(2)2 0 0 1
 11.001001001001 3(1)1 0 0 1
 total NATs = 30
L= 13
 1.0000000000000 1(1)7061 6306 11 2862
 2.0000000000101 13(4)67 44 3 28
 3.0000000100011 1(39)52 13 1 2
 4.0000000100111 1(39)78 39 1 2
 5.0000000110001 1(39)52 13 1 2
 6.0000000111001 1(39)78 39 1 2
```

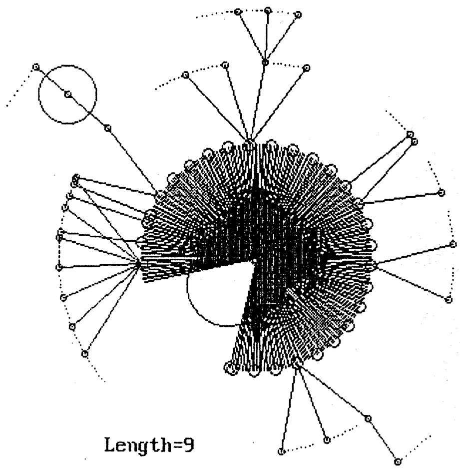
λ ratio = .3135 Z = .3125

1111111111111110-1111111011101001-rule 4294901481
= 5-code 61 -111101 Length=3 -13

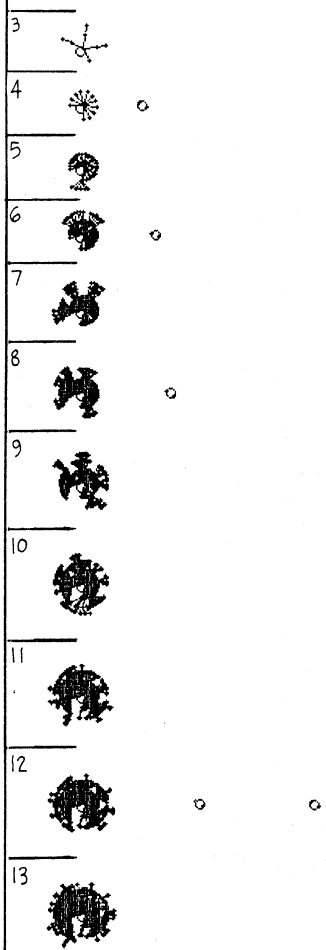








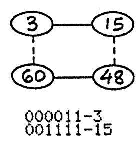
Length=9

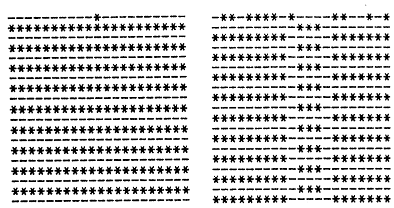


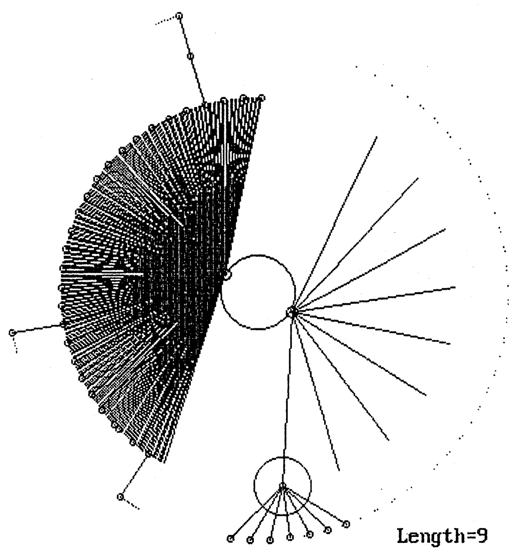

```
5-code  61 =111101

ty. at no(p)s g ml mp

L= 3
 1.111 1(1)8 4 2 5
 total NATs = 1
L= 4
 1.1111 1(1)12 11 1 12
 2.0001 2(2)2 0 0 1
 total NATs = 3
L= 5
 1.11111 1(1)32 30 2 27
 total NATs = 1
L= 6
 1.111111 1(1)58 42 2 43
 2.000001 3(2)2 0 0 1
 total NATs = 4
L= 7
 1.1111111 1(1)128 85 3 72
 total NATs = 1
L= 8
 1.11111111 1(1)252 187 3 132
 2.00010001 2(2)2 0 0 1
 total NATs = 3
L= 9
 1.111111111 1(1)512 391 4 248
 total NATs = 1
L= 10
 1.1111111111 1(1)1024 832 3 459
 total NATs = 1
L= 11
 1.11111111111 1(1)2048 1673 4 849
 total NATs = 1
L= 12
 1.111111111111 1(1)4086 3440 4 1551
 2.000001000001 3(2)2 0 0 1
 3.000100010001 2(2)2 0 0 1
 total NATs = 6
L= 13
 1.1111111111111 1(1)8192 6982 5 2862
 total NATs = 1
```

λ ratio = .375 Z = .25

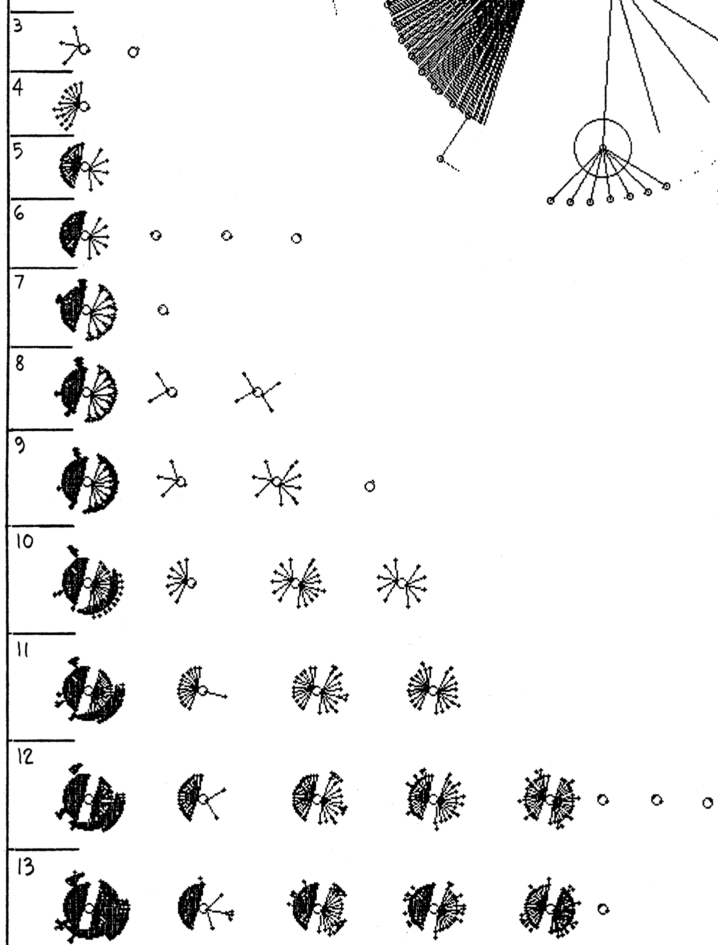
0000000000000001-0000000100010111-rule 65815
= 5-code 3 -000011 Length=3 -13







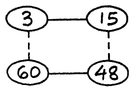
Length=9



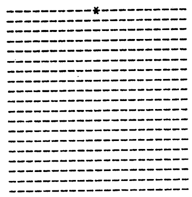

```
5-code  3 =000011

ty. at no(p)s g ml mp

L= 3
 1.000 1(2)5 3 1 4
 2.001 3(1)1 0 0 1
 total NATs = 4
L= 4
 1.0000 1(2)16 10 2 11
 total NATs = 1
L= 5
 1.00000 1(2)32 30 1 26
 total NATs = 1
L= 6
 1.000000 1(2)49 47 1 42
 2.000011 3(2)2 0 0 1
 3.000101 3(2)2 0 0 1
 4.001001 3(1)1 0 0 1
 total NATs = 10
L= 7
 1.0000000 1(2)114 91 2 71
 2.0000011 7(2)2 0 0 1
 total NATs = 8
L= 8
 1.00000000 1(2)200 162 3 131
 2.00000011 8(2)4 2 1 3
 3.00000111 4(2)6 4 1 3
 total NATs = 13
L= 9
 1.000000000 1(2)356 309 3 247
 2.000000011 9(2)5 3 1 4
 3.000000111 9(2)12 8 2 6
 4.001001001 3(1)1 0 0 1
 total NATs = 22
L= 10
 1.0000000000 1(2)684 602 4 458
 2.0000000011 10(2)9 7 1 8
 3.0000000111 10(2)19 15 2 10
 4.0000001111 5(2)12 10 1 6
 total NATs = 26
L= 11
 1.00000000000 1(2)1278 1155 3 848
 2.00000000011 11(2)16 14 1 14
 3.00000000111 11(2)29 23 2 13
 4.00000001111 11(2)25 20 2 12
 total NATs = 34
L= 12
 1.000000000000 1(2)2377 2227 3 1550
 2.000000000011 12(2)30 28 1 27
 3.000000000111 12(2)43 37 2 24
 4.000000001111 12(2)46 33 2 22
 5.011111000000 6(2)46 36 2 18
 6.000011000011 3(2)2 0 0 1
 7.000101000101 3(2)2 0 0 1
 8.001001001001 3(1)1 0 0 1
 total NATs = 52
L= 13
 1.0000000000000 1(2)4500 4238 4 2861
 2.0000000000011 13(2)58 54 2 50
 3.0000000000111 13(2)81 71 2 47
 4.0000000001111 13(2)72 59 2 41
 5.0111111000000 13(2)71 56 2 32
 6.0000011000011 13(2)2 0 0 1
```

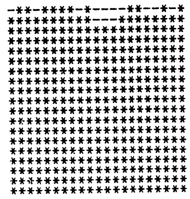
λ ratio = .375 Z = .25

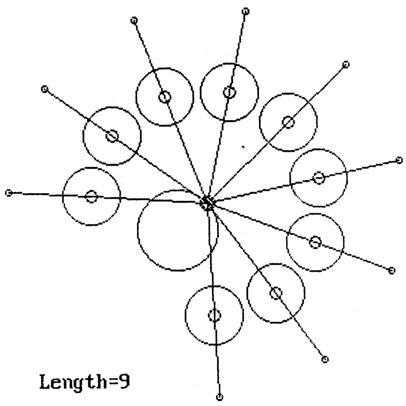
1111111111111110-1111111011101000-rule 4294901480
= 5-code 60 -111100
Length=3 -10



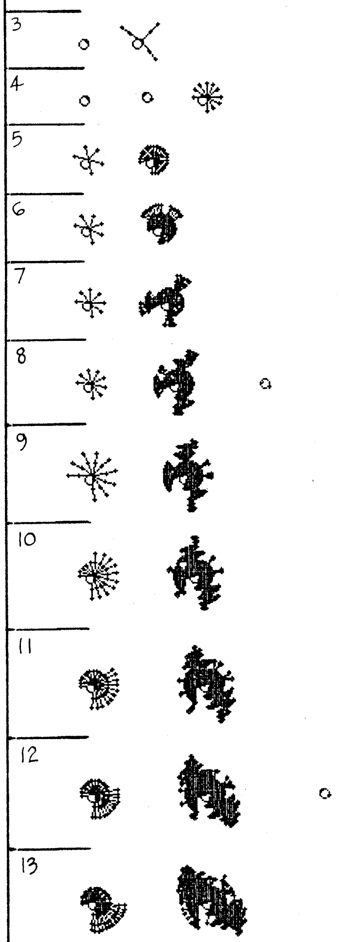
111100-60
110000-48







Length=9



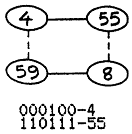
5-code 60 =111100

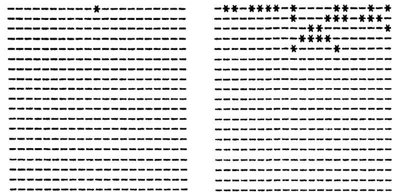
ty. at no(p)s g ml mp

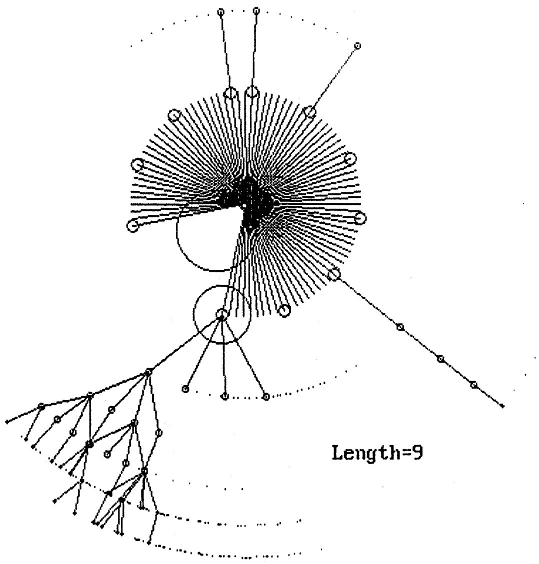
```
L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)7 3 2 4
 total NATs = 2
L= 4
 1.0000 1(1)1 0 0 1
 2.0001 2(2)2 0 0 1
 3.1111 1(1)11 10 1 11
 total NATs = 4
L= 5
 1.00000 1(1)6 5 1 6
 2.11111 1(1)26 25 1 26
 total NATs = 2
L= 6
 1.000000 1(1)7 6 1 7
 2.111111 1(1)57 41 2 42
 total NATs = 2
L= 7
 1.0000000 1(1)8 7 1 8
 2.1111111 1(1)120 84 3 71
 total NATs = 2
L= 8
 1.00000000 1(1)9 8 1 9
 2.11111111 1(1)243 186 4 131
 3.00010001 2(2)2 0 0 1
 total NATs = 4
L= 9
 1.000000000 1(1)19 9 2 10
 2.111111111 1(1)493 399 4 247
 total NATs = 2
L= 10
 1.0000000000 1(1)26 15 2 16
 2.1111111111 1(1)998 857 5 458
 total NATs = 2
L= 11
 1.00000000000 1(1)34 22 2 23
 2.11111111111 1(1)2014 1760 5 848
 total NATs = 2
L= 12
 1.000000000000 1(1)43 30 2 31
 2.111111111111 1(1)4049 3589 6 1550
 3.000100010001 2(2)2 0 0 1
 total NATs = 4
L= 13
 1.0000000000000 1(1)66 39 3 40
 2.1111111111111 1(1)8126 7319 6 2861
 total NATs = 2
```

λ ratio = .625 Z = .625

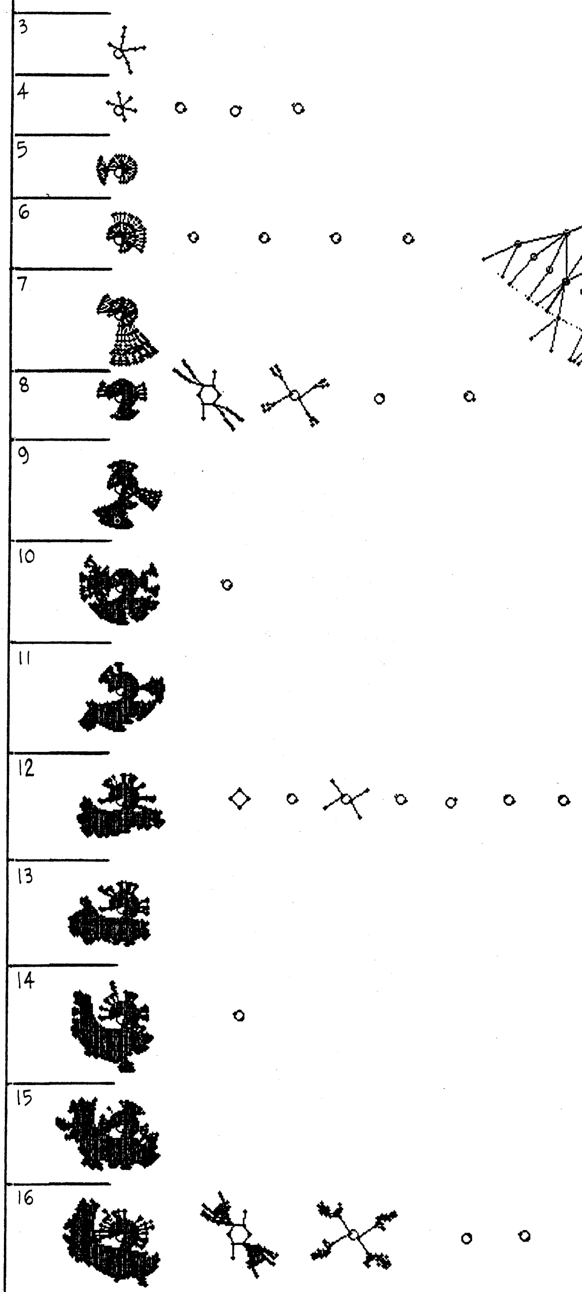
0000000100010110-0001011001101000-rule 18224744
= 5-code 4 -000100 Length=3 -13







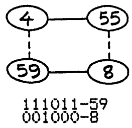
Length=9

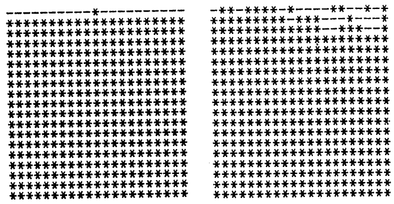


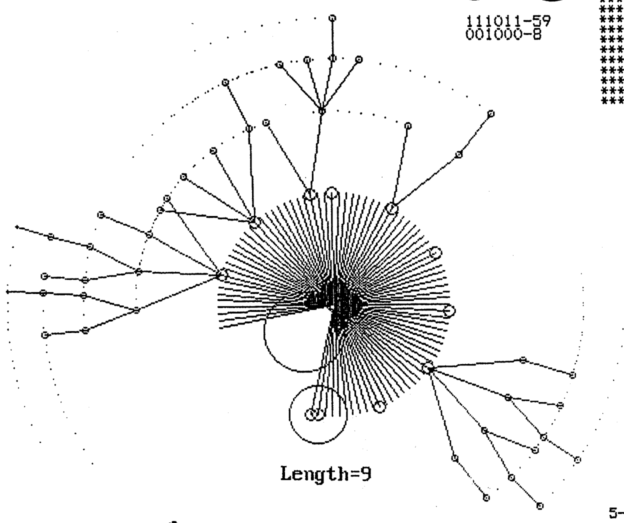

```
5-code  4 =000100

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)8 4 2 5
 total NATs = 1
L= 4
 1.0000 1(1)6 5 1 6
 2.0001 2(2)2 0 0 1
 3.0011 4(1)1 0 0 1
 4.0101 1(2)2 0 0 1
 total NATs = 8
L= 5
 1.00000 1(1)32 30 2 22
 total NATs = 1
L= 6
 1.000000 1(1)44 28 2 29
 2.000111 3(2)2 0 0 1
 3.001011 3(2)2 0 0 1
 4.001101 3(2)2 0 0 1
 5.010101 1(2)2 0 0 1
 total NATs = 11
L= 7
 1.0000000 1(1)128 57 9 37
 total NATs = 1
L= 8
 1.00000000 1(1)94 69 2 54
 2.00000011 4(6)28 6 5 3
 3.10001000 2(2)22 8 3 3
 4.00110011 4(1)1 0 0 1
 5.01010101 1(2)2 0 0 1
 total NATs = 12
L= 9
 1.000000000 1(1)512 274 6 86
 total NATs = 1
L= 10
 1.0000000000 1(1)1022 800 6 142
 2.0101010101 1(2)2 0 0 1
 total NATs = 2
L= 11
 1.00000000000 1(1)2048 1343 10 233
 total NATs = 1
L= 12
 1.000000000000 1(1)4008 2546 9 369
 2.000010110011 12(4)4 0 0 1
 3.000100010001 2(2)2 0 0 1
 4.110001110001 3(2)6 4 1 3
 5.001011001011 3(2)2 0 0 1
 6.001100110011 4(1)1 0 0 1
 7.001101001101 3(2)2 0 0 1
 8.010101010101 1(2)2 0 0 1
 total NATs = 29
L= 13
 1.0000000000000 1(1)8192 5097 11 574
 total NATs = 1
L= 14
 1.00000000000000 1(1)16382 10697 10 919
 2.01010101010101 1(2)2 0 0 1
 total NATs = 2
L= 15
 1.000000000000000 1(1)32768 24498 14 1480
 total NATs = 1
L= 16
 1.0000000000000000 1(1)64686 46421 11 2390
 2.1000000110000001 4(6)168 82 6 25
 3.0100010001000100 2(2)86 48 6 6
 4.0011001100110011 4(1)1 0 0 1
 5.0101010101010101 1(2)2 0 0 1
 total NATs = 12
```

λ ratio = .625   Z = .625

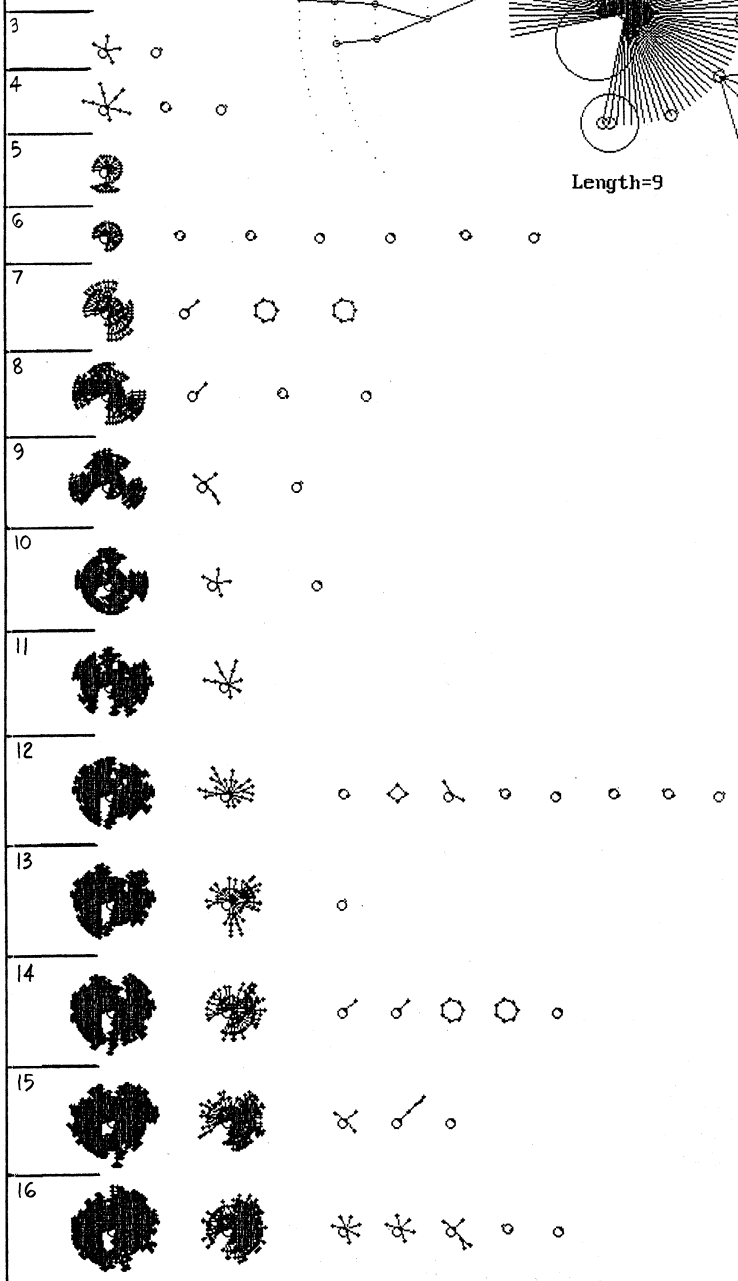
1111111011101001-1110100110010111-rule 4276742551
= 5-code 59 -111011
Length=3 -16







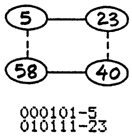
Length=9

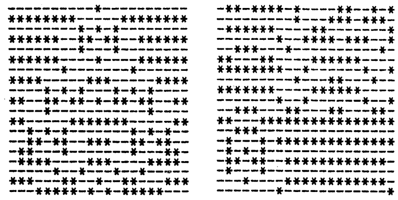


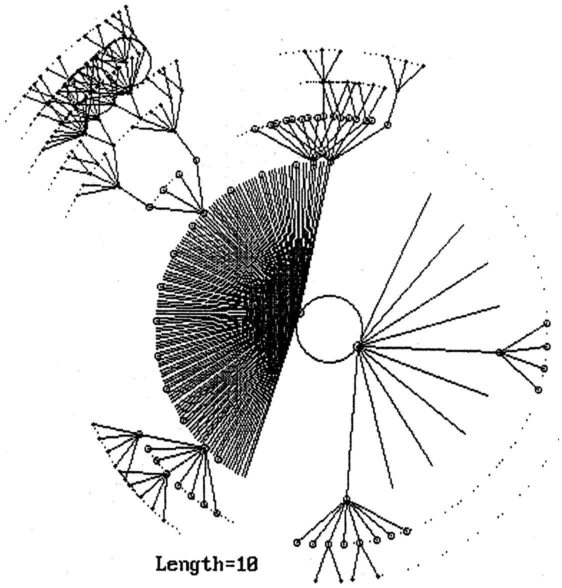

```
5-code  59 =111011

ty. at no(p)s g ml mp

L= 3
 1.111 1(1)5 4 1 5
 2.001 3(1)1 0 0 1
 total NATs = 4
L= 4
 1.1111 1(1)10 5 2 6
 2.0011 2(2)2 0 0 1
 3.0101 2(1)1 0 0 1
 total NATs = 5
L= 5
 1.11111 1(1)32 30 2 22
 total NATs = 1
L= 6
 1.111111 1(1)29 28 1 29
 2.000011 3(2)2 0 0 1
 3.000101 3(2)2 0 0 1
 4.000111 6(1)1 0 0 1
 5.001001 3(1)1 0 0 1
 6.001011 6(2)2 0 0 1
 7.010101 2(1)1 0 0 1
 total NATs = 24
L= 7
 1.1111111 1(1)100 50 3 37
 2.1111000 7(1)2 1 1 2
 3.0001011 1(7)7 0 0 1
 4.0001101 1(7)7 0 0 1
 total NATs = 10
L= 8
 1.11111111 1(1)234 101 4 54
 2.11111000 8(1)2 1 1 2
 3.00110011 2(2)2 0 0 1
 4.01010101 2(1)1 0 0 1
 total NATs = 13
L= 9
 1.111111111 1(1)464 247 5 86
 2.111111000 9(1)5 3 2 4
 3.001001001 3(1)1 0 0 1
 total NATs = 13
L= 10
 1.1111111111 1(1)972 760 4 142
 2.1111111000 10(1)5 4 1 5
 3.0101010101 2(1)1 0 0 1
 total NATs = 13
L= 11
 1.11111111111 1(1)1938 1277 5 233
 2.11111111000 11(1)10 6 2 7
 total NATs = 12
L= 12
 1.111111111111 1(1)3709 2366 5 369
 2.111111111000 12(1)24 15 3 13
 3.000011000011 3(2)2 0 0 1
 4.000011001011 12(4)4 0 0 1
 5.001110001110 6(1)3 2 1 3
 6.000101000101 3(2)2 0 0 1
 7.001001001001 3(1)1 0 0 1
 8.001011001011 6(2)2 0 0 1
 9.001100110011 2(2)2 0 0 1
 10.010101010101 2(1)1 0 0 1
 total NATs = 50
L= 13
 1.1111111111111 1(1)7399 4655 6 574
 2.1111111111000 13(1)60 34 4 20
 3.0001110001111 13(1)1 0 0 1
 total NATs = 27
L= 14
 1.11111111111111 1(1)14674 9640 6 919
 2.11111111111000 14(1)118 74 4 32
 3.11110001111000 7(1)2 1 1 2
 4.00111000111110 14(1)2 1 1 2
 5.00010110001011 1(7)7 0 0 1
 6.00011010001101 1(7)7 0 0 1
 7.01010101010101 2(1)1 0 0 1
 total NATs = 40
L= 15
 1.111111111111111 1(1)28970 21873 7 1480
 2.111111111111000 15(1)244 171 4 51
 3.111110001111000 15(1)4 3 1 4
 4.001110001111110 15(1)5 1 4 2
 5.001001001001001 3(1)1 0 0 1
 total NATs = 49
L= 16
 1.1111111111111111 1(1)57866 41477 6 2390
 2.1111111111111000 16(1)463 323 5 79
 3.1111110001111000 16(1)7 6 1 7
 4.1111100011111000 8(1)6 5 1 6
 5.0111000111111100 16(1)6 4 2 4
 6.0011001100110011 2(2)2 0 0 1
 7.0101010101010101 2(1)1 0 0 1
 total NATs = 61
```

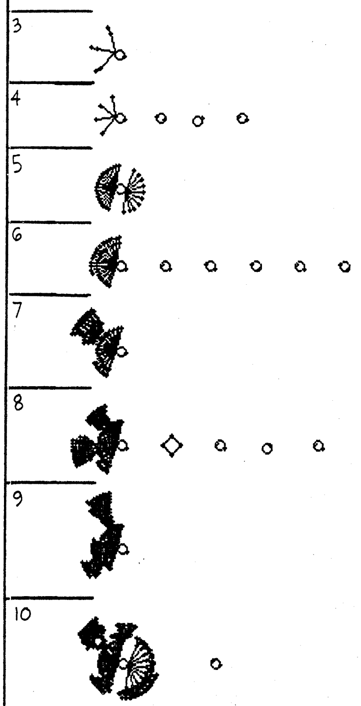
λ ratio = .6875 Z = .6875

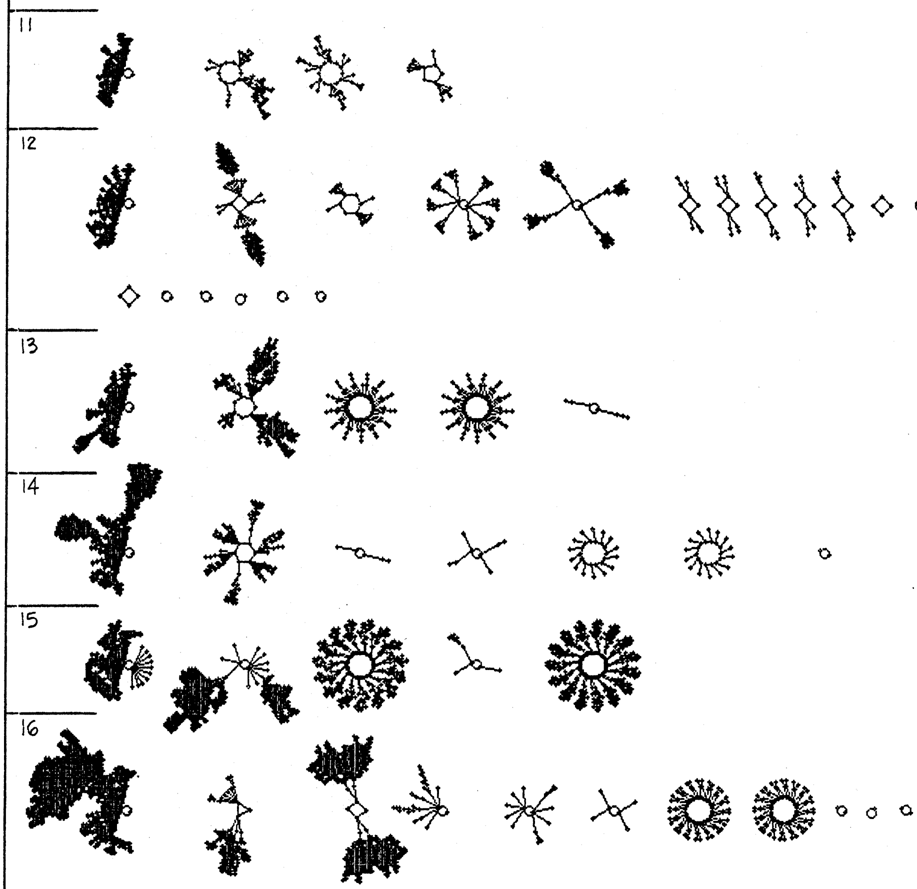
0000000100010110-0001011001101001-rule 18224745
= 5-code 5 -000101 Length=3 -16









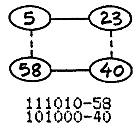


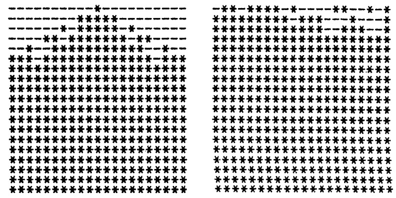

```
5-code  5 =000101

ty. at no(p)s g ml mp
L= 3
 1.000 1(2)8 3 2 4
 total NATs = 1
L= 4
 1.0000 1(2)6 4 1 5
 2.0001 2(2)2 0 0 1
 3.0011 4(1)1 0 0 1
 4.0101 1(2)2 0 0 1
 total NATs = 8
L= 5
 1.00000 1(2)32 30 1 21
 total NATs = 1
L= 6
 1.000000 1(2)38 21 2 22
 2.000001 3(2)2 0 0 1
 3.000111 3(2)2 0 0 1
 4.001011 3(2)2 0 0 1
 5.001101 3(2)2 0 0 1
 6.010101 1(2)2 0 0 1
 total NATs = 14
L= 7
 1.0000000 1(2)128 56 8 29
 total NATs = 1
L= 8
 1.00000000 1(2)230 84 7 45
 2.00000101 4(4)4 0 0 1
 3.00010001 2(2)2 0 0 1
 4.00110011 4(1)1 0 0 1
 5.01010101 1(2)2 0 0 1
 total NATs = 12
L= 9
 1.000000000 1(2)512 219 10 76
 total NATs = 1
L= 10
 1.0000000000 1(2)1022 760 8 131
 2.0101010101 1(2)2 0 0 1
 total NATs = 2
L= 11
 1.00000000000 1(2)475 286 4 210
 2.01000000101 11(10)82 42 10 10
 3.00000010011 11(12)44 20 4 5
 4.00000011011 11(5)17 10 2 6
 total NATs = 34
L= 12
 1.000000000000 1(2)1230 721 8 338
 2.010000000100 6(4)292 168 12 13
 3.000000001001 6(6)20 14 1 7
 4.100000100000 3(2)74 48 3 5
 5.110001110001 3(2)110 52 8 6
 6.000000100111 6(4)10 4 2 3
 7.000000101101 6(4)12 4 2 3
 8.100000110100 6(4)12 4 3 2
 9.000000111001 6(4)10 4 2 3
 10.000001001011 6(4)12 4 3 2
 11.000001010101 6(4)4 0 0 1
 12.000001101011 6(2)2 0 0 1
 13.000010110011 12(4)4 0 0 1
 14.000100010001 2(2)2 0 0 1
 15.001011001011 3(2)2 0 0 1
 16.001100110011 4(1)1 0 0 1
 17.001101001101 3(2)2 0 0 1
 18.010101010101 1(2)2 0 0 1
 total NATs = 86
L= 13
 1.0000000000000 1(2)2004 1170 14 534
 2.0100000000101 13(7)451 227 17 25
 3.0000000101111 1(52)117 39 3 3
 4.0000000111101 1(52)117 39 3 3
 5.0000001101011 13(2)7 2 3 2
 total NATs = 29
L= 14
 1.00000000000000 1(2)14870 8372 23 869
 2.00000000001111 7(6)190 108 8 26
 3.00000001101011 14(2)5 2 2 2
 4.00000011011011 7(2)8 4 2 3
 5.00000101001111 1(14)28 14 1 2
 6.00000111100101 1(14)28 14 1 2
 7.01010101010101 1(2)2 0 0 1
 total NATs = 32
L= 15
 1.000000000000000 1(2)5168 4335 6 1419
 2.011000000001101 15(2)1771 1277 28 64
 3.101110010000001 1(75)435 210 6 4
 4.000000011011011 15(2)11 6 4 3
 5.001110110000001 1(75)435 210 6 4
 total NATs = 33
L= 16
 1.0000000000000000 1(2)54022 33124 28 2301
 2.0010000001001111 16(3)278 173 13 42
 3.1010000010100000 4(4)1512 852 16 36
 4.0000000001101011 16(2)29 12 7 7
 5.0000000011011011 16(2)17 11 3 6
 6.0000000110111011 8(2)6 4 1 3
 7.0000000101100101 1(48)112 48 2 3
 8.0000001101001101 1(48)112 48 2 3
 9.0001000100010001 2(2)2 0 0 1
 10.0011001100110011 4(1)1 0 0 1
 11.0101010101010101 1(2)2 0 0 1
 total NATs = 70
```

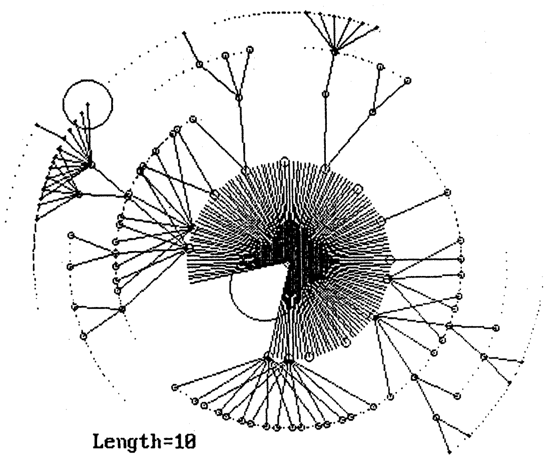
λ ratio = .6875 Z = .6875

1111111011101001-1110100110010110-rule   4276742550

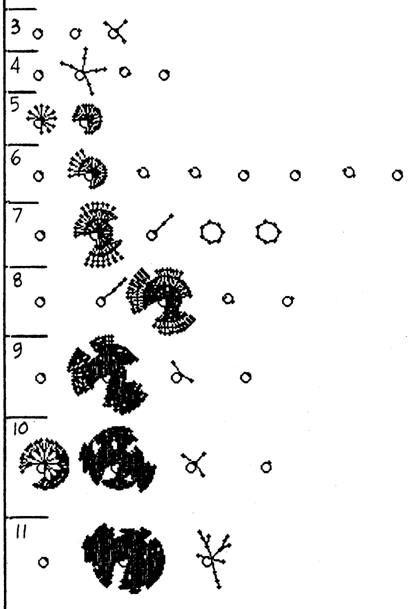
= 5-code 58 -111010   Length=3 -11

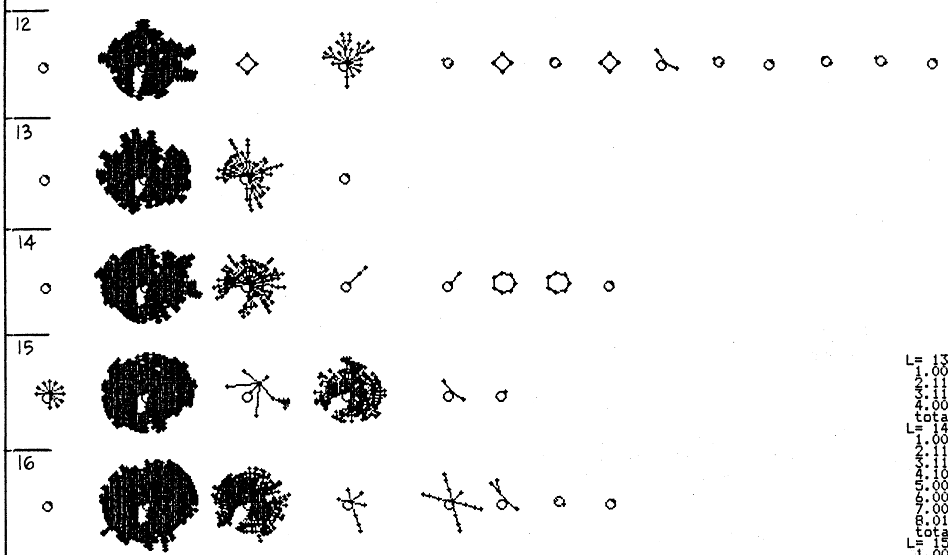






Length=10



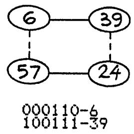


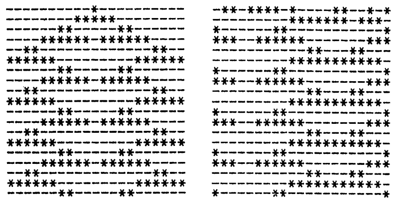

```
5-code  58 =111010

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)1 0 0 1
 2.001 3(1)1 0 0 1
 3.111 1(1)4 3 1 4
 total NATs = 5
L= 4
 1.0000 1(1)1 0 0 1
 2.1111 1(1)9 4 2 5
 3.0011 2(2)2 0 0 1
 4.0101 2(1)1 0 0 1
 total NATs = 6
L= 5
 1.00000 1(1)11 10 1 11
 2.11111 1(1)21 20 1 21
 total NATs = 2
L= 6
 1.000000 1(1)1 0 0 1
 2.111111 1(1)28 21 2 22
 3.000011 3(2)2 0 0 1
 4.000101 3(2)2 0 0 1
 5.000111 6(1)1 0 0 1
 6.001001 3(1)1 0 0 1
 7.001011 6(2)2 0 0 1
 8.010101 2(1)1 0 0 1
 total NATs = 25
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111111 1(1)92 49 4 29
 3.1000111 7(1)3 1 2 2
 4.0001011 1(7)7 0 0 1
 5.0001101 1(7)7 0 0 1
 total NATs = 11
L= 8
 1.00000000 1(1)1 0 0 1
 2.11000111 8(1)4 1 3 2
 3.11111111 1(1)217 76 5 45
 4.00110011 2(2)2 0 0 1
 5.01010101 2(1)1 0 0 1
 total NATs = 14
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111111 1(1)481 201 6 76
 3.000111111 9(1)3 2 1 3
 4.001001001 3(1)1 0 0 1
 total NATs = 14
L= 10
 1.0000000000 1(1)71 60 2 11
 2.1111111111 1(1)911 670 5 131
 3.0001111111 10(1)4 3 1 4
 4.0101010101 2(1)1 0 0 1
 total NATs = 14
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111111 1(1)1893 1001 7 210
 3.11000111111 11(1)14 7 3 6
 total NATs = 13
L= 12
 1.000000000000 1(1)1 0 0 1
 2.111111111111 1(1)3612 2017 8 338
 3.000000000101 6(4)4 0 0 1
 4.110001111111 12(1)26 17 3 12
 5.000000010101 12(2)2 0 0 1
 6.000000110011 6(4)4 0 0 1
 7.000011000011 3(2)2 0 0 1
 8.000011001011 12(4)4 0 0 1
 9.001110001110 6(1)3 2 1 3
 10.000101000101 3(2)2 0 0 1
 11.001001001001 3(1)1 0 0 1
 12.001011001011 6(2)2 0 0 1
 13.001100110011 2(2)2 0 0 1
 14.010101010101 2(1)1 0 0 1
 total NATs = 75
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.1111111111111 1(1)7437 3835 8 534
 3.1111000111111 13(1)57 30 5 18
 4.0001110001111 13(1)1 0 0 1
 total NATs = 28
L= 14
 1.00000000000000 1(1)1 0 0 1
 2.11111111111111 1(1)14764 8295 9 869
 3.11111000111111 14(1)111 64 4 28
 4.10001111000111 7(1)3 1 2 2
 5.00111000111110 14(1)2 1 1 2
 6.00010110001011 1(7)7 0 0 1
 7.00011010001101 1(7)7 0 0 1
 8.01010101010101 2(1)1 0 0 1
 total NATs = 41
L= 15
 1.000000000000000 1(1)11 10 1 11
 2.111111111111111 1(1)29064 21485 7 1419
 3.001111110001110 15(1)16 11 5 6
 4.111110001111111 15(1)227 154 4 46
 5.000111100011111 15(1)3 2 1 3
 6.001001001001001 3(1)1 0 0 1
 total NATs = 50
L= 16
 1.0000000000000000 1(1)1 0 0 1
 2.1111111111111111 1(1)57865 35196 10 2301
 3.1111000111111111 16(1)456 277 5 73
 4.1100011111000111 8(1)8 5 3 6
 5.0011111110001110 16(1)14 5 3 6
 6.0011111110001110 16(1)5 3 2 3
 7.0011001100110011 2(2)2 0 0 1
 8.0101010101010101 2(1)1 0 0 1
 total NATs = 62
```

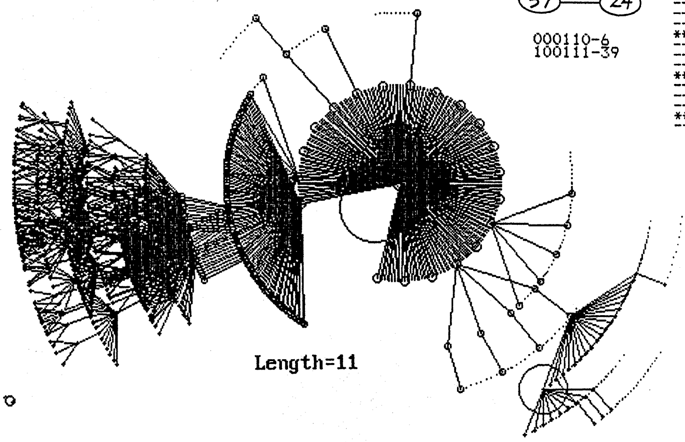
λ ratio = .9375 Z = .4375

0000000100010111-0001011101111110-rule 18290558

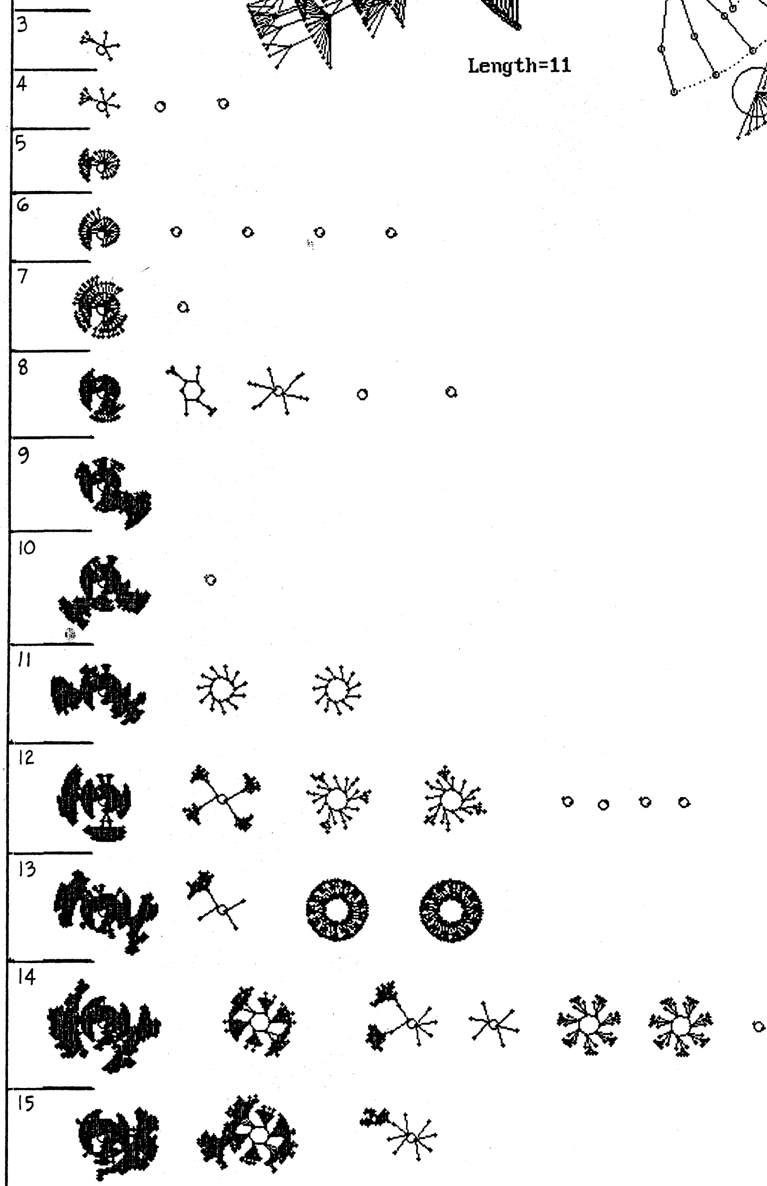
= 5-code 6 -000110   Length=3 -16

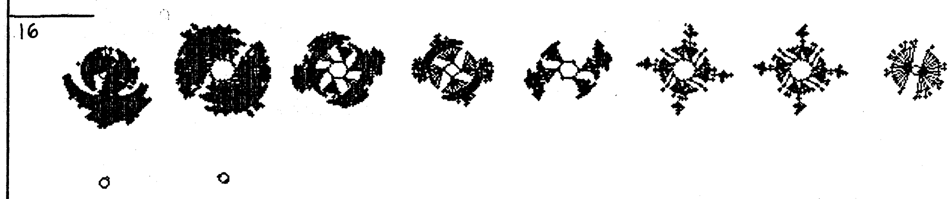






Length=11





5-code 6 =000110

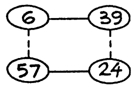
ty. at no(p)s g ml mp

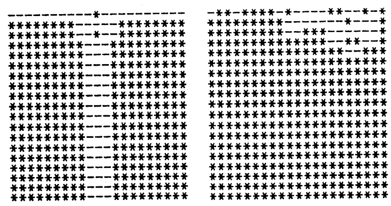
L= 3
 1.000 1(1)8 6 2 5
 total NATs = 1
L= 4
 1.0000 1(1)10 8 2 6
 2.0011 4(1)1 0 0 1
 3.0101 1(2)2 0 0 1
 total NATs = 6
L= 5
 1.00000 1(1)32 30 2 17
 total NATs = 1
L= 6
 1.000000 1(1)44 36 2 23
 2.000111 3(2)2 0 0 1
 3.001011 3(2)2 0 0 1
 4.001101 3(2)2 0 0 1
 5.010101 1(2)2 0 0 1
 total NATs = 11
L= 7
 1.0000000 1(1)114 70 3 30
 2.0000111 7(2)2 0 0 1
 total NATs = 8
L= 8
 1.00000000 1(1)138 112 3 46
 2.11000111 4(6)16 8 2 3
 3.00001111 4(2)12 6 2 4
 4.00110011 4(1)1 0 0 1
 5.01010101 1(2)2 0 0 1
 total NATs = 14
L= 9
 1.000000000 1(1)512 339 9 77
 total NATs = 1
L= 10
 1.0000000000 1(1)1022 765 9 127
 2.0101010101 1(2)2 0 0 1
 total NATs = 2
L= 11
 1.00000000000 1(1)2004 1507 9 211
 2.11011100000 1(11)22 11 1 2
 3.00000111011 1(11)22 11 1 2
 total NATs = 3
L= 12
 1.000000000000 1(1)3400 2696 7 339
 2.111000111000 3(2)114 88 4 14
 3.110011100000 4(9)42 24 3 4
 4.000000111011 4(9)42 24 3 4
 5.001011001011 3(2)2 0 0 1
 6.001100110011 4(1)1 0 0 1
 7.001101001101 3(2)2 0 0 1
 8.010101010101 1(2)2 0 0 1
 total NATs = 23
L= 13
 1.0000000000000 1(1)6801 5395 9 535
 2.1110001110000 13(2)69 55 5 29
 3.1100111100000 1(39)247 156 2 5
 4.0000001110011 1(39)247 156 2 5
 total NATs = 16
L= 14
 1.00000000000000 1(1)12532 10164 10 870
 2.00110001100000 7(6)286 226 4 35
 3.11100011110001 7(2)216 176 7 51
 4.11000011110001 14(2)8 6 1 4
 5.11001111000000 2(7)56 42 2 4
 6.00000011110011 2(7)56 42 2 4
 7.01010101010101 1(2)2 0 0 1
 total NATs = 34
L= 15
 1.000000000000000 1(1)17138 14331 9 1415
 2.001100001100000 15(6)976 810 11 93
 3.000111100011110 15(2)66 48 8 10
 total NATs = 31
L= 16
 1.0000000000000000 1(1)21978 18544 11 2302
 2.0000001110000000 8(14)3228 2678 9 206
 3.0011000011000000 8(6)1502 1292 6 138
 4.0011001100000000 8(4)420 372 4 117
 5.1100000111000001 4(6)248 200 5 16
 6.1100000111100001 4(12)144 92 6 11
 7.0011110000111000 4(12)144 92 6 11
 8.1111000011110000 4(2)52 38 2 16
 9.0011001100110011 4(1)1 0 0 1
 10.0101010101010101 1(2)2 0 0 1
 total NATs = 46

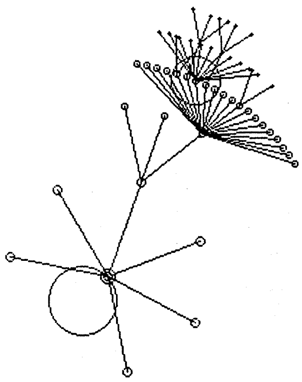
λ ratio = .9375 Z = .4375

1111111011101000-1110100010000001-rule 4276676737

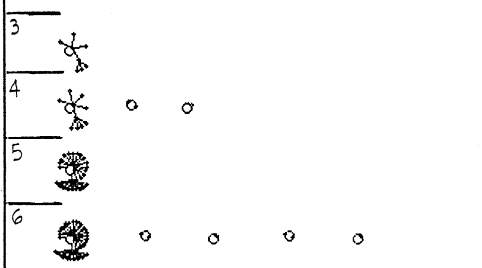
= 5-code 57 -111001 Length=3 -16

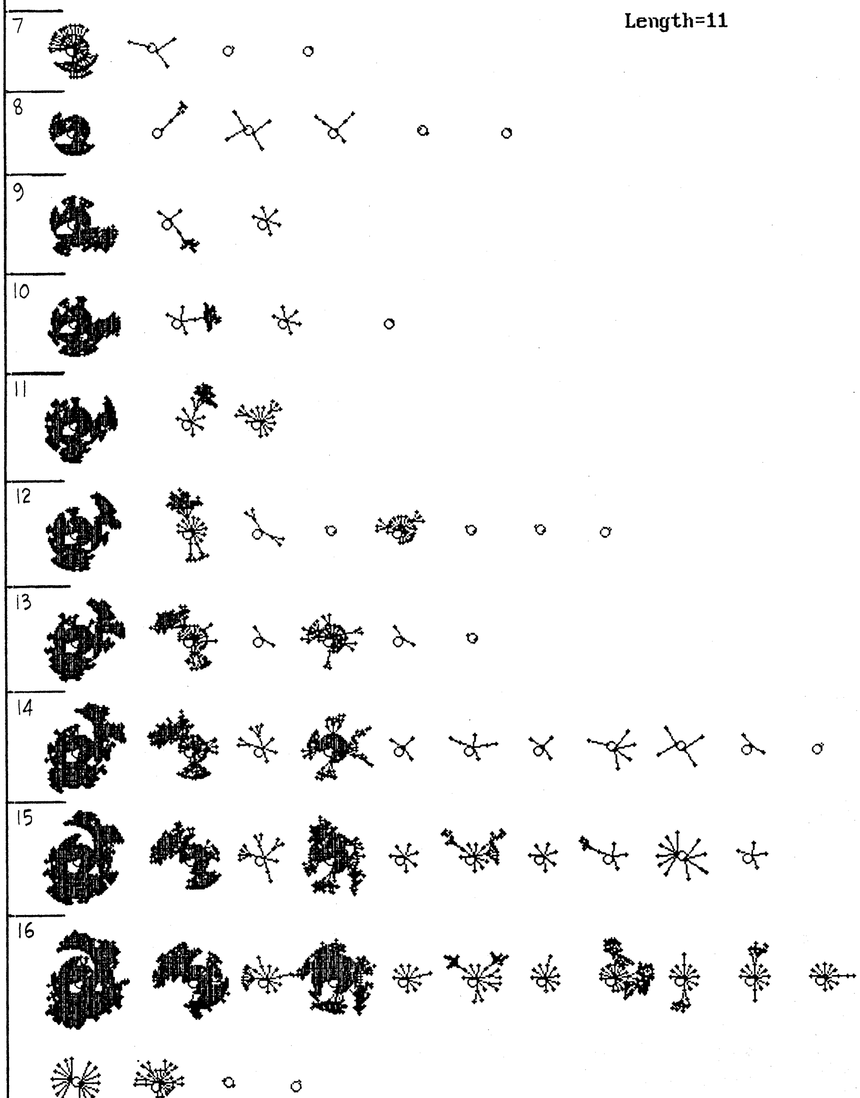






Length=11





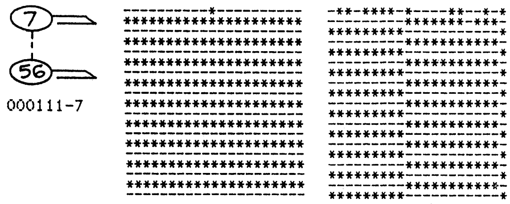

```
5-code  57 =111001

ty. at no(p)s g ml mp

L= 3
 1.111 1(1)8 6 2 5
 total NATs = 1
L= 4
 1.1111 1(1)10 8 2 6
 2.0011 2(2)2 0 0 1
 3.0101 2(1)1 0 0 1
 total NATs = 5
L= 5
 1.11111 1(1)32 30 2 17
 total NATs = 1
L= 6
 1.111111 1(1)38 36 2 23
 2.000001 3(2)2 0 0 1
 3.000111 6(1)1 0 0 1
 4.001011 6(2)2 0 0 1
 5.010101 2(1)1 0 0 1
 total NATs = 18
L= 7
 1.1111111 1(1)79 49 3 30
 2.0000001 7(2)5 3 1 3
 3.0000111 7(1)1 0 0 1
 4.0001111 7(1)1 0 0 1
 total NATs = 22
L= 8
 1.11111111 1(1)106 88 2 46
 2.01111100 8(1)9 5 4 4
 3.00000011 4(2)6 4 1 3
 4.00001111 8(1)6 3 2 4
 5.00110011 2(2)2 0 0 1
 6.01010101 2(1)1 0 0 1
 total NATs = 25
L= 9
 1.111111111 1(1)314 195 6 77
 2.011111100 9(1)16 11 5 6
 3.000011111 9(1)6 5 1 6
 total NATs = 19
L= 10
 1.1111111111 1(1)592 425 6 127
 2.0111111100 10(1)37 29 5 20
 3.0000111111 10(1)6 5 1 6
 4.0101010101 2(1)1 0 0 1
 total NATs = 23
L= 11
 1.11111111111 1(1)1267 891 5 211
 2.01111111100 11(1)53 43 5 20
 3.00001111111 11(1)18 15 2 12
 total NATs = 23
L= 12
 1.111111111111 1(1)2626 2024 6 339
 2.011111111100 12(1)92 70 6 27
 3.001110001110 6(1)7 4 2 3
 4.000001000001 3(2)2 0 0 1
 5.000011111111 12(1)25 22 2 19
 6.001011001011 6(2)2 0 0 1
 7.001100110011 2(2)2 0 0 1
 8.010101010101 2(1)1 0 0 1
 total NATs = 44
L= 13
 1.1111111111111 1(1)5189 4043 7 535
 2.0111111111100 13(1)169 136 6 44
 3.0011110001110 13(1)3 2 1 3
 4.1111100001111 13(1)54 43 3 32
 5.0011100001110 13(1)3 2 1 3
 6.0000001000001 13(2)2 0 0 1
 total NATs = 66
L= 14
 1.11111111111111 1(1)9935 8085 8 870
 2.01111111111100 14(1)305 250 7 70
 3.00111110001110 14(1)11 8 2 6
 4.11111100001111 14(1)122 94 6 56
 5.00111100001110 14(1)4 3 1 4
 6.00111100011110 7(1)7 4 2 5
 7.00111100011100 14(1)4 3 1 4
 8.00000010000001 7(2)7 5 1 5
 9.00000010000011 14(2)6 4 1 3
 10.00001110000111 7(1)3 2 1 3
 11.01010101010101 2(1)1 0 0 1
 total NATs = 108
L= 15
 1.111111111111111 1(1)19673 16266 7 1415
 2.011111111111100 15(1)513 445 6 126
 3.001111110001110 15(1)15 10 2 6
 4.111111100001111 15(1)266 216 6 86
 5.001111100001110 15(1)7 6 1 7
 6.001111100011110 15(1)35 25 5 10
 7.001111100011100 15(1)7 6 1 7
 8.011110000111100 15(1)12 7 4 5
 9.000000100000011 15(2)13 10 2 7
 10.000011100001111 15(1)5 4 1 5
 total NATs = 136
L= 16
 1.1111111111111111 1(1)38026 31848 8 2302
 2.0111111111111100 16(1)984 851 6 206
 3.0011111110001110 16(1)21 16 2 10
 4.1111111100001111 16(1)526 442 6 138
 5.0011111100001110 16(1)11 9 2 10
 6.0011111100011110 16(1)38 24 5 10
 7.0011111100011100 16(1)11 9 2 10
 8.0011111000111110 8(1)95 73 5 16
 9.0011110000111110 16(1)25 17 4 11
 10.0111110000111100 16(1)25 17 4 11
 11.0011100001111100 16(1)13 10 3 11
 12.0000001100000011 4(2)18 16 1 9
 13.0000111100001111 8(1)26 19 2 16
 14.0011001100110011 2(2)2 0 0 1
 15.0101010101010101 2(1)1 0 0 1
 total NATs = 169
```

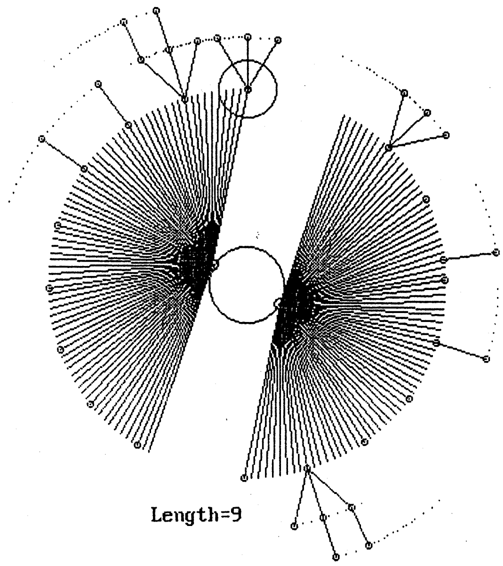
λ ratio = 1 $Z = .375$

minority rule

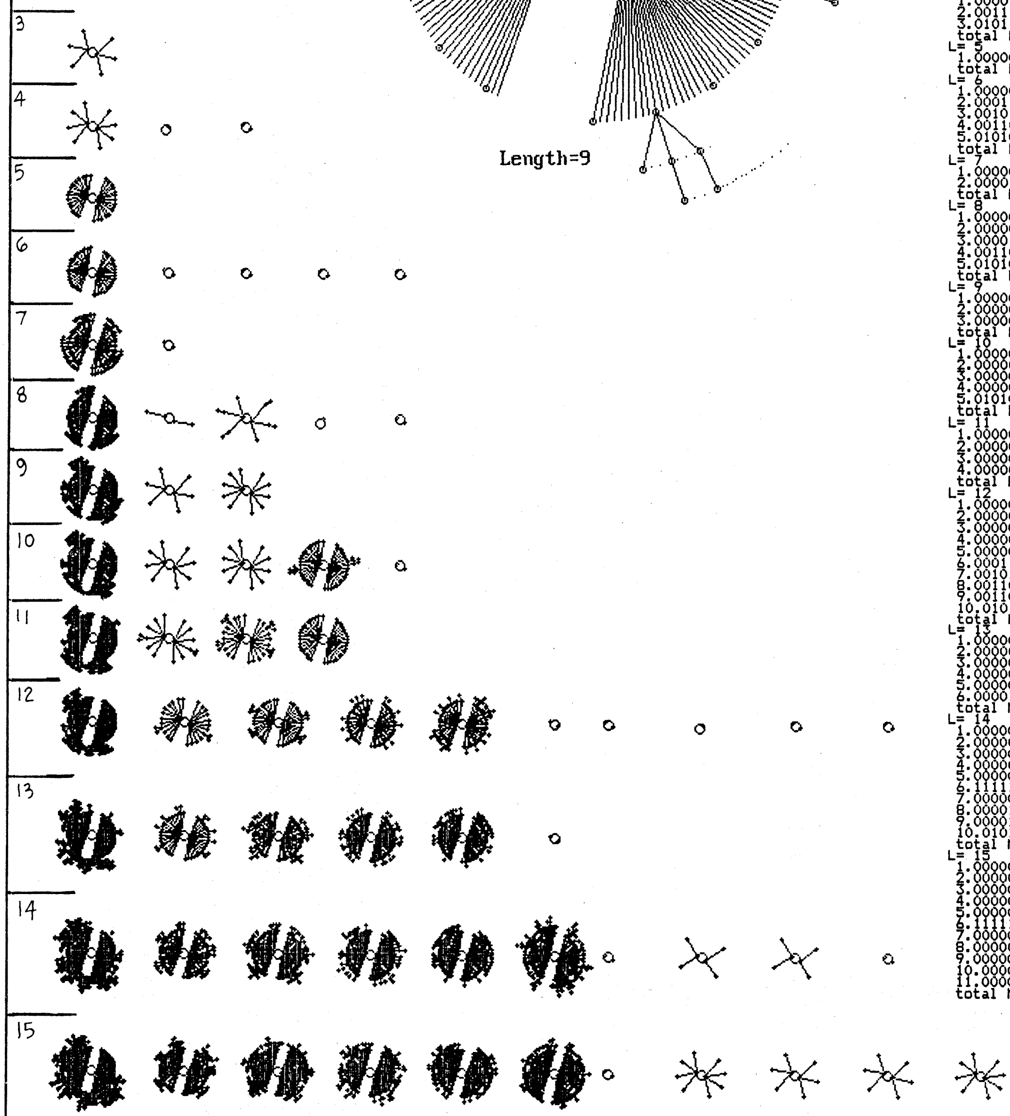
0000000100010111-0001011101111111-rule 18290559

= 5-code 7 -000111 Length=3 -15



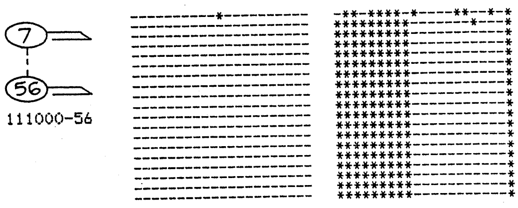


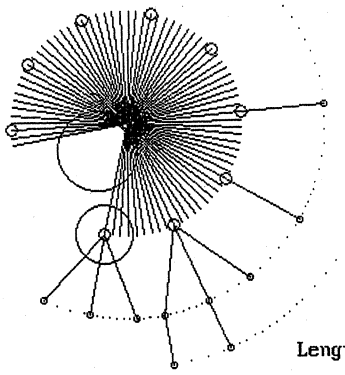

```
5-code  7 =000111

ty. at no(p)s g ml mp

L= 3
 1.000 1(2)8 6 1 4
 total NATs = 1
L= 4
 1.0000 1(2)10 8 1 5
 2.0011 4(1)1 0 0 1
 3.0101 1(2)2 0 0 1
 total NATs = 6
L= 5
 1.00000 1(2)32 30 1 16
 total NATs = 1
L= 6
 1.000000 1(2)44 42 1 22
 2.000111 3(2)2 0 0 1
 3.001011 3(2)2 0 0 1
 4.001101 3(2)2 0 0 1
 5.010101 1(2)2 0 0 1
 total NATs = 11
L= 7
 1.0000000 1(2)114 70 2 29
 2.0000111 7(2)2 0 0 1
 total NATs = 8
L= 8
 1.00000000 1(2)170 136 2 45
 2.00000111 8(2)4 2 1 2
 3.00001111 4(2)12 6 2 4
 4.00110011 4(1)1 0 0 1
 5.01010101 1(2)2 0 0 1
 total NATs = 18
L= 9
 1.000000000 1(2)332 222 3 76
 2.000000111 9(2)8 6 1 4
 3.000001111 9(2)12 10 1 6
 total NATs = 19
L= 10
 1.0000000000 1(2)552 430 3 126
 2.0000000111 10(2)10 8 1 5
 3.0000001111 10(2)12 10 1 6
 4.0000011111 5(2)50 44 3 20
 5.0101010101 1(2)2 0 0 1
 total NATs = 27
L= 11
 1.00000000000 1(2)1014 770 3 210
 2.00000000111 11(2)18 14 2 7
 3.00000001111 11(2)36 30 2 12
 4.00000011111 11(2)40 38 1 20
 total NATs = 34
L= 12
 1.000000000000 1(2)1756 1514 3 338
 2.000000000111 12(2)30 26 2 13
 3.000000001111 12(2)50 44 2 19
 4.000000011111 12(2)68 54 2 28
 5.000000111111 6(2)90 68 3 29
 6.000111000111 3(2)2 0 0 1
 7.001011001011 3(2)2 0 0 1
 8.001100110011 4(1)1 0 0 1
 9.001101001101 3(2)2 0 0 1
 10.010101010101 1(2)2 0 0 1
 total NATs = 57
L= 13
 1.0000000000000 1(2)3174 2626 4 534
 2.0000000000111 13(2)54 44 2 21
 3.0000000001111 13(2)96 82 3 32
 4.0000000011111 13(2)120 104 2 47
 5.0000000111111 13(2)114 96 2 39
 6.0000111000111 13(2)2 0 0 1
 total NATs = 66
L= 14
 1.00000000000000 1(2)5546 4690 4 869
 2.00000000000111 14(2)96 82 2 35
 3.00000000001111 14(2)164 148 3 56
 4.00000000011111 14(2)202 174 3 76
 5.00000000111111 14(2)182 162 2 62
 6.11111110000000 7(2)238 190 4 56
 7.00000111000111 14(2)2 0 0 1
 8.00001110000111 7(2)6 4 1 3
 9.00001110001111 14(2)6 4 1 3
 10.01010101010101 1(2)2 0 0 1
 total NATs = 100
L= 15
 1.000000000000000 1(2)9788 8466 5 1414
 2.000000000000111 15(2)168 146 3 57
 3.000000000001111 15(2)312 276 3 86
 4.000000000011111 15(2)364 328 3 136
 5.000000000111111 15(2)330 292 3 102
 6.111111110000000 15(2)320 278 3 88
 7.000000111000111 15(2)2 0 0 1
 8.000001110000111 15(2)10 8 1 5
 9.000001110001111 15(2)8 6 1 4
 10.000001111000111 15(2)8 6 1 4
 11.000011100001111 15(2)10 8 1 5
 total NATs = 151
```

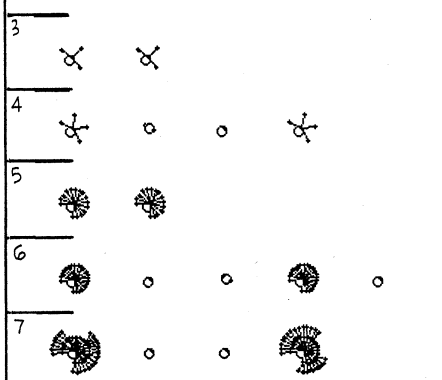


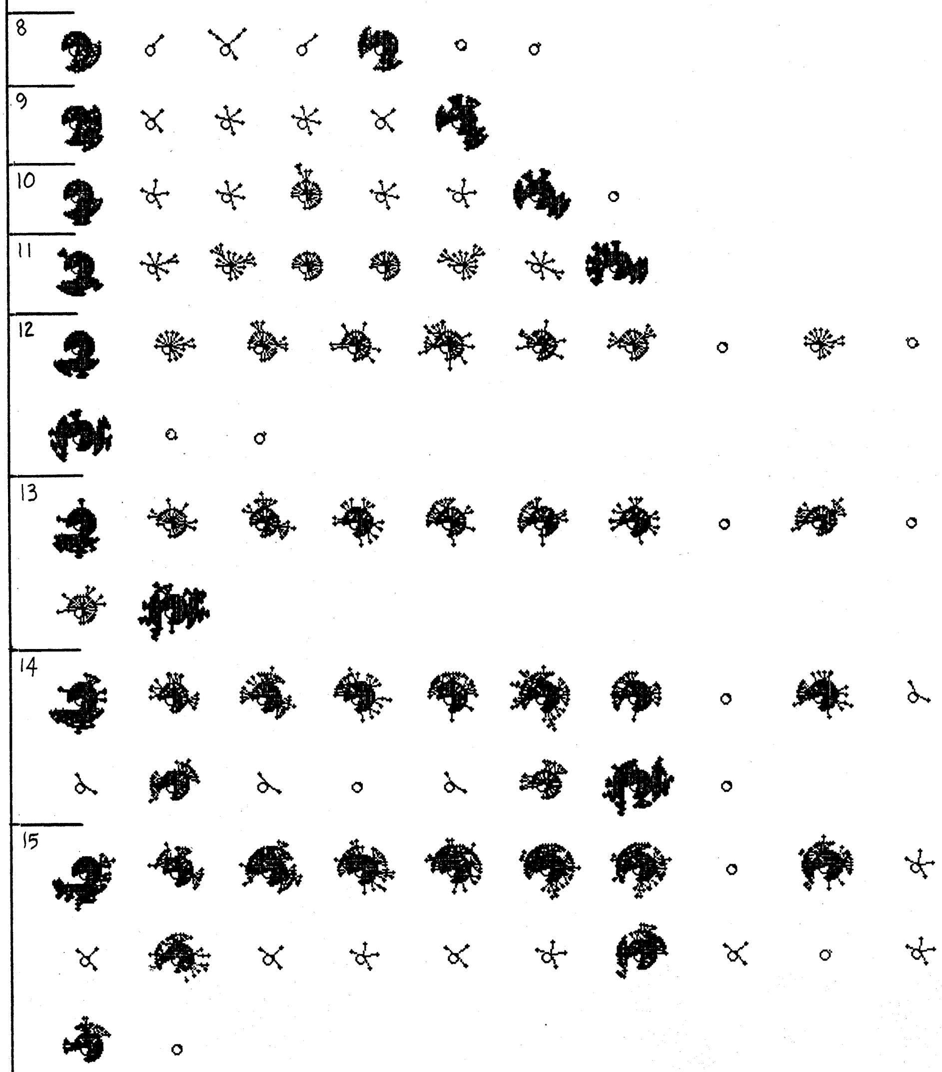
λ ratio = 1   Z = .375
majority rule

1111111011101000-1110100010000000-rule 4276676736
= 5-code 56 -111000 Length=3 -15





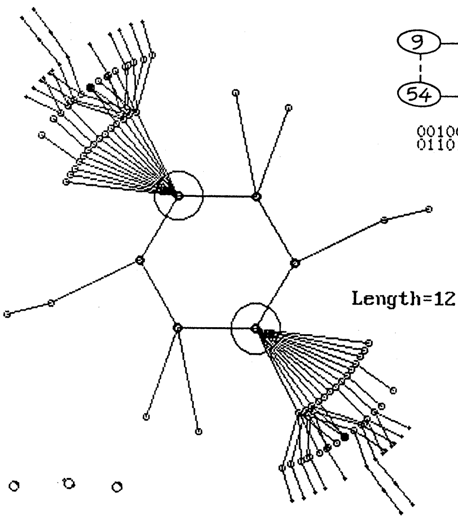


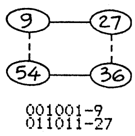


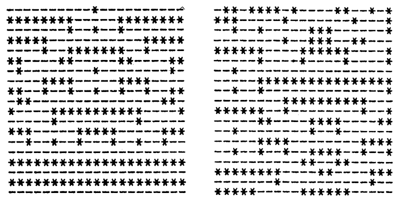

```
5-code  56 =111000

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)4 3 1 4
 2.111 1(1)4 3 1 4
 total NATs = 2
L= 4
 1.0000 1(1)5 4 1 5
 2.0011 2(2)2 0 0 1
 3.0101 2(1)1 0 0 1
 4.1111 1(1)5 4 1 5
 total NATs = 6
L= 5
 1.00000 1(1)16 15 1 16
 2.11111 1(1)16 15 1 16
 total NATs = 2
L= 6
 1.000000 1(1)22 21 1 22
 2.000111 6(1)1 0 0 1
 3.001011 6(2)2 0 0 1
 4.111111 1(1)22 21 1 22
 5.010101 2(1)1 0 0 1
 total NATs = 16
L= 7
 1.0000000 1(1)57 35 2 29
 2.0000111 7(1)1 0 0 1
 3.0001111 7(1)1 0 0 1
 4.1111111 1(1)57 35 2 29
 total NATs = 16
L= 8
 1.00000000 1(1)85 68 2 45
 2.00000111 8(1)2 1 1 2
 3.00001111 8(1)6 3 2 4
 4.00011111 8(1)2 1 1 2
 5.11111111 1(1)85 68 2 45
 6.00110011 2(2)2 0 0 1
 7.01010101 2(1)1 0 0 1
 total NATs = 30
L= 9
 1.000000000 1(1)166 111 3 76
 2.000000111 9(1)4 3 1 4
 3.000001111 9(1)6 5 1 6
 4.000011111 9(1)6 5 1 6
 5.000111111 9(1)4 3 1 4
 6.111111111 1(1)166 111 3 76
 total NATs = 38
L= 10
 1.0000000000 1(1)276 215 3 126
 2.0000000111 10(1)5 4 1 5
 3.0000001111 10(1)6 5 1 6
 4.0000011111 10(1)25 22 3 20
 5.0000111111 10(1)6 5 1 6
 6.0001111111 10(1)5 4 1 5
 7.1111111111 1(1)276 215 3 126
 8.0101010101 2(1)1 0 0 1
 total NATs = 54
L= 11
 1.00000000000 1(1)507 385 3 210
 2.00000000111 11(1)9 7 2 7
 3.00000001111 11(1)18 15 2 12
 4.00000011111 11(1)20 19 1 20
 5.00000111111 11(1)20 19 1 20
 6.00001111111 11(1)18 15 2 12
 7.00011111111 11(1)9 7 2 7
 8.11111111111 1(1)507 385 3 210
 total NATs = 68
L= 12
 1.000000000000 1(1)878 757 3 338
 2.000000000111 12(1)15 13 2 13
 3.000000001111 12(1)25 22 2 19
 4.000000011111 12(1)34 27 2 28
 5.000000111111 12(1)45 34 3 29
 6.000001111111 12(1)34 27 2 28
 7.000011111111 12(1)25 22 2 19
 8.000111000111 6(1)1 0 0 1
 9.000111111111 12(1)15 13 2 13
 10.001011001011 6(2)2 0 0 1
 11.111111111111 1(1)878 757 3 338
 12.001100110011 2(2)2 0 0 1
 13.010101010101 2(1)1 0 0 1
 total NATs = 102
L= 13
 1.0000000000000 1(1)1587 1313 4 534
 2.0000000000111 13(1)27 22 2 21
 3.0000000001111 13(1)48 41 3 32
 4.0000000011111 13(1)60 52 2 47
 5.0000000111111 13(1)57 48 2 39
 6.0000001111111 13(1)57 48 2 39
 7.0000011111111 13(1)60 52 2 47
 8.0000111000111 13(1)1 0 0 1
 9.0000111111111 13(1)48 41 3 32
 10.0001110001111 13(1)1 0 0 1
 11.0001111111111 13(1)27 22 2 21
 12.1111111111111 1(1)1587 1313 4 534
 total NATs = 132
L= 14
 1.00000000000000 1(1)2773 2345 4 869
 2.00000000000111 14(1)48 41 2 35
 3.00000000001111 14(1)82 74 3 56
 4.00000000011111 14(1)101 87 3 76
 5.00000000111111 14(1)91 81 2 62
 6.00000001111111 14(1)119 95 4 56
 7.00000011111111 14(1)91 81 2 62
 8.00000111000111 14(1)1 0 0 1
 9.00000111111111 14(1)101 87 3 76
 10.00001110000111 7(1)3 2 1 3
 11.00001110001111 14(1)3 2 1 3
 12.00001111111111 14(1)82 74 3 56
 13.00001111000111 14(1)3 2 1 3
 14.00011100011111 14(1)1 0 0 1
 15.00011110001111 7(1)3 2 1 3
 16.00011111111111 14(1)48 41 2 35
 17.11111111111111 1(1)2773 2345 4 869
 18.01010101010101 2(1)1 0 0 1
 total NATs = 200
```

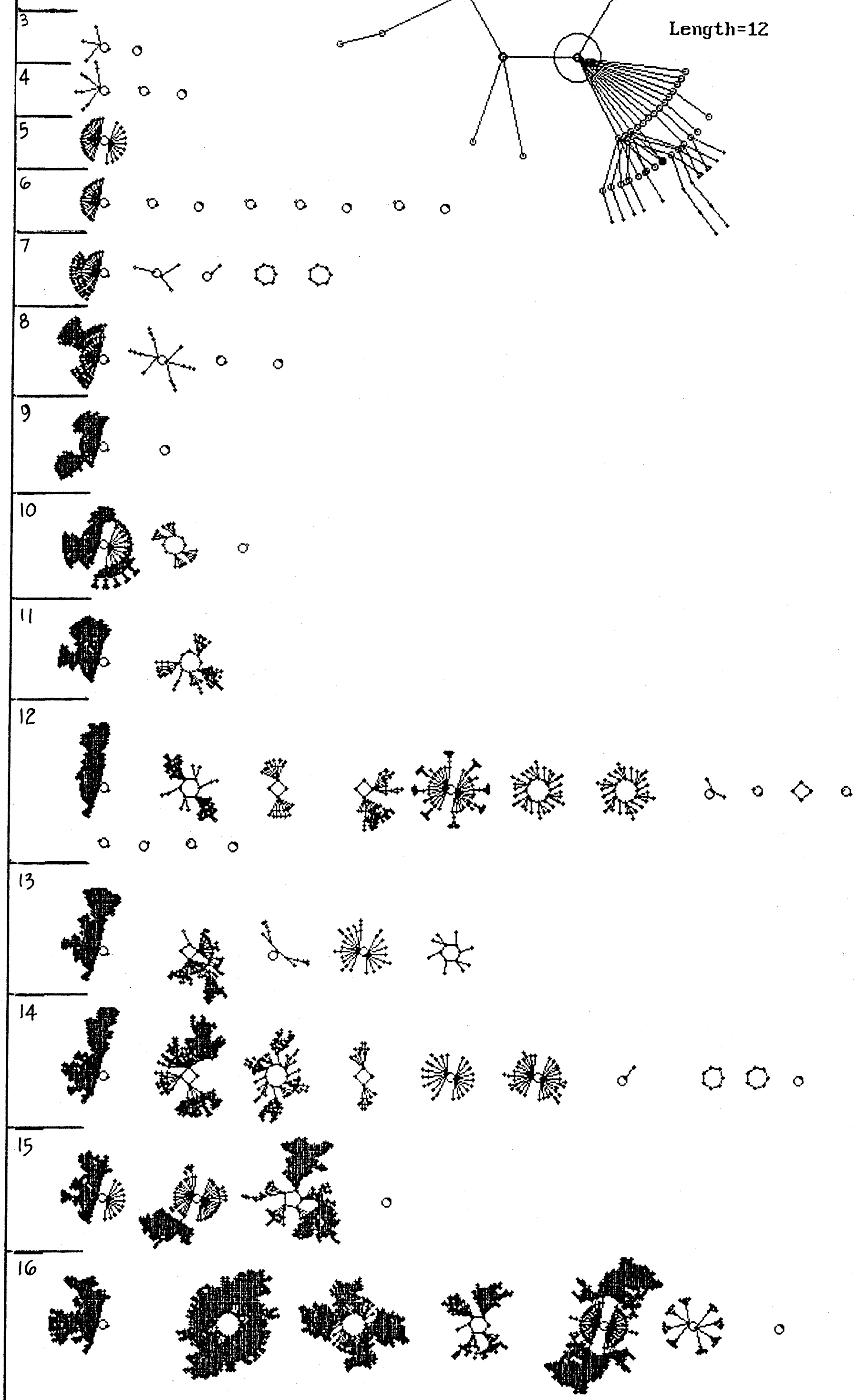
λ ratio = .6875 Z = .6875

0001011001101000-0110100010000001-rule 375941249
= 5-code 9 -001001
Length=3 -16









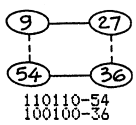
5-code 9 =001001

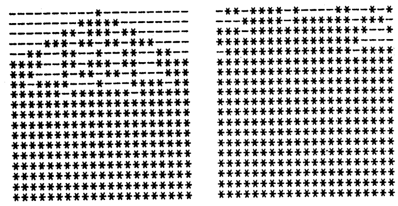
ty. at no(p)s g ml mp

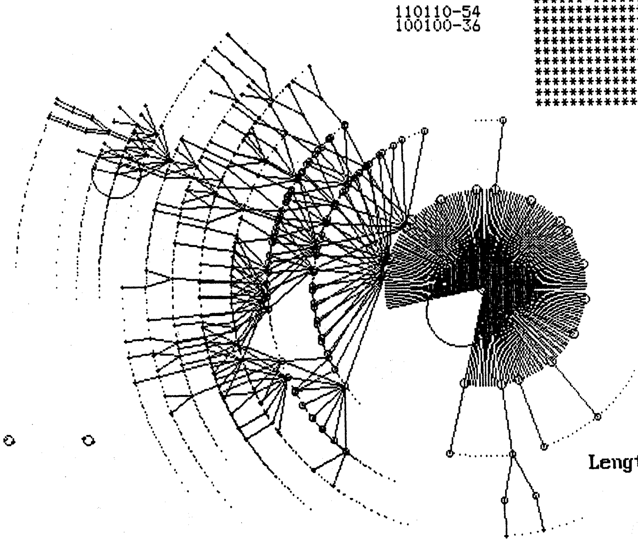
L= 3
1.000 1(2)5 3 1 4
2.011 3(1)1 0 0 1
total NATs = 4
L= 4
1.0000 1(2)10 4 2 5
2.0011 2(2)2 0 0 1
3.0101 2(1)1 0 0 1
total NATs = 5
L= 5
1.00000 1(2)32 30 1 21
total NATs = 1
L= 6
1.000000 1(2)23 21 1 22
2.000001 3(2)2 0 0 1
3.000111 6(1)1 0 0 1
4.001011 6(2)2 0 0 1
5.001111 3(2)2 0 0 1
6.010101 2(1)1 0 0 1
7.010111 3(2)2 0 0 1
8.011011 3(1)1 0 0 1
total NATs = 27
L= 7
1.0000000 1(2)65 28 3 22
2.0000001 7(2)5 3 1 3
3.0000111 7(1)2 1 1 2
4.0010111 1(7)7 0 0 1
5.0011101 1(7)7 0 0 1
total NATs = 17
L= 8
1.00000000 1(2)186 76 8 29
2.00000011 4(2)16 6 3 4
3.00110011 2(2)2 0 0 1
4.01010101 2(1)1 0 0 1
total NATs = 9
L= 9
1.000000000 1(2)509 252 9 49
2.011011011 3(1)1 0 0 1
total NATs = 4
L= 10
1.0000000000 1(2)902 670 6 91
2.0000000011 5(8)24 16 1 5
3.0101010101 2(1)1 0 0 1
total NATs = 8
L= 11
1.00000000000 1(2)1234 770 7 122
2.00111110000 11(8)74 38 5 11
total NATs = 12
L= 12
1.000000000000 1(2)1759 979 13 170
2.000000000001 6(6)114 64 5 15
3.001111100000 6(4)30 16 2 8
4.000000000111 12(4)76 37 5 9
5.000010000010 3(2)98 68 4 15
6.000000100111 2(12)42 24 2 4
7.000000111001 2(12)42 24 2 4
8.000111000111 6(1)3 2 1 3
9.001011001011 6(2)2 0 0 1
10.001011001111 12(4)4 0 0 1
11.001100110011 2(2)2 0 0 1
12.001111001111 3(2)2 0 0 1
13.010101010101 2(1)1 0 0 1
14.010111010111 3(2)2 0 0 1
15.011011011011 3(1)1 0 0 1
total NATs = 69
L= 13
1.0000000000000 1(2)5163 2808 13 235
2.0000000000111 13(4)175 97 11 16
3.0011100001110 13(1)13 6 4 3
4.0000001000001 13(2)31 19 3 12
5.0000001000111 13(6)14 8 1 3
total NATs = 53
L= 14
1.00000000000000 1(2)6253 3710 15 358
2.00111111111000 14(4)591 342 12 38
3.00000000001001 7(10)130 72 7 9
4.00111000000111 7(4)28 16 3 5
5.00000010000001 7(2)27 19 2 11
6.00000010000011 14(2)38 28 3 14
7.00001110000111 7(1)2 1 1 2
8.00101110010111 1(7)7 0 0 1
9.00111010011101 1(7)7 0 0 1
10.01010101010101 2(1)1 0 0 1
total NATs = 61
L= 15
1.000000000000000 1(2)2120 1473 7 579
2.001100000011000 15(2)800 585 14 49
3.001111111110000 15(5)1243 878 14 56
4.011011011011011 3(1)1 0 0 1
total NATs = 34
L= 16
1.0000000000000000 1(2)4474 2860 11 845
2.0000000000000001 8(24)3434 2208 13 80
3.0000000000000011 8(16)1480 990 10 56
4.0011111111100000 16(6)347 218 7 46
5.0110000001100000 4(2)4028 2794 17 70
6.0011001100110011 2(2)42 32 2 5
7.0101010101010101 2(1)1 0 0 1
total NATs = 41

λ ratio = .6875 Z = .6875

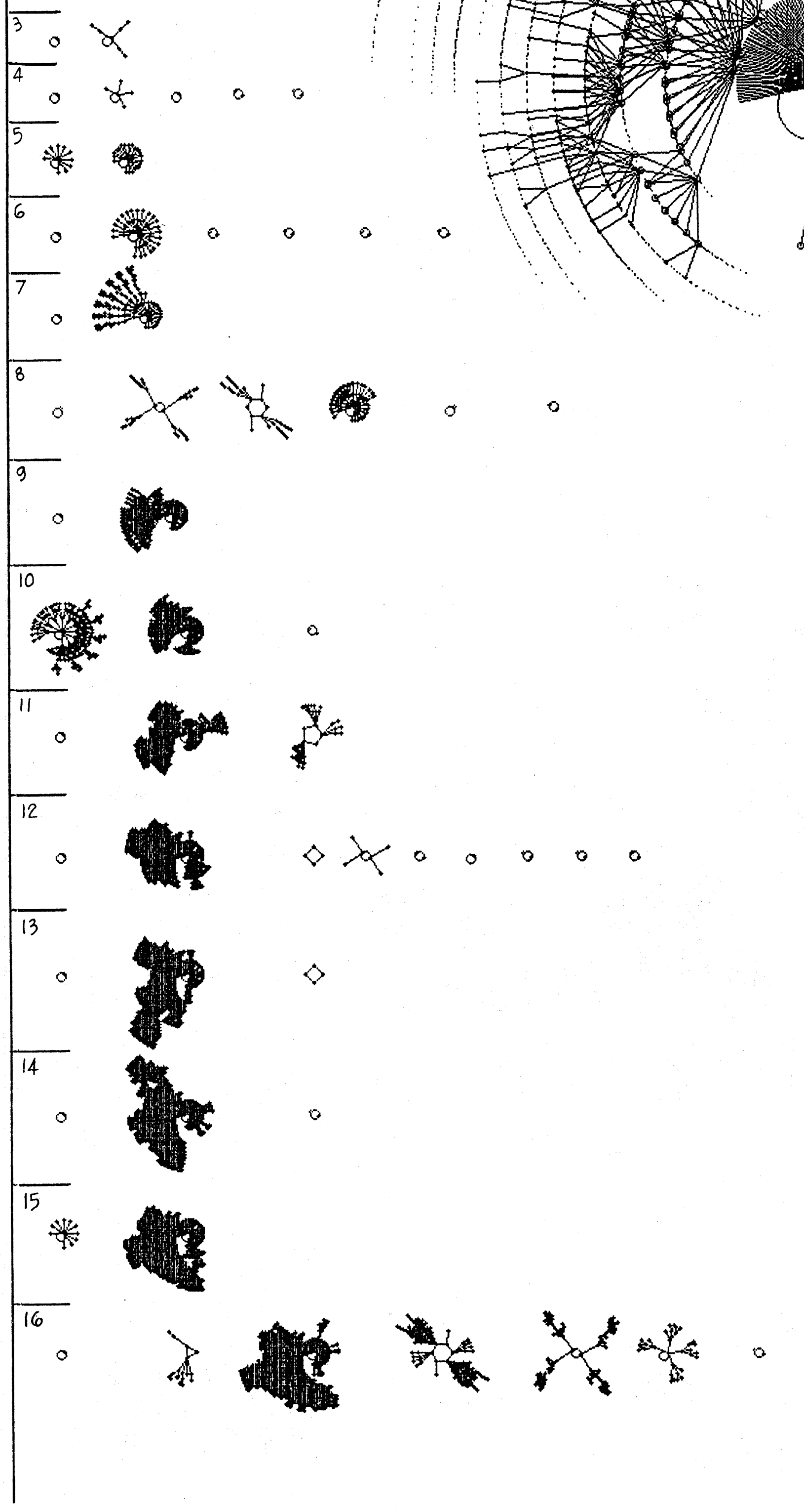
1110100110010111-1001011101111110-rule 3919026046
= 5-code 54 -110110
Length=3-16







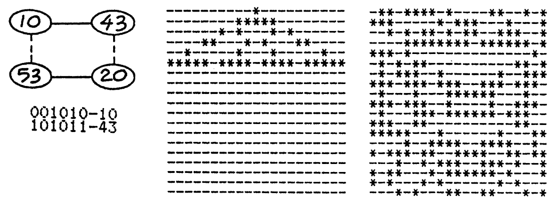
Length=12

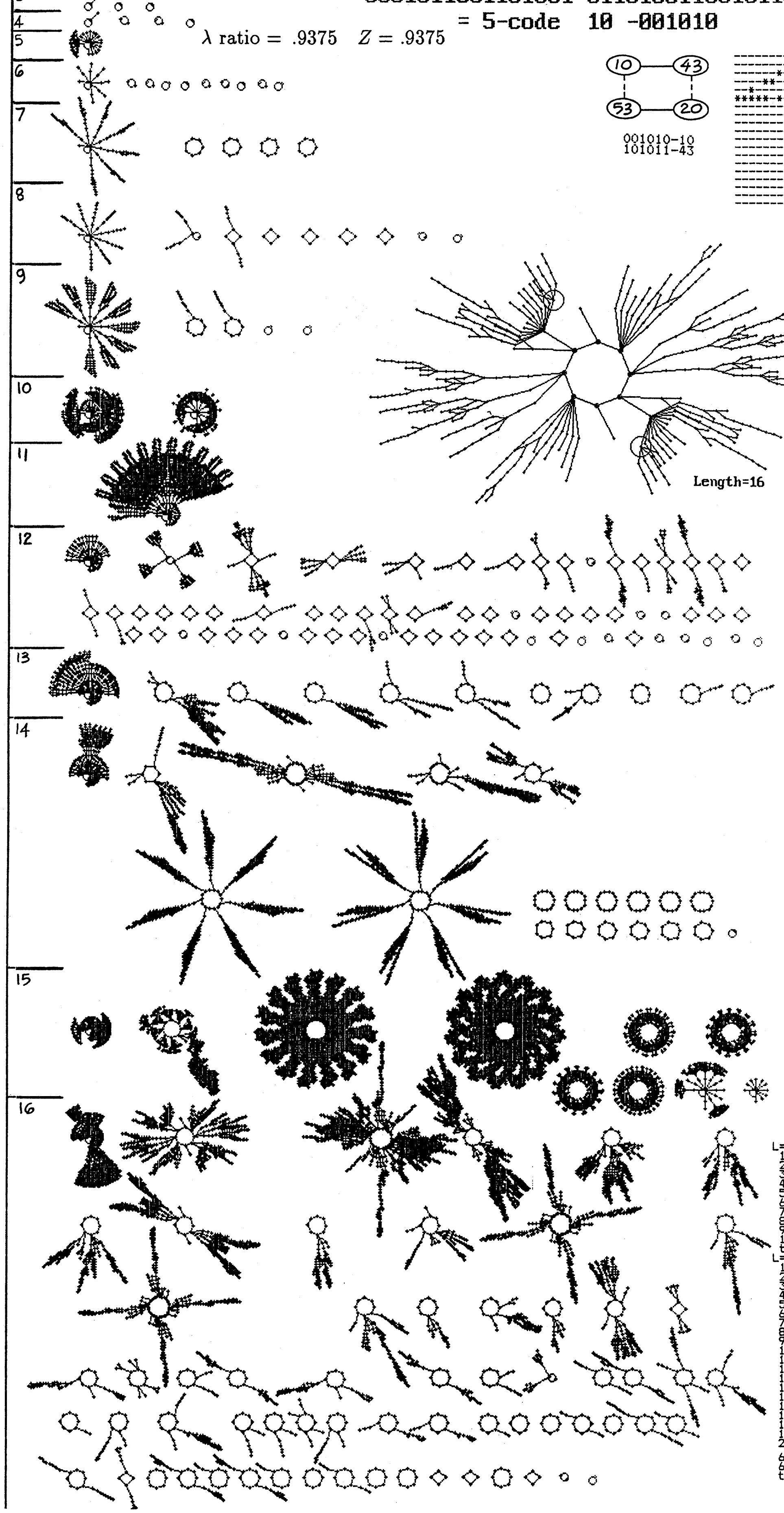



```
5-code  54 =110110

ty. at no(p)s g ml mp
L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)7 3 2 4
 total NATs = 2
L= 4
 1.0000 1(1)1 0 0 1
 2.1111 1(1)5 4 1 5
 3.0011 4(1)1 0 0 1
 4.0101 1(2)2 0 0 1
 5.0111 2(2)2 0 0 1
 total NATs = 9
L= 5
 1.00000 1(1)11 10 1 11
 2.11111 1(1)21 20 1 21
 total NATs = 2
L= 6
 1.000000 1(1)1 0 0 1
 2.111111 1(1)43 21 2 22
 3.000111 3(2)2 0 0 1
 4.001011 3(2)2 0 0 1
 5.001101 3(2)2 0 0 1
 6.010101 1(2)2 0 0 1
 total NATs = 12
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111111 1(1)127 56 9 22
 total NATs = 2
L= 8
 1.00000000 1(1)1 0 0 1
 2.10111011 2(2)30 8 5 3
 3.11001111 4(6)30 8 5 4
 4.11111111 1(1)69 52 2 29
 5.00110011 4(1)1 0 0 1
 6.01010101 1(2)2 0 0 1
 total NATs = 13
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111111 1(1)511 252 9 49
 total NATs = 2
L= 10
 1.0000000000 1(1)191 120 6 12
 2.1111111111 1(1)831 630 6 91
 3.0101010101 1(2)2 0 0 1
 total NATs = 3
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111111 1(1)1541 869 10 122
 3.00000000011 11(5)46 29 3 11
 total NATs = 13
L= 12
 1.000000000000 1(1)1 0 0 1
 2.111111111111 1(1)4007 2203 13 170
 3.000110110111 12(4)4 0 0 1
 4.000111000111 3(2)6 4 1 3
 5.001011001011 3(2)2 0 0 1
 6.001100110011 4(1)1 0 0 1
 7.001101001101 3(2)2 0 0 1
 8.010101010101 1(2)2 0 0 1
 9.011101110111 2(2)2 0 0 1
 total NATs = 30
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.1111111111111 1(1)8139 4498 22 235
 3.0000010111011 13(4)4 0 0 1
 total NATs = 15
L= 14
 1.00000000000000 1(1)1 0 0 1
 2.11111111111111 1(1)16381 9646 19 358
 3.01010101010101 1(2)2 0 0 1
 total NATs = 3
L= 15
 1.000000000000000 1(1)11 10 1 11
 2.111111111111111 1(1)32757 23408 14 579
 total NATs = 2
L= 16
 1.0000000000000000 1(1)1 0 0 1
 2.1011000000011011 16(3)18 9 3 5
 3.1111111111111111 1(1)63957 42324 19 845
 4.1001111110011111 4(6)226 116 8 26
 5.1101110111011101 2(2)126 80 8 6
 6.1100110011001100 4(1)33 20 3 5
 7.0101010101010101 1(2)2 0 0 1
 total NATs = 29
```

0001011001101001-0110100110010110-rule 376007062
= 5-code 10 -001010
Length=3 -16

λ ratio = .9375 Z = .9375

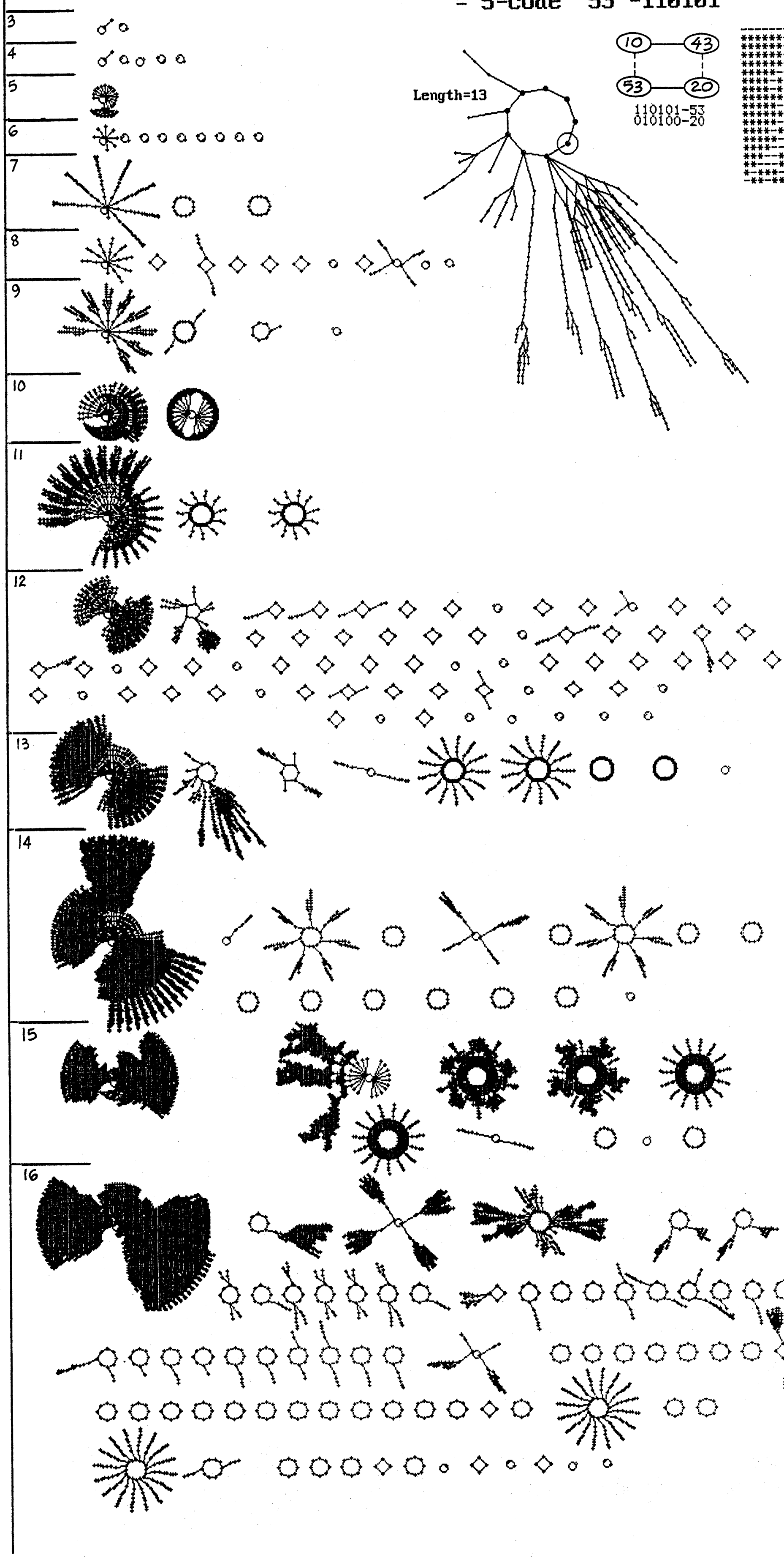


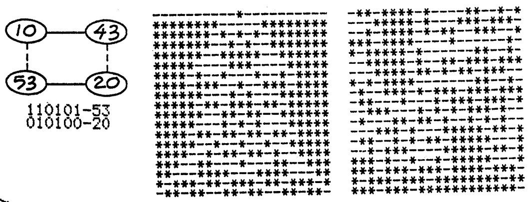



```
5-code  10 =001010

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)2 1 1 2
 2.001 3(1)1 0 0 1
 3.011 3(1)1 0 0 1
 total NATs = 7
L= 4
 1.0000 1(1)2 1 1 2
 2.0001 4(2)2 0 0 1
 3.0011 2(2)2 0 0 1
 4.0101 2(1)1 0 0 1
 total NATs = 9
L= 5
 1.00000 1(1)32 30 2 17
 total NATs = 1
L= 6
 1.000000 1(1)14 7 2 8
 2.000011 3(2)2 0 0 1
 3.000101 3(2)2 0 0 1
 4.000111 6(1)1 0 0 1
 5.001001 3(1)1 0 0 1
 6.001011 6(2)2 0 0 1
 7.001111 3(2)2 0 0 1
 8.010101 2(1)1 0 0 1
 9.010111 3(2)2 0 0 1
 10.011011 3(1)1 0 0 1
 total NATs = 33
L= 7
 1.0000000 1(1)100 15 12 9
 2.0001011 1(7)7 0 0 1
 3.0001101 1(7)7 0 0 1
 4.0010111 1(7)7 0 0 1
 5.0011101 1(7)7 0 0 1
 total NATs = 5
L= 8
 1.00000000 1(1)50 9 6 10
 2.01000100 4(2)10 2 4 3
 3.01111000 4(4)12 2 4 2
 4.00000101 4(4)4 0 0 1

 8.00110011 2(2)2 0 0 1
 9.01010101 2(1)1 0 0 1
 total NATs = 41
L= 9
 1.000000000 1(1)254 37 12 11
 2.000001011 9(7)14 1 7 2
 3.000001101 9(7)14 1 7 2
 4.001001001 3(1)1 0 0 1
 5.011011011 3(1)1 0 0 1
 total NATs = 25
L= 10
 1.0000000000 1(1)712 550 5 27
 2.1010101010 2(1)156 130 3 13
 total NATs = 3
L= 11
 1.00000000000 1(1)2048 342 26 24
 total NATs = 1
L= 12
 1.000000000000 1(1)62 31 3 32
 2.010100010100 3(2)74 24 4 5
 3.000111100000 6(4)62 20 7 4
 4.000000000101 6(4)32 8 4 4
 5.000000000111 12(4)13 3 4 3

 65.010111010111 3(2)2 0 0 1
 66.011011011011 3(1)1 0 0 1
 total NATs = 525
L= 13
 1.0000000000000 1(1)223 79 8 41
 2.0101000010100 13(9)259 50 24 5
 3.0011101011000 13(9)100 12 26 3
 4.1010111000001 13(9)100 12 26 3
 5.0101001000000 13(9)44 6 18 3
 6.0000000100101 13(9)44 6 18 3
 7.0000000101111 13(9)9 0 0 1
 8.0000000110011 13(9)22 4 9 3
 9.0000000111101 13(9)9 0 0 1
 10.0000011001111 13(9)13 1 4 2
 11.0000011110011 13(9)13 1 4 2
 total NATs = 131
L= 14
 1.00000000000000 1(1)268 92 12 51
 2.01000000000100 14(6)224 35 31 6
 3.01000000001000 7(36)594 98 53 5
 4.11011011011000 14(14)262 49 42 5
 5.01000000101001 7(8)100 22 11 5
 6.01010000000100 2(14)1057 182 34 5
 7.00000101000100 2(14)1057 182 34 5
 8.00000011001111 2(14)14 0 0 1

 19.00111010011101 1(7)7 0 0 1
 20.01010101010101 2(1)1 0 0 1
 total NATs = 69
L= 15
 1.000000000000000 1(1)302 285 2 77
 2.101000000001010 15(8)1336 1031 27 19
 3.011001110001111 1(75)5490 4110 16 14
 4.000111001101111 1(75)5490 4110 16 14
 5.010000111011001 1(15)225 165 2 12
 6.110000101001101 1(15)225 165 2 12
 7.010001111010110 1(15)225 165 2 12
 8.110001001101011 1(15)225 165 2 12
 9.110110110110110 3(1)171 135 4 13
 10.010010010010010 3(1)11 10 1 11
 total NATs = 28
L= 16
 1.0000000000000000 1(1)690 185 12 90
 2.0101000000010100 8(8)482 116 18 13
 3.0100000000001000 8(22)1430 352 39 10
 4.0100000000010101 8(8)1232 304 34 8
 5.0000000000000111 16(8)302 76 19 8
 6.0000000000001011 16(8)182 41 21 6
 7.0000000000001101 16(8)182 41 21 6
 8.0011000100000010 8(8)570 130 25 7
 9.0000000000010101 16(8)100 21 18 4
 10.0000000000011011 16(8)68 14 11 6
 11.0000000000100101 4(40)364 80 28 6
 12.0000000000100111 16(8)118 21 35 5
 13.0000000000101001 4(40)364 80 28 6
 14.0000000000111001 16(8)118 21 35 5
 15.0000000001010111 16(8)27 6 9 4
 16.0111011101100011 16(8)71 20 13 6
 17.0000000001110101 16(8)27 6 9 4
 18.0000000001110111 8(8)182 42 12 4
 19.0100000101000001 4(4)16 8 2 3
 20.0000000010001011 16(8)55 12 16 3

 63.0011001100110011 2(2)2 0 0 1
 64.0101010101010101 2(1)1 0 0 1
 total NATs = 769
```

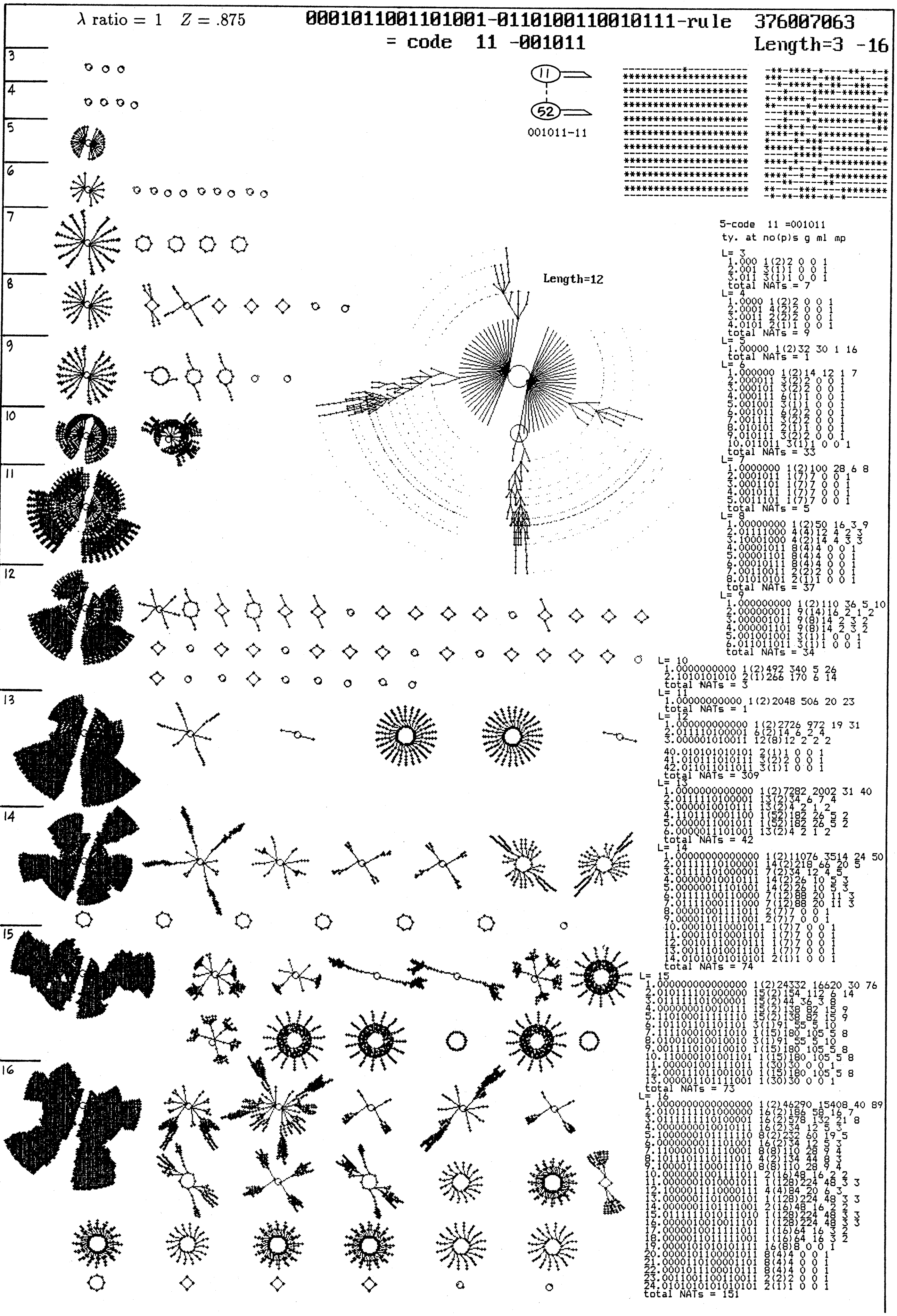
λ ratio = .9375 Z = .9375

11101001100101110-10010110011101001-rule 3918960233
= 5-code 53 -110101
Length=3 -6

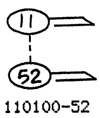


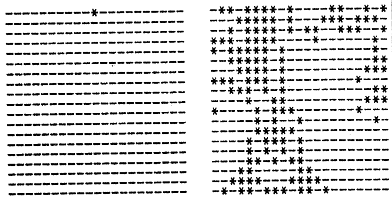


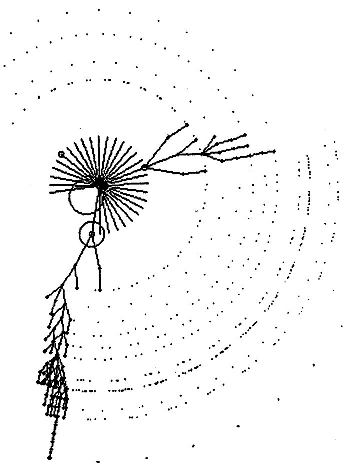

```
5-code  53 =110101
ty. at no(p)s g ml mp
L= 3
 1.111 1(1)2 1 1 2
 2.001 3(2)2 0 0 1
 total NATs = 4
L= 4
 1.1111 1(1)2 1 1 2
 2.0001 2(2)2 0 0 1
 3.0011 4(1)1 0 0 1
 4.0101 1(2)2 0 0 1
 5.0111 2(2)2 0 0 1
 total NATs = 10
L= 5
 1.11111 1(1)32 30 2 17
 total NATs = 1
L= 6
 1.111111 1(1)8 7 1 8
 2.000001 3(2)2 0 0 1
 3.000011 6(2)2 0 0 1
 4.000101 6(2)2 0 0 1
 5.000111 3(2)2 0 0 1
 6.001001 3(2)2 0 0 1
 7.001011 3(2)2 0 0 1
 8.001101 3(2)2 0 0 1
 9.010101 1(2)2 0 0 1
 total NATs = 29
L= 7
 1.1111111 1(1)100 15 13 9
 2.0001011 1(14)14 0 0 1
 3.0001101 1(14)14 0 0 1
 total NATs = 3
L= 8
 1.11111111 1(1)26 9 3 10
 2.00000001 8(4)4 0 0 1
 3.10000111 4(4)12 2 4 2
 4.00000101 4(4)4 0 0 1
 5.00001011 8(4)4 0 0 1
 6.00001101 8(4)4 0 0 1
 7.00010001 2(2)2 0 0 1
 8.00010111 8(4)4 0 0 1
 9.11101110 2(2)14 4 3 3
 10.00110011 4(1)1 0 0 1
 11.01010101 1(2)2 0 0 1
 total NATs = 50
L= 9
 1.111111111 1(1)182 28 11 11
 2.000001011 9(22)28 2 3 2
 3.000011111 9(7)8 1 1 2
 4.001001001 3(2)2 0 0 1
 total NATs = 22
L= 10
 1.1111111111 1(1)742 550 7 27
 2.1010101010 1(2)282 260 2 13
 total NATs = 2
L= 11
 1.11111111111 1(1)1762 320 22 24
 2.10111001001 1(121)143 11 2 2
 3.00111011001 1(121)143 11 2 2
 total NATs = 3
L= 12
 1.111111111111 1(1)476 187 8 32
 2.010111110101 12(5)111 34 8 5
 3.000000001011 12(4)8 1 4 2

 65.010101010101 1(2)2 0 0 1
 66.011101110111 2(2)2 0 0 1
 total NATs = 529
L= 13
 1.1111111111111 1(1)2342 443 17 41
 2.0000000000001 13(9)359 50 35 5
 3.0001000011111 13(6)30 10 6 2
 4.0111110110100 13(2)14 2 6 2
 5.0000001000101 1(143)208 13 5 2
 6.0000001010001 1(143)208 13 5 2
 7.0000001011111 1(91)91 0 0 1
 8.0000001111101 1(91)91 0 0 1
 9.0000101111101 13(1)1 0 0 1
 total NATs = 57
L= 14
 1.11111111111111 1(1)14730 2626 43 51
 2.01000001011111 14(1)6 1 5 2
 3.11001100000011 2(14)112 14 10 2
 4.00000001000101 2(14)14 0 0 1
 5.01111110110100 14(2)64 10 13 3
 6.00000001010001 2(14)14 0 0 1
 7.01100111100000 2(14)112 14 10 2
 8.00000100010101 2(14)14 0 0 1
 9.00000101010001 2(14)14 0 0 1
 10.00000110110111 2(14)14 0 0 1
 11.00000111011011 2(14)14 0 0 1
 12.00001001111011 1(14)14 0 0 1
 13.00001101111001 1(14)14 0 0 1
 14.00010110001011 1(14)14 0 0 1
 15.00011010001101 1(14)14 0 0 1
 16.01010101010101 1(2)2 0 0 1
 total NATs = 50
L= 15
 1.111111111111111 1(1)10412 7470 17 77
 2.110110110110110 3(2)5227 4175 34 19
 3.101111101101100 3(55)925 730 7 14
 4.101111101001101 3(55)925 730 7 14
 5.101111000100110 1(30)435 330 5 12
 6.001111010110010 1(30)435 330 5 12
 7.101101111111000 15(2)14 2 6 2
 8.000010011111011 1(15)15 0 0 1
 9.000010111111101 15(1)1 0 0 1
 10.000011011111001 1(15)15 0 0 1
 total NATs = 44
L= 16
 1.1111111111111111 1(1)36010 8585 40 90
 2.0100101110100100 16(8)727 164 23 13
 3.0111011101110111 2(2)606 148 14 8
 4.0110101101111111 8(26)674 192 19 6
 5.0011110110010100 16(8)39 9 10 4
 6.1101111000010100 16(8)39 9 10 4
 7.0000000010011001 8(8)16 4 2 3

 71.0011001100110011 4(1)1 0 0 1
 72.0101010101010101 1(2)2 0 0 1
 total NATs = 860
```


λ ratio = 1   Z = .875
0001011001101001-0110100110010111-rule 376007063
= code 11 -001011
Length=3 -16
3
4
5
6
7
8
9
10
11
12
13
14
15
16
11
52
001011-11
Length=12
5-code 11 =001011
ty. at no(p)s g ml mp
L= 3
1.000 1(2)2 0 0 1
2.001 3(1)1 0 0 1
3.011 3(1)1 0 0 1
total NATs = 7
L= 4
1.0000 1(2)2 0 0 1
2.0001 4(2)2 0 0 1
3.0011 2(2)2 0 0 1
4.0101 2(1)1 0 0 1
total NATs = 9
L= 5
1.00000 1(2)32 30 1 16
total NATs = 1
L= 6
1.000000 1(2)14 12 1 7
2.000011 3(2)2 0 0 1
3.000101 3(2)2 0 0 1
4.000111 6(1)1 0 0 1
5.001001 3(1)1 0 0 1
6.001011 6(2)2 0 0 1
7.001111 3(2)2 0 0 1
8.010101 2(1)1 0 0 1
9.010111 3(2)2 0 0 1
10.011011 3(1)1 0 0 1
total NATs = 33
L= 7
1.0000000 1(2)100 28 6 8
2.0001011 1(7)7 0 0 1
3.0001101 1(7)7 0 0 1
4.0010111 1(7)7 0 0 1
5.0011101 1(7)7 0 0 1
total NATs = 5
L= 8
1.00000000 1(2)50 16 3 9
2.01111000 4(4)12 4 2 3
3.10001000 4(2)14 4 3 3
4.00001011 8(4)4 0 0 1
5.00001101 8(4)4 0 0 1
6.00010111 8(4)4 0 0 1
7.00110011 2(2)2 0 0 1
8.01010101 2(1)1 0 0 1
total NATs = 37
L= 9
1.000000000 1(2)110 36 5 10
2.000000011 9(14)16 2 1 2
3.000001011 9(8)14 2 3 2
4.000001101 9(8)14 2 3 2
5.001001001 3(1)1 0 0 1
6.011011011 3(1)1 0 0 1
total NATs = 34
L= 10
1.0000000000 1(2)492 340 5 26
2.1010101010 2(1)266 170 6 14
total NATs = 3
L= 11
1.00000000000 1(2)2048 506 20 23
total NATs = 1
L= 12
1.000000000000 1(2)2726 972 19 31
2.011110100001 6(2)14 6 2 4
3.000001010011 12(8)12 2 2 2
40.010101010101 2(1)1 0 0 1
41.010111010111 3(2)2 0 0 1
42.011011011011 3(1)1 0 0 1
total NATs = 309
L= 13
1.0000000000000 1(2)7282 2002 31 40
2.0111110100001 13(2)34 6 7 4
3.0000010010111 13(2)4 2 1 2
4.1101110001100 1(52)182 26 5 2
5.0000011001011 1(52)182 26 5 2
6.0000011101001 13(2)4 2 1 2
total NATs = 42
L= 14
1.00000000000000 1(2)11076 3514 24 50
2.01111110100001 14(2)218 66 20 5
3.01111101000001 7(2)34 12 4 5
4.00000010010111 14(2)26 10 5 3
5.00000011101001 14(2)26 10 5 3
6.01111100110000 7(12)88 20 11 3
7.01111000111000 7(12)88 20 11 3
8.00001001111011 2(7)7 0 0 1
9.00001101111001 2(7)7 0 0 1
10.00010110001011 1(7)7 0 0 1
11.00011010001101 1(7)7 0 0 1
12.00101110010111 1(7)7 0 0 1
13.00111010011101 1(7)7 0 0 1
14.01010101010101 2(1)1 0 0 1
total NATs = 74
L= 15
1.000000000000000 1(2)24332 16620 30 76
2.010111101000000 15(2)154 112 6 14
3.011111101000001 15(2)44 36 3 8
4.000000010010111 15(2)138 82 15 9
5.110100011111110 15(2)138 82 15 9
6.101101101101101 3(1)91 55 5 10
7.111100010011010 1(15)180 105 5 8
8.010010010010010 3(1)91 55 5 10
9.001111010110010 1(15)180 105 5 8
10.110000101001101 1(15)180 105 5 8
11.000001001111011 1(30)30 0 0 1
12.000111011001010 1(15)180 105 5 8
13.000001101111001 1(30)30 0 0 1
total NATs = 73
L= 16
1.0000000000000000 1(2)46290 15408 40 89
2.0101111101000000 16(2)186 58 16 7
3.0111111110100001 16(2)578 132 21 8
4.0000000010010111 16(2)34 12 5 3
5.1000000101111110 8(2)232 60 19 5
6.0000000011101001 16(2)34 12 5 3
7.1100001011110001 8(8)110 28 9 4
8.1011101110111011 4(2)134 44 8 3
9.1000011100011110 8(8)110 28 9 4
10.0000001001111011 2(16)48 16 2 2
11.0000001010001011 1(128)224 48 3 3
12.1000011110000111 4(4)84 20 6 3
13.0000001101000101 1(128)224 48 3 3
14.0000001101111001 2(16)48 16 2 2
15.0111111010111010 1(128)224 48 3 3
16.0000010010011101 1(128)224 48 3 3
17.0000010011111011 1(16)64 16 3 2
18.0000011011111001 1(16)64 16 3 2
19.0000101010101111 16(8)8 0 0 1
20.0000101100001011 8(4)4 0 0 1
21.0000110100001101 8(4)4 0 0 1
22.0001011100010111 8(4)4 0 0 1
23.0011001100110011 2(2)2 0 0 1
24.0101010101010101 2(1)1 0 0 1
total NATs = 151

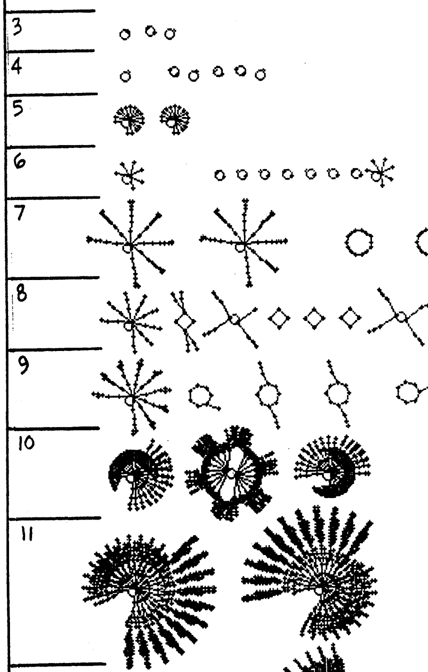
λ ratio = 1 $Z = .875$ 1110100110010110-1001011001101000-rule 3918960232
= 5-code 52 -110100 Length=1 -16

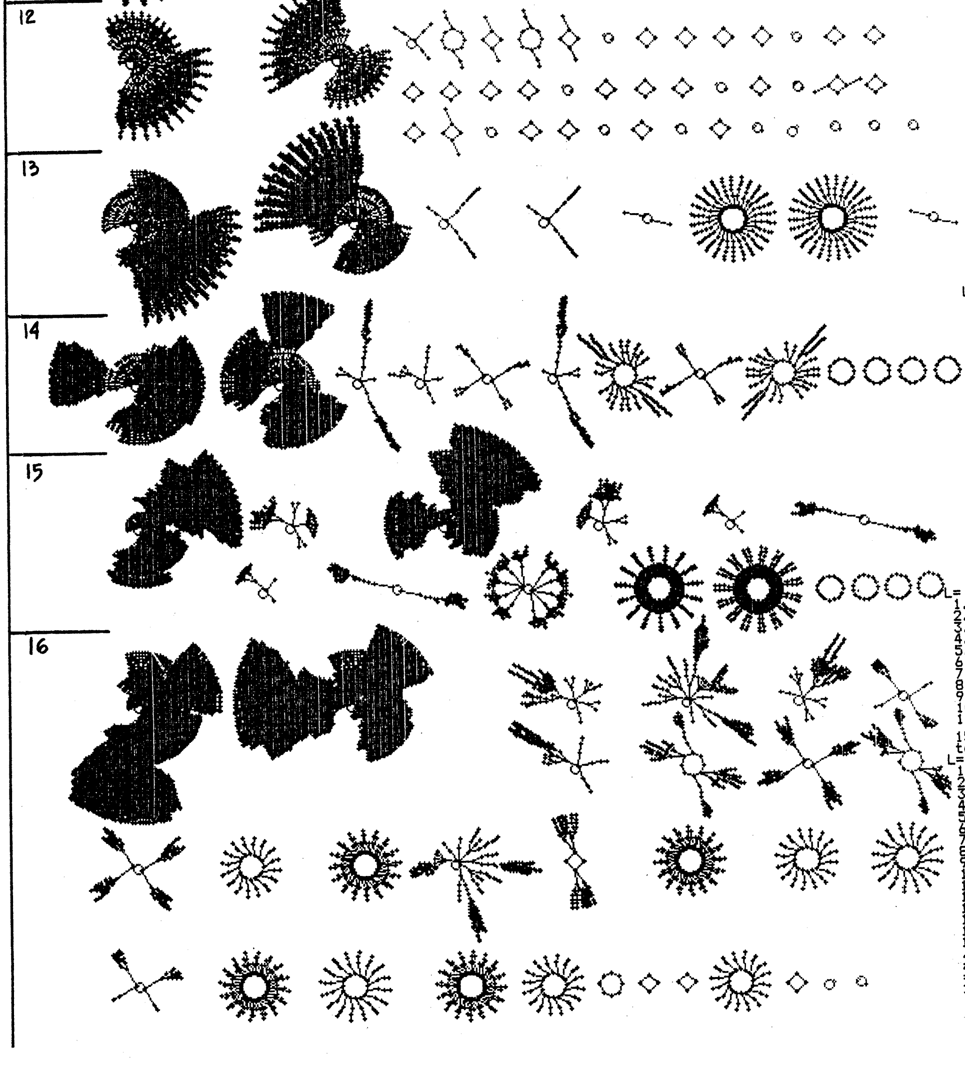






Length=12






```
5-code  52 =110100
ty. at no(p)s g ml mp
L= 3
 1.000 1(1)1 0 0 1
 2.001 3(2)2 0 0 1
 3.111 1(1)1 0 0 1
 total NATs = 5
L= 4
 1.0000 1(1)1 0 0 1
 2.0001 2(2)2 0 0 1
 3.0011 4(1)1 0 0 1
 4.0101 1(2)2 0 0 1
 5.0111 2(2)2 0 0 1
 6.1111 1(1)1 0 0 1
 total NATs = 11
L= 5
 1.00000 1(1)16 15 1 16
 2.11111 1(1)16 15 1 16
 total NATs = 2
L= 6
 1.000000 1(1)7 6 1 7
 2.000011 6(2)2 0 0 1

 8.010101 1(2)2 0 0 1
 9.111111 1(1)7 6 1 7
 total NATs = 27
L= 7
 1.0000000 1(1)50 14 6 8
 2.1111111 1(1)50 14 6 8
 3.0001011 1(14)14 0 0 1
 4.0001101 1(14)14 0 0 1
 total NATs = 4
L= 8
 1.00000000 1(1)25 8 3 9
 2.10000111 4(4)12 4 2 3
 3.10001000 2(2)14 4 3 3
 4.00001011 8(4)4 0 0 1
 5.00001101 8(4)4 0 0 1
 6.00010111 8(4)4 0 0 1
 7.11101110 2(2)14 4 3 3
 8.11111111 1(1)25 8 3 9
 9.00110011 4(1)1 0 0 1
 10.01010101 1(2)2 0 0 1
 total NATs = 39
L= 9
 1.000000000 1(1)55 18 5 10
 2.000000011 9(7)8 1 1 2
 3.000001011 9(8)14 2 3 2
 4.000001101 9(8)14 2 3 2
 5.000011111 9(7)8 1 1 2
 6.111111111 1(1)55 18 5 10
 7.001001001 3(2)2 0 0 1
 total NATs = 41
L= 10
 1.0000000000 1(1)246 170 5 26
 2.1010101010 1(2)532 340 6 14
 3.1111111111 1(1)246 170 5 26
 total NATs = 3
L= 11
 1.00000000000 1(1)1024 253 20 23
 2.11111111111 1(1)1024 253 20 23
 total NATs = 2
L= 12
 1.000000000000 1(1)1363 486 19 31
 2.111111111111 1(1)1363 486 19 31
 3.100001011110 12(1)7 3 2 4
 4.000001010011 12(8)12 2 2 2

 40.001101001101 3(2)2 0 0 1
 41.010101010101 1(2)2 0 0 1
 42.011101110111 2(2)2 0 0 1
 total NATs = 311
L= 13
 1.0000000000000 1(1)3641 1001 31 40
 2.1111111111111 1(1)3641 1001 31 40
 3.1000001011110 13(1)17 3 7 4
 4.1000001011110 13(1)17 3 7 4
 5.0000010010111 13(2)4 2 1 2
 6.1101110001100 1(52)182 26 5 2
 7.0000011001011 1(52)182 26 5 2
 8.0001011011111 13(2)4 2 1 2
 total NATs = 56
L= 14
 1.00000000000000 1(1)5538 1757 24 50
 2.11111111111111 1(1)5538 1757 24 50
 3.10000001011110 14(1)109 33 20 5
 4.10000001011110 14(1)17 6 4 5
 5.00000010010111 14(2)26 10 5 3
 6.01011111101000 14(1)109 33 20 5
 7.10000011001111 7(12)88 20 11 3
 8.11000101101111 14(2)26 10 5 3
 9.00001111000111 7(12)88 20 11 3
 10.00001001111011 1(14)14 0 0 1
 11.00001101111001 1(14)14 0 0 1
 12.00010110001011 1(14)14 0 0 1
 13.00011010001101 1(14)14 0 0 1
 14.01010101010101 1(2)2 0 0 1
 total NATs = 91
L= 15
 1.000000000000000 1(1)12166 8310 30 76
 2.101000001011111 15(1)77 56 6 14
 3.111111111111111 1(1)12166 8310 30 76
 4.100000001011110 15(1)77 56 6 14
 5.100000001011110 15(1)22 18 3 8
 6.000000010010111 15(2)138 82 15 9
 7.100000101111110 15(1)22 18 3 8
 8.000101101111111 15(2)138 82 15 9
 9.010010010010010 3(2)182 110 5 10
 10.111100010011010 1(30)360 210 5 8
 11.001111010110010 1(30)360 210 5 8

 15.000011011111001 1(15)15 0 0 1
 total NATs = 101
L= 16
 1.0000000000000000 1(1)23145 7704 40 89
 2.1111111111111111 1(1)23145 7704 40 89
 3.1010000001011111 16(1)93 29 16 7
 4.1000000001011110 16(1)289 66 21 8
 5.1000000001011110 16(1)93 29 16 7
 6.0000000010010111 16(2)34 12 5 3
 7.1000000101111110 16(1)116 30 19 5
 8.1100001011110001 8(8)110 28 9 4
 9.1011101110111011 2(2)134 44 8 3
 10.1000011100011110 8(8)110 28 9 4
 11.0100010001000100 2(2)134 44 8 3
 12.0000001001111011 1(16)48 16 2 2
 13.0000001010001011 1(128)224 48 3 3
 14.1111010000101111 16(1)289 66 21 8
 15.1000011110000111 4(4)84 20 6 3
 16.0000001101000101 1(128)224 48 3 3

 28.0001011100010111 8(4)4 0 0 1
 29.0011001100110011 4(1)1 0 0 1
 30.0101010101010101 1(2)2 0 0 1
 total NATs = 193
```

206

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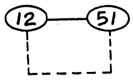
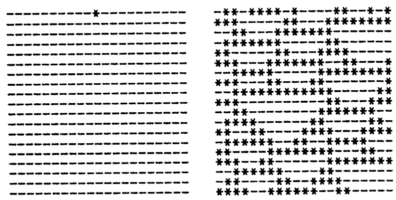
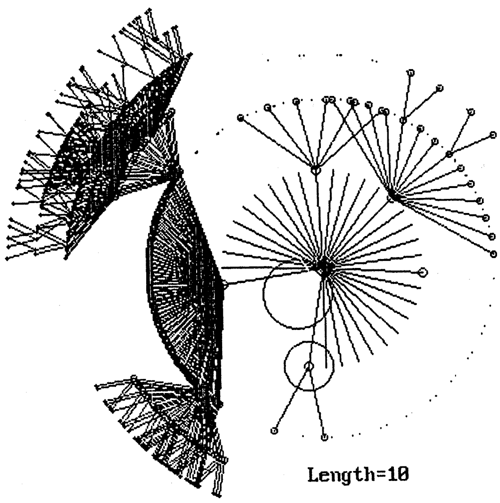
λ ratio = .75 $Z = .5$ 0001011101111110-0111111011101000-rule 394165992

= 5-code 12 -001100 Length=3 -15

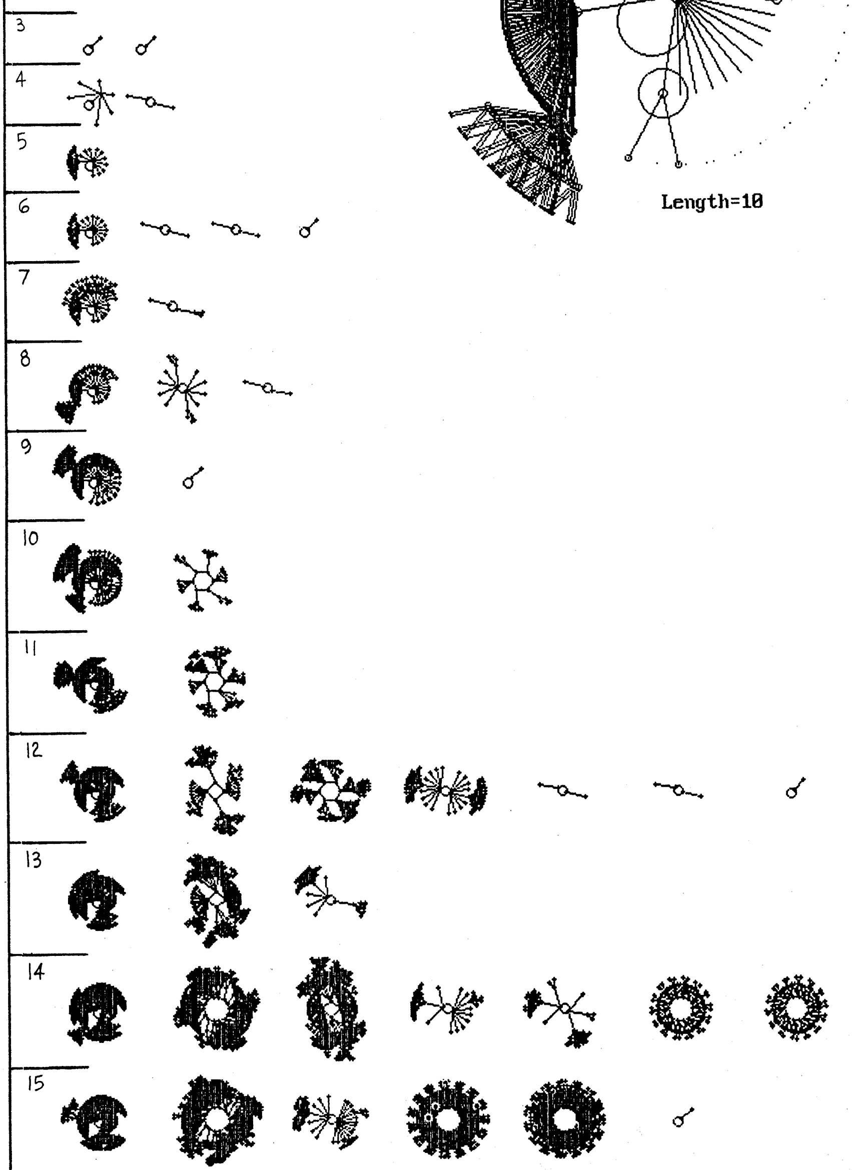


001100-12
110011-51





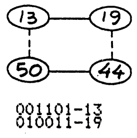
Length=10

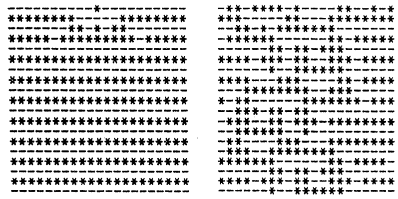


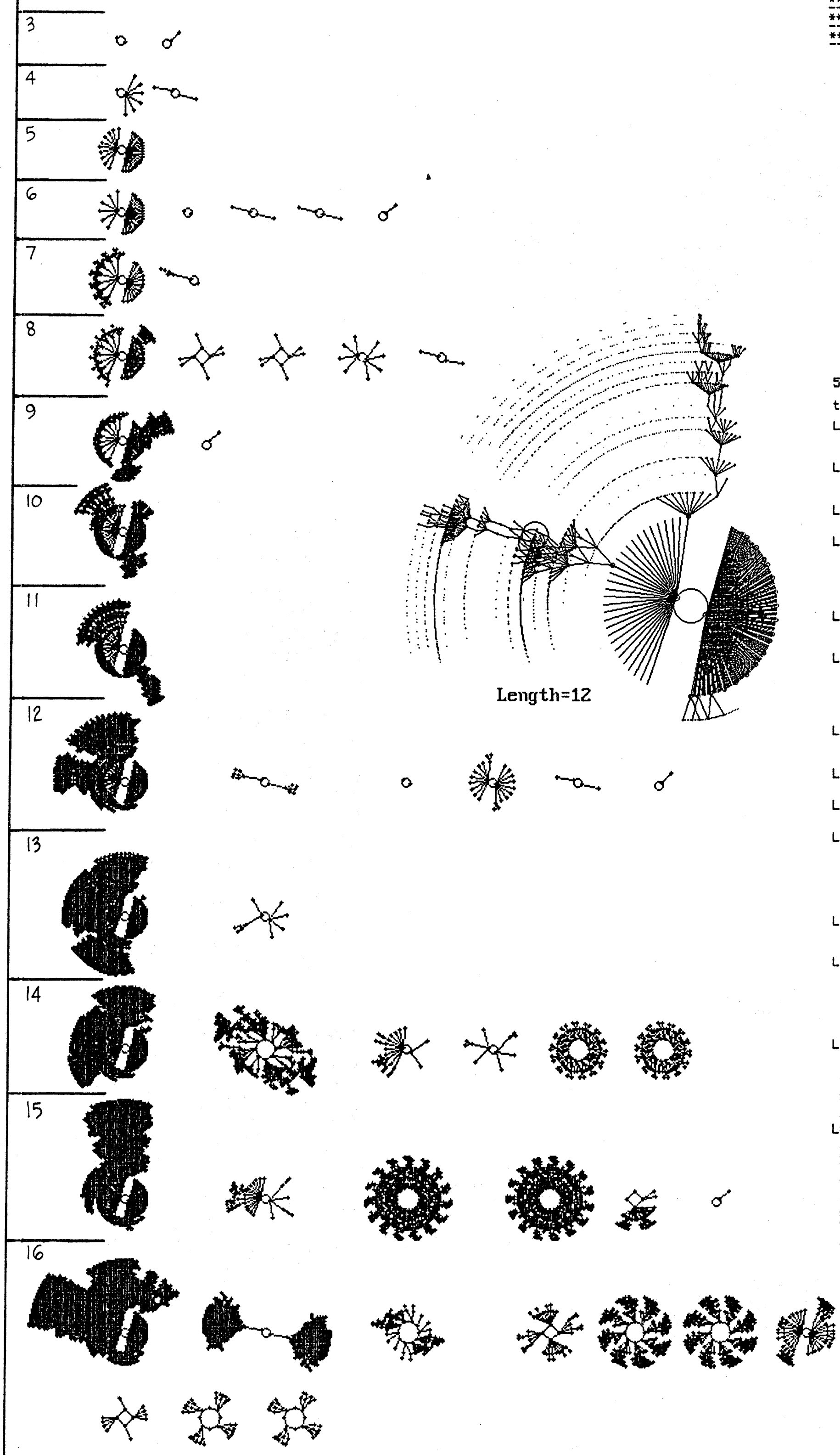

```
5-code  12 =001100

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)2 1 1 2
 2.110 3(1)2 1 1 2
 total NATs = 4
L= 4
 1.0000 1(1)8 6 2 6
 2.0001 2(2)4 2 1 2
 total NATs = 3
L= 5
 1.00000 1(1)32 30 2 20
 total NATs = 1
L= 6
 1.000000 1(1)34 32 2 20
 2.100111 3(2)4 2 1 2
 3.010111 3(2)4 2 1 2
 4.110110 3(1)2 1 1 2
 total NATs = 10
L= 7
 1.0000000 1(1)86 70 3 16
 2.1000111 7(2)6 3 2 2
 total NATs = 8
L= 8
 1.00000000 1(1)168 122 6 30
 2.10001111 4(2)20 14 3 6
 3.00010001 2(2)4 2 1 2
 total NATs = 7
L= 9
 1.000000000 1(1)506 369 6 54
 2.110110110 3(1)2 1 1 2
 total NATs = 4
L= 10
 1.0000000000 1(1)824 672 6 122
 2.1000000111 5(6)40 28 3 6
 total NATs = 6
L= 11
 1.00000000000 1(1)750 594 6 154
 2.10000000111 11(6)118 94 3 20
 total NATs = 12
L= 12
 1.000000000000 1(1)986 870 5 204
 2.110000001100 6(4)168 122 6 18
 3.100000001111 6(6)264 212 4 22
 4.111001111001 3(2)164 132 4 20
 5.000100010001 2(2)4 2 1 2
 6.010111010111 3(2)4 2 1 2
 7.110110110110 3(1)2 1 1 2
 total NATs = 24
L= 13
 1.0000000000000 1(1)1380 1274 3 312
 2.1100000001100 13(4)422 333 7 34
 3.1110001111001 13(2)102 86 4 24
 total NATs = 27
L= 14
 1.00000000000000 1(1)2398 2242 5 548
 2.10000000000111 7(14)888 726 5 62
 3.11000000011000 7(4)844 708 8 46
 4.11100011110001 7(2)82 71 3 22
 5.11100111100011 14(2)76 58 4 10
 6.10000111111001 1(14)112 84 2 6
 7.10000110011111 1(14)112 84 2 6
 total NATs = 38
L= 15
 1.000000000000000 1(1)3962 3780 5 950
 2.100000000000111 15(14)1668 1419 6 102
 3.111000111100011 15(2)104 85 4 16
 4.110000110011111 1(45)1110 870 4 14
 5.100000111111001 1(45)1110 870 4 14
 6.110110110110110 3(1)2 1 1 2
 total NATs = 36
```

λ ratio = .6875   Z = .5625   0001011101111110-0111111011101001-rule   394165993

= 5-code 13 -001101   Length=3 -16

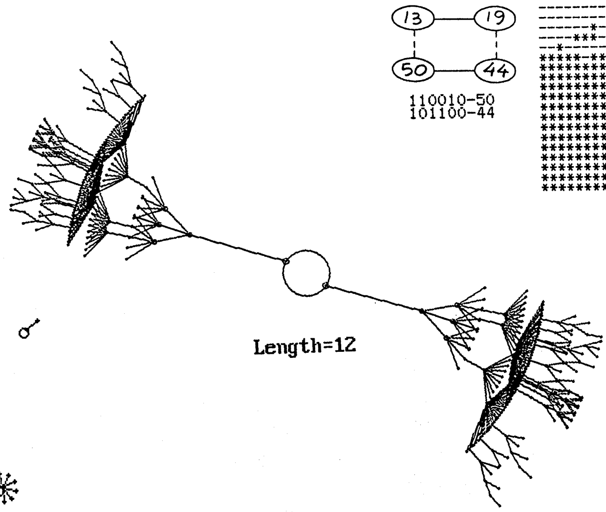


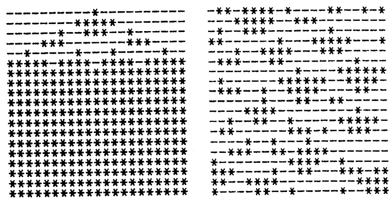




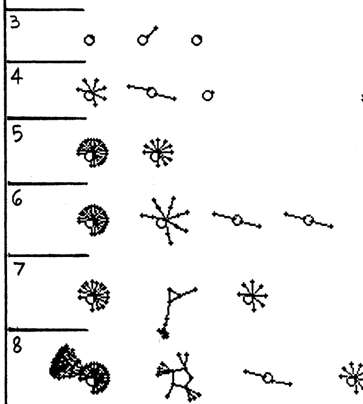

```
5-code  13 =001101

ty. at no(p)s g ml mp

L= 3
 1.000 1(2)2 0 0 1
 2.110 3(1)2 1 1 2
 total NATs = 4
L= 4
 1.0000 1(2)8 6 1 7
 2.0001 2(2)4 2 1 2
 total NATs = 3
L= 5
 1.00000 1(2)32 30 1 21
 total NATs = 1
L= 6
 1.000000 1(2)28 26 1 21
 2.000001 3(2)2 0 0 1
 3.100111 3(2)4 2 1 2
 4.010111 3(2)4 2 1 2
 5.110110 3(1)2 1 1 2
 total NATs = 13
L= 7
 1.0000000 1(2)79 63 3 15
 2.1100111 7(2)7 2 4 2
 total NATs = 8
L= 8
 1.00000000 1(2)128 90 4 31
 2.00111000 4(4)10 6 1 3
 3.00000011 4(4)10 6 1 3
 4.10001111 4(2)10 8 1 5
 5.00010001 2(2)4 2 1 2
 total NATs = 15
L= 9
 1.000000000 1(2)506 324 9 55
 2.110110110 3(1)2 1 1 2
 total NATs = 4
L= 10
 1.0000000000 1(2)1024 697 9 123
 total NATs = 1
L= 11
 1.00000000000 1(2)2048 1419 14 155
 total NATs = 1
L= 12
 1.000000000000 1(2)3968 2778 17 205
 2.000100010001 2(2)16 6 3 3
 3.000001000001 3(2)2 0 0 1
 4.100111100111 3(2)24 20 2 10
 5.010111010111 3(2)4 2 1 2
 6.110110110110 3(1)2 1 1 2
 total NATs = 15
L= 13
 1.0000000000000 1(2)8036 5837 14 313
 2.0011110011111 13(2)12 7 3 5
 total NATs = 14
L= 14
 1.00000000000000 1(2)9685 7268 14 549
 2.00000000000001 7(8)872 654 12 36
 3.00111110011111 7(2)33 20 4 12
 4.00111100011111 14(2)12 8 2 4
 5.00110000111111 1(14)98 70 2 5
 6.00110011111100 1(14)98 70 2 5
 total NATs = 31
L= 15
 1.000000000000000 1(2)29702 23280 32 951
 2.001111100011111 15(2)56 36 5 15
 3.011111100110000 1(30)675 480 6 8
 4.011111100000110 1(30)675 480 6 8
 5.000000011111111 15(4)58 46 3 20
 6.110110110110110 3(1)2 1 1 2
 total NATs = 36
L= 16
 1.0000000000000000 1(2)47280 36938 30 1423
 2.0100010001000100 2(2)7436 5672 14 79
 3.0011111111110011 8(10)120 92 4 22
 4.0011100000011000 4(4)50 30 3 10
 5.1010110000011111 2(8)360 264 4 11
 6.0110101111110000 2(8)360 264 4 11
 7.0001111100011111 4(2)102 68 4 15
 8.0000001100000011 4(4)14 10 1 5
 9.0000001100000111 4(8)40 20 2 4
 10.0000001110000011 4(8)40 20 2 4
 total NATs = 35
```

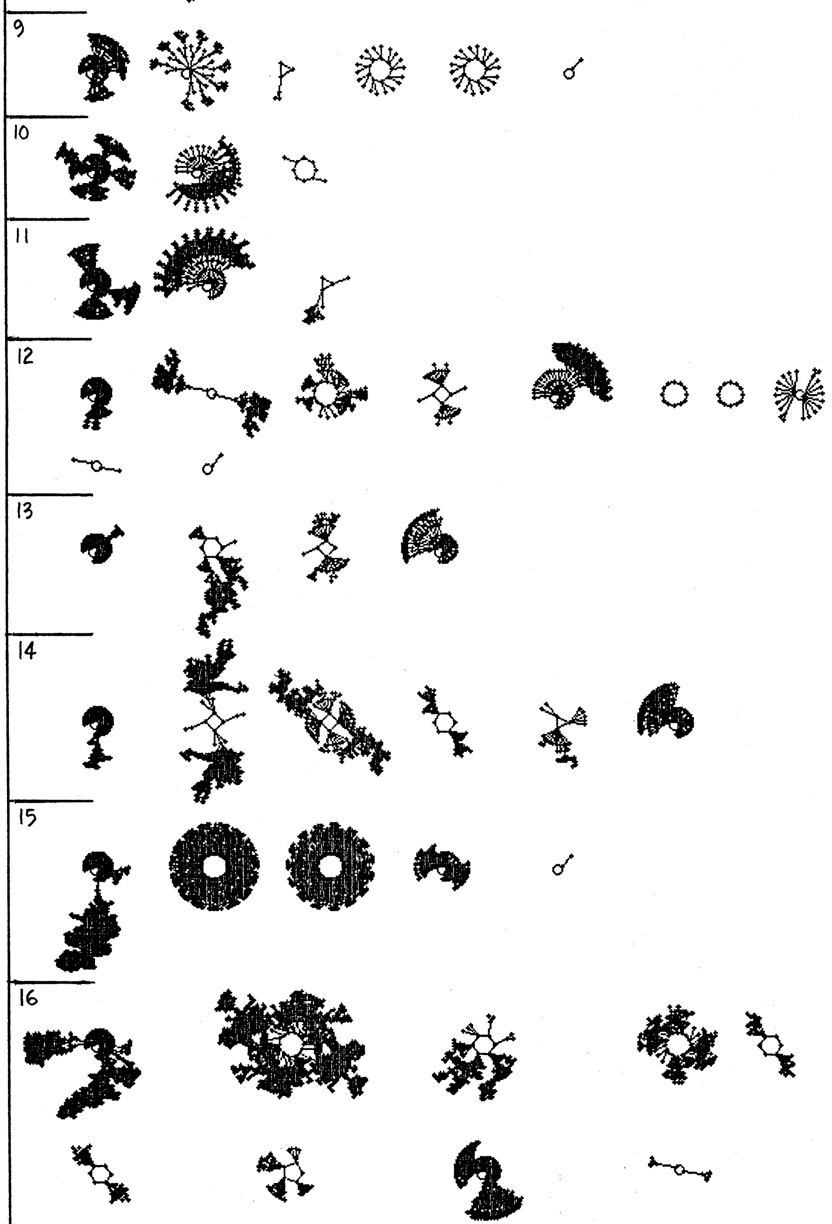
λ ratio = .6875 Z = .5625   1110100010000001-1000000100010110-rule   3900801302
= 5-code 50 -110010   Length=3 -16





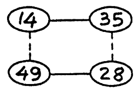
Length=12



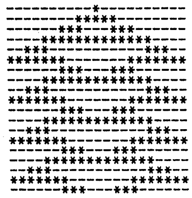


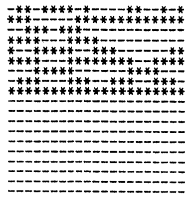

```
5-code  50 =110010

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)1 0 0 1
 2.001 3(1)2 1 1 2
 3.111 1(1)1 0 0 1
 total NATs = 5
L= 4
 1.0000 1(1)7 6 1 7
 2.1011 2(2)4 2 1 2
 3.1111 1(1)1 0 0 1
 total NATs = 4
L= 5
 1.00000 1(1)21 20 1 21
 2.11111 1(1)11 10 1 11
 total NATs = 2
L= 6
 1.000000 1(1)21 20 1 21
 2.111111 1(1)13 6 2 7
 3.000011 3(2)4 2 1 2
 4.000101 3(2)4 2 1 2
 5.001001 3(1)2 1 1 2
 total NATs = 11
L= 7
 1.0000000 1(1)15 14 1 15
 2.0011000 7(3)15 8 5 6
 3.1111111 1(1)8 7 1 8
 total NATs = 9
L= 8
 1.00000000 1(1)119 82 6 31
 2.00000001 8(5)15 10 1 5
 3.10111011 2(2)4 2 1 2
 4.11111111 1(1)9 8 1 9
 total NATs = 12
L= 9
 1.000000000 1(1)289 207 4 55
 2.111111111 1(1)109 63 5 10
 3.000000111 9(3)6 2 2 2
 4.000001011 1(9)27 18 1 3
 5.000001101 1(9)27 18 1 3
 6.001001001 3(1)2 1 1 2
 total NATs = 16
L= 10
 1.0000000000 1(1)613 467 5 123
 2.1111111111 1(1)311 210 6 21
 3.0000001011 10(8)10 2 1 2
 total NATs = 12
L= 11
 1.00000000000 1(1)936 693 6 155
 2.11111111111 1(1)749 506 10 23
 3.01101100000 11(3)33 20 5 8
 total NATs = 13
L= 12
 1.000000000000 1(1)289 234 4 205
 2.111011101110 2(2)508 372 10 34
 3.000000000011 12(13)120 79 4 18
 4.000000000101 6(4)28 20 2 10
 5.111111111111 1(1)1069 744 9 31
 6.000000100111 1(12)12 0 0 1
 7.000000111001 1(12)12 0 0 1
 8.011000011000 3(2)24 20 2 10
 9.000101000101 3(2)4 2 1 2
 10.001001001001 3(1)2 1 1 2
 total NATs = 33
L= 13
 1.0000000000000 1(1)352 325 3 313
 2.0000000000001 13(6)534 382 23 52
 3.0000000000101 13(4)55 39 3 17
 4.1111111111111 1(1)183 130 5 40
 total NATs = 28
L= 14
 1.00000000000000 1(1)1011 884 7 549
 2.00011101110000 7(4)1284 980 20 70
 3.11100000011100 7(4)748 528 15 47
 4.01000001110000 7(6)70 48 4 13
 5.11000001100000 7(3)53 41 6 20
 6.11111111111111 1(1)288 175 5 50
 total NATs = 30
L= 15
 1.000000000000000 1(1)30381 23915 33 951
 2.000000000010111 1(90)1110 720 5 11
 3.000000000011101 1(90)1110 720 5 11
 4.111111111111111 1(1)161 115 3 71
 5.001001001001001 3(1)2 1 1 2
 total NATs = 7
L= 16
 1.0000000000000000 1(1)16743 13090 21 1423
 2.0010011111001000 8(14)4036 3056 21 180
 3.0000000000000011 16(6)540 446 10 90
 4.0100000001110000 16(16)328 262 4 25
 5.1100000001100000 8(6)68 44 4 12
 6.1100000001100001 8(6)68 44 4 12
 7.0000000100000001 8(5)87 68 3 24
 8.1111111111111111 1(1)809 536 8 89
 9.1110111011101110 2(2)12 8 2 4
 total NATs = 68
```

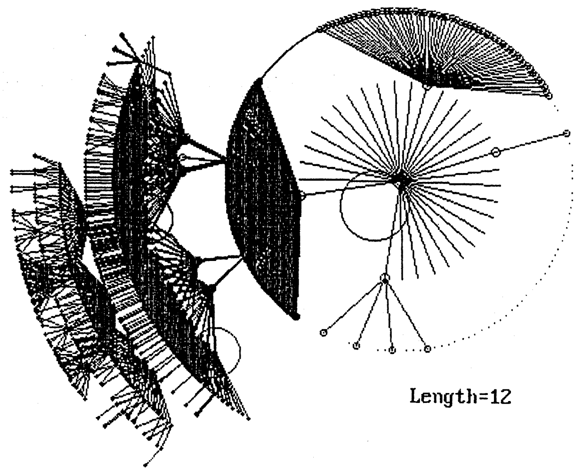
λ ratio = .4375   Z = .3125   0001011101111111-0111111111111110-rule 394231806
= 5-code 14 -001110   Length=3 -13



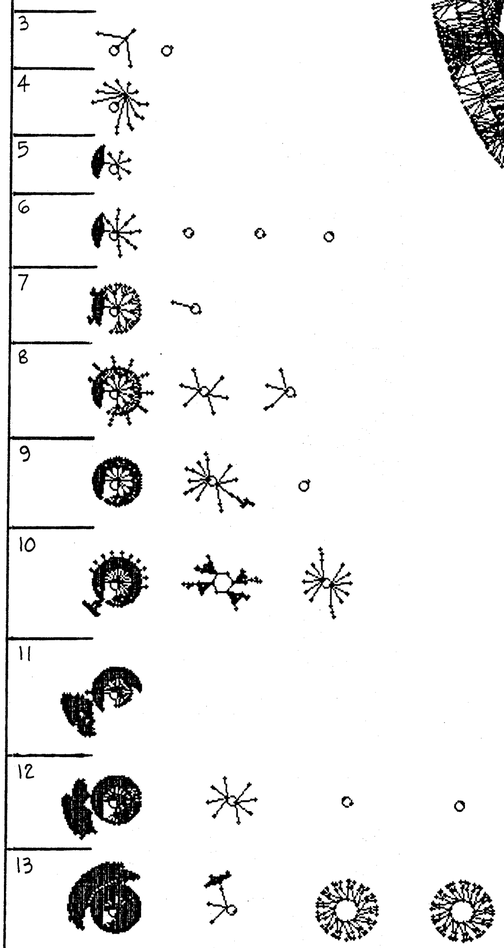
001110-14
100011-35





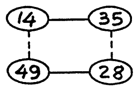


Length=12

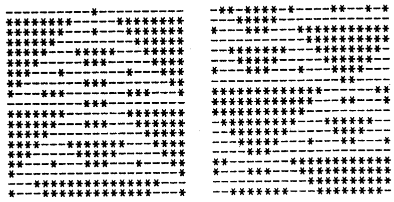


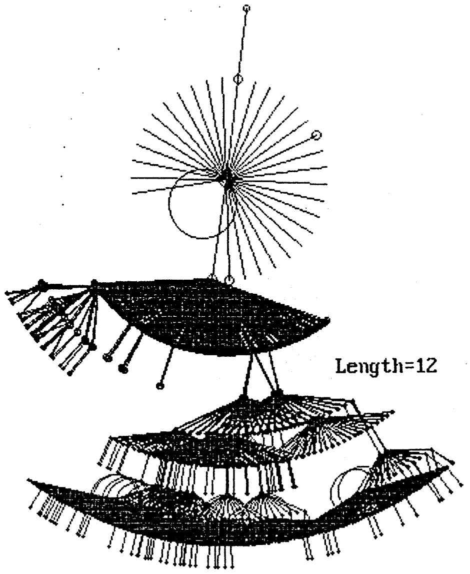

```
5-code  14 =001110

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)5 3 2 3
 2.011 3(1)1 0 0 1
 total NATs = 4
L= 4
 1.0000 1(1)16 10 3 10
 total NATs = 1
L= 5
 1.00000 1(1)32 30 2 25
 total NATs = 1
L= 6
 1.000000 1(1)49 41 2 35
 2.001111 3(2)2 0 0 1
 3.010111 3(2)2 0 0 1
 4.011011 3(1)1 0 0 1
 total NATs = 10
L= 7
 1.0000000 1(1)107 84 3 56
 2.1100111 7(2)3 1 1 2
 total NATs = 8
L= 8
 1.00000000 1(1)176 146 4 98
 2.11000111 4(2)8 6 1 4
 3.11001111 8(2)6 3 2 4
 total NATs = 13
L= 9
 1.000000000 1(1)284 273 2 183
 2.110000111 9(2)25 14 6 7
 3.011011011 3(1)1 0 0 1
 total NATs = 13
L= 10
 1.0000000000 1(1)664 612 5 347
 2.1100000111 5(6)54 36 4 11
 3.1100001111 5(2)18 12 3 7
 total NATs = 11
L= 11
 1.00000000000 1(1)2048 1727 10 605
 total NATs = 1
L= 12
 1.000000000000 1(1)4057 3451 9 1063
 2.110011110011 3(2)10 8 1 5
 3.010111010111 3(2)2 0 0 1
 4.011011011011 3(1)1 0 0 1
 total NATs = 10
L= 13
 1.0000000000000 1(1)7555 6656 7 1898
 2.1110001111001 13(2)37 30 4 28
 3.1101111000001 1(13)78 39 2 3
 4.0000011110111 1(13)78 39 2 3
 total NATs = 16
```

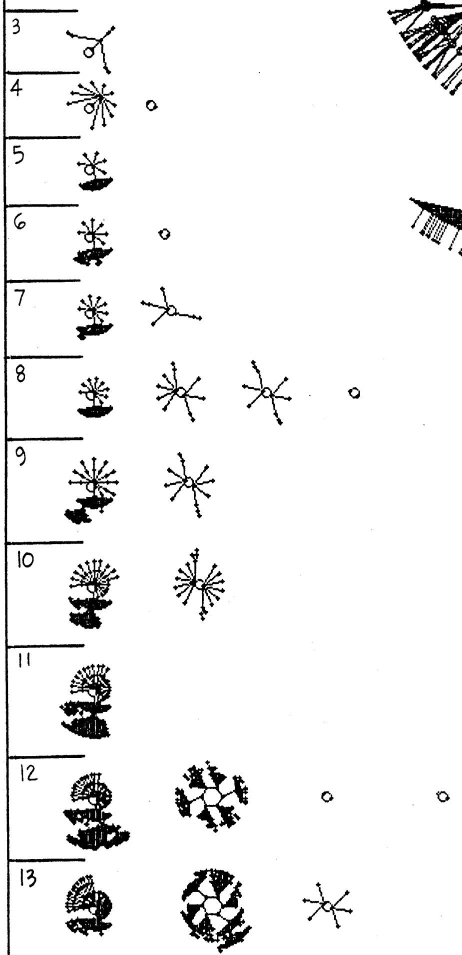
λ ratio = .4375 Z = .3125 1110100010000000-1000000000000001-rule 3900735489
= 5-code 49 -110001 Length=3 -13



110001-49
011100-28



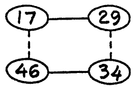




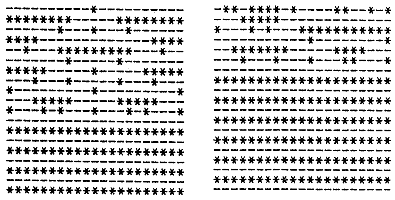

```
5-code  49 =110001

ty. at no(p)s g ml mp

L= 3
 1.111 1(1)8 3 3 3
 total NATs = 1
L= 4
 1.1111 1(1)12 10 2 10
 2.0111 2(2)2 0 0 1
 total NATs = 3
L= 5
 1.11111 1(1)32 30 2 25
 total NATs = 1
L= 6
 1.111111 1(1)58 41 3 35
 2.000001 3(2)2 0 0 1
 total NATs = 4
L= 7
 1.1111111 1(1)72 63 3 56
 2.0000001 7(2)8 4 2 4
 total NATs = 8
L= 8
 1.11111111 1(1)108 106 2 98
 2.00000001 8(2)12 8 2 6
 3.00000011 4(2)12 6 3 4
 4.01110111 2(2)2 0 0 1
 total NATs = 15
L= 9
 1.111111111 1(1)395 327 6 183
 2.000000011 9(2)13 8 3 5
 total NATs = 10
L= 10
 1.1111111111 1(1)914 772 6 347
 2.0011100000 5(2)22 16 3 8
 total NATs = 6
L= 11
 1.11111111111 1(1)2048 1727 8 605
 total NATs = 1
L= 12
 1.111111111111 1(1)2898 2539 9 1063
 2.000000000011 6(6)198 156 3 33
 3.000001000001 3(2)2 0 0 1
 4.011101110111 2(2)2 0 0 1
 total NATs = 12
L= 13
 1.1111111111111 1(1)2199 2002 4 1898
 2.1100011000111 13(6)453 388 6 100
 3.0000001000001 13(2)8 6 1 4
 total NATs = 27
```

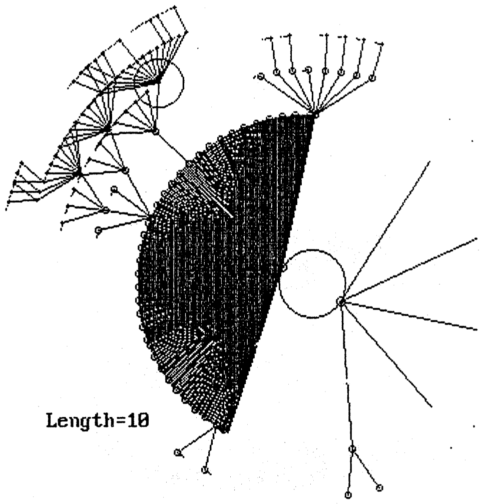
λ ratio = .375 Z = .375

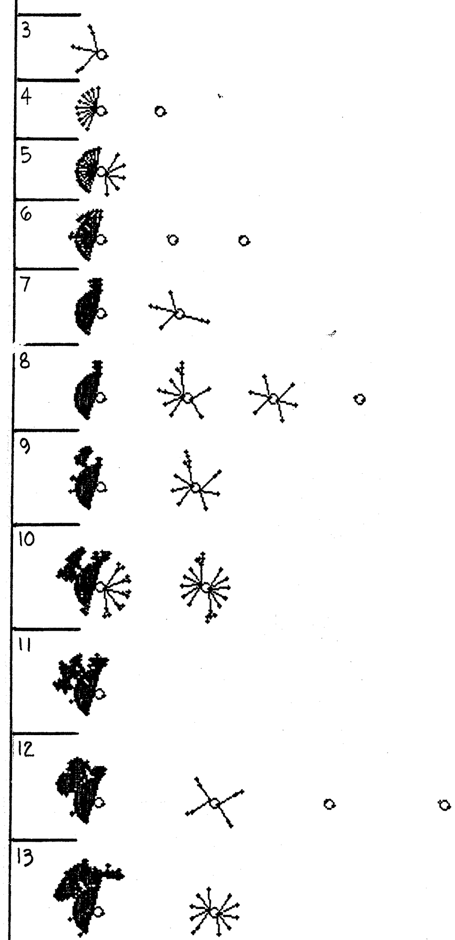
01101000100000000-1000000000000001-rule 1753251841
= 5-code 17 -010001 Length=3 -13



010001-17
011101-29

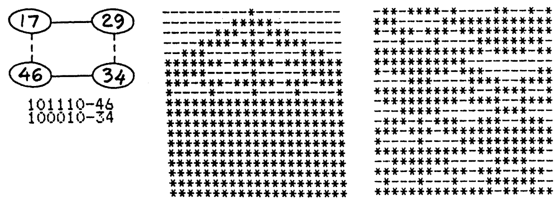


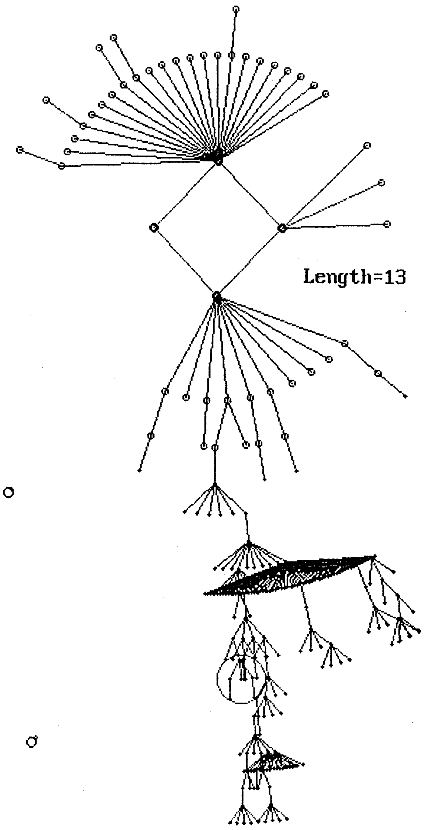


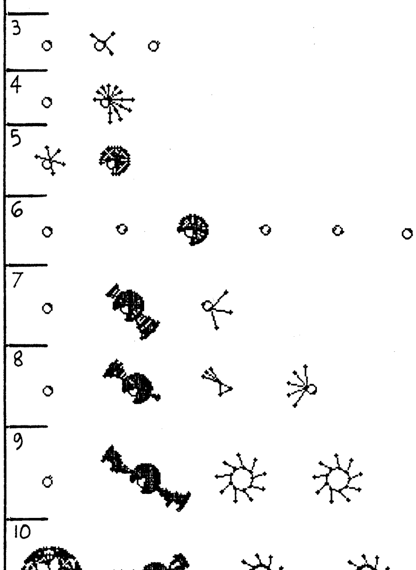


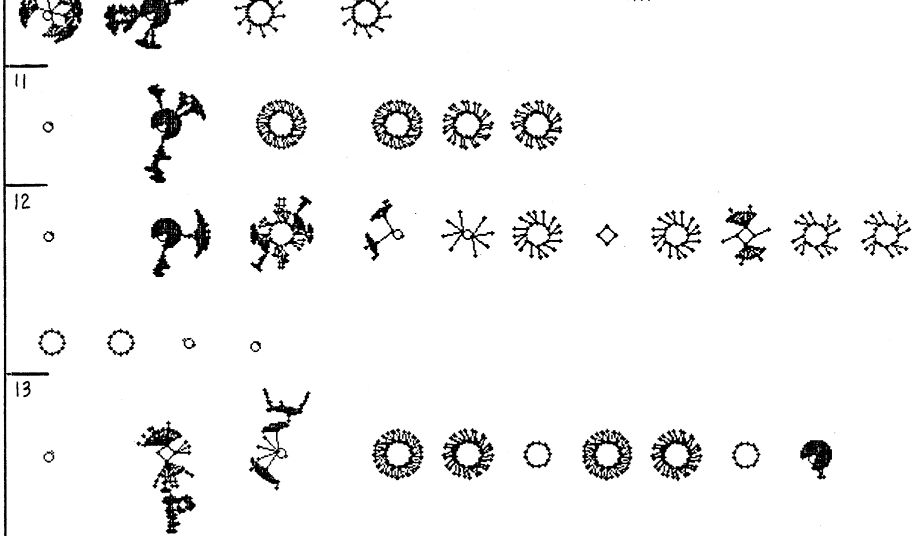

```
5-code  17 =010001

ty. at no(p)s g ml mp

L= 3
 1.000 1(2)8 3 2 4
 total NATs = 1
L= 4
 1.0000 1(2)12 10 1 11
 2.0111 2(2)2 0 0 1
 total NATs = 3
L= 5
 1.00000 1(2)32 30 1 26
 total NATs = 1
L= 6
 1.000000 1(2)52 35 2 36
 2.000001 3(2)2 0 0 1
 3.011111 3(2)2 0 0 1
 total NATs = 7
L= 7
 1.0000000 1(2)72 56 3 57
 2.0000001 7(2)8 4 2 4
 total NATs = 8
L= 8
 1.00000000 1(2)116 98 3 99
 2.00000001 8(2)13 8 3 6
 3.00000011 4(2)8 6 1 4
 4.01110111 2(2)2 0 0 1
 total NATs = 15
L= 9
 1.000000000 1(2)386 309 5 184
 2.000000011 9(2)14 8 4 5
 total NATs = 10
L= 10
 1.0000000000 1(2)914 742 5 348
 2.0011100000 5(2)22 16 3 7
 total NATs = 6
L= 11
 1.00000000000 1(2)2048 1595 7 606
 total NATs = 1
L= 12
 1.000000000000 1(2)4056 3291 7 1064
 2.000001000001 3(2)10 4 2 3
 3.011101110111 2(2)2 0 0 1
 4.011111011111 3(2)2 0 0 1
 total NATs = 9
L= 13
 1.0000000000000 1(2)8023 6604 8 1899
 2.0000001000001 13(2)13 10 2 6
 total NATs = 14
```

λ ratio = .375 Z = .375 **1001011101111111-01111111111111110-rule** 2541715454
= 5-code 46 -101110 Length=3 -13





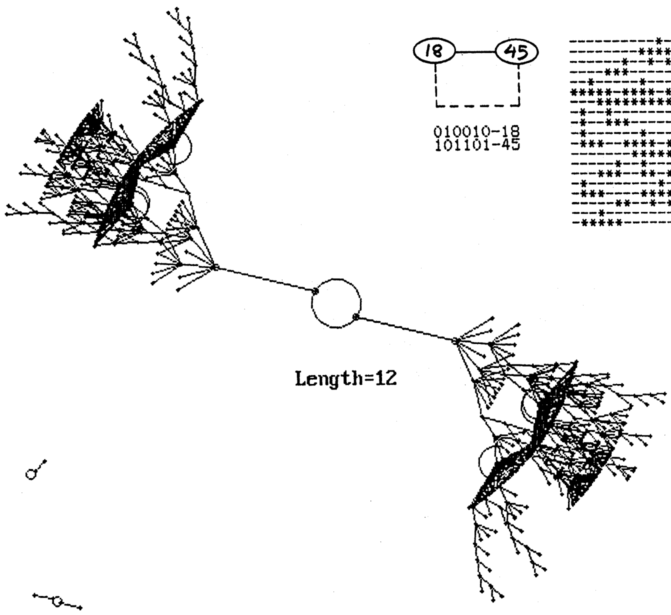


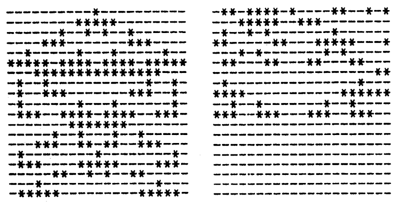


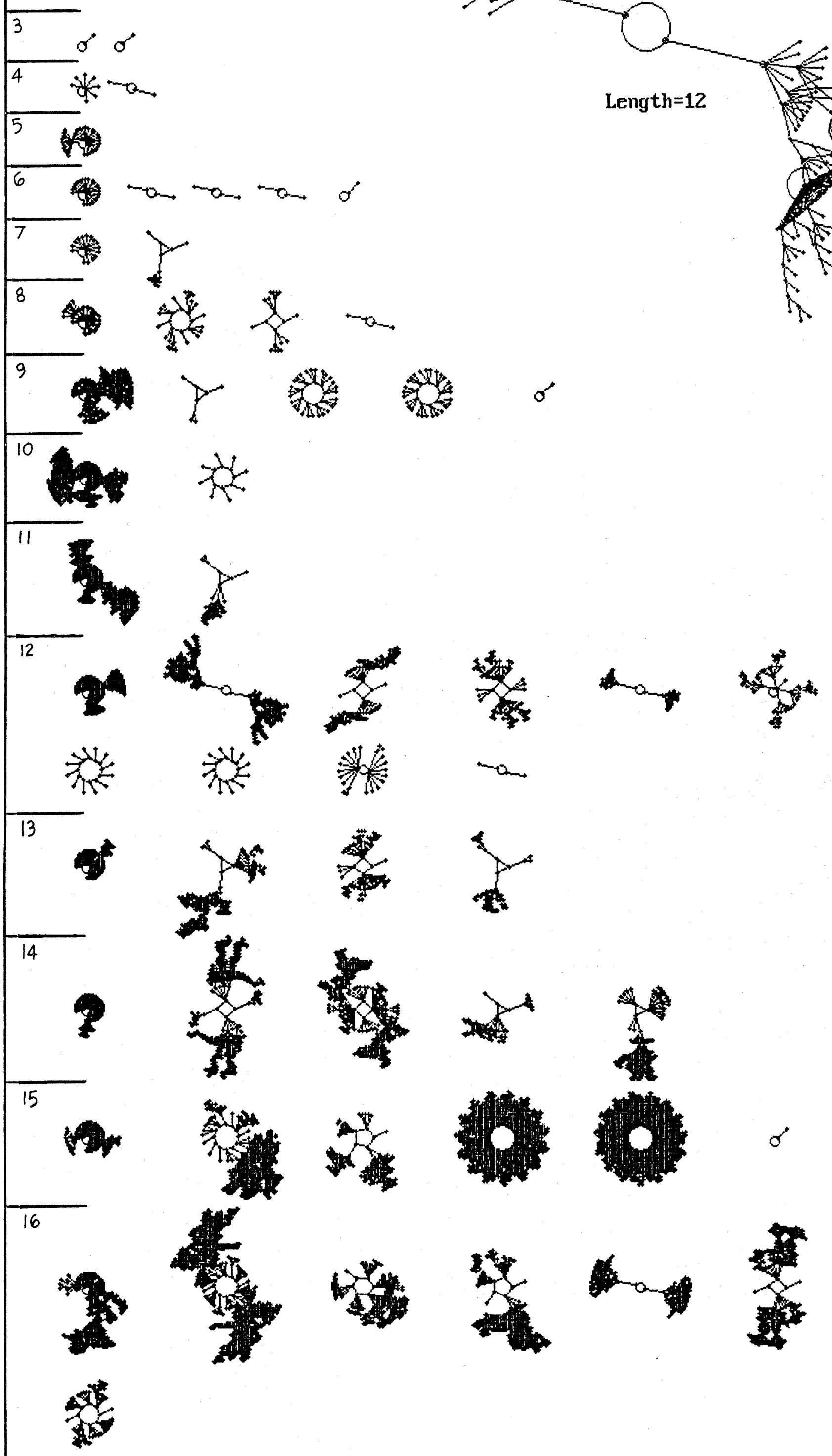

```
5-code  46 =101110

ty. at no(p)s g ml  mp

L= 3
 1.000 1(1)1 0 0 1
 2.111 1(1)4 3 1 4
 3.011 3(1)1 0 0 1
 total NATs = 5
L= 4
 1.0000 1(1)1 0 0 1
 2.1111 1(1)15 10 2 11
 total NATs = 2
L= 5
 1.00000 1(1)6 5 1 6
 2.11111 1(1)26 25 1 26
 total NATs = 2
L= 6
 1.000000 1(1)1 0 0 1
 2.000001 6(2)2 0 0 1
 3.111111 1(1)36 35 1 36
 4.001111 3(2)2 0 0 1
 5.010111 3(2)2 0 0 1
 6.011011 3(1)1 0 0 1
 total NATs = 17
L= 7
 1.0000000 1(1)1 0 0 1
 2.1111111 1(1)92 63 4 57
 3.0000011 7(2)5 3 1 4
 total NATs = 9
L= 8
 1.00000000 1(1)1 0 0 1
 2.11111111 1(1)151 122 4 99
 3.00000011 8(3)6 3 1 4
 4.11011111 8(2)7 5 1 6
 total NATs = 18
L= 9
 1.000000000 1(1)1 0 0 1
 2.111111111 1(1)472 363 7 184
 3.001011111 1(9)18 9 1 2
 4.001111101 1(9)18 9 1 2
 5.011011011 3(1)1 0 0 1
 total NATs = 7
L= 10
 1.0000000000 1(1)146 130 3 10
 2.1111111111 1(1)798 672 8 348
 3.0001011111 1(30)40 10 1 2
 4.0001111101 1(30)40 10 1 2
 total NATs = 4
L= 11
 1.00000000000 1(1)1 0 0 1
 2.11111111111 1(1)1805 1485 11 606
 3.00001011111 1(33)66 33 1 4
 4.00001111101 1(33)66 33 1 4
 5.00010111111 1(33)55 22 1 2
 6.00011111011 1(33)55 22 1 2
 total NATs = 6
L= 12
 1.000000000000 1(1)1 0 0 1
 2.111111111111 1(1)2148 1935 6 1064
 3.000000000011 6(16)178 136 5 33
 4.011111011111 6(2)42 34 3 17
 5.110011110011 3(2)10 8 1 5
 6.110111111001 3(12)28 16 1 3
 7.000001011111 12(4)4 0 0 1
 8.110111001111 3(12)28 16 1 3
 9.111111110101 6(4)34 26 2 13
 10.000001011111 3(12)24 12 1 3
 11.000001111101 3(12)24 12 1 3
 12.000110111111 1(12)12 0 0 1
 13.000111111011 1(12)12 0 0 1
 14.010111010111 3(2)2 0 0 1
 15.011011011011 3(1)1 0 0 1
 total NATs = 55
L= 13
 1.0000000000000 1(1)1 0 0 1
 2.1111011011111 13(4)254 185 19 57
 3.1011111011111 13(2)200 173 13 100
 4.1101111111001 1(39)78 39 1 4
 5.1101111110001 1(39)91 52 1 3
 6.0000011101111 1(13)13 0 0 1
 7.1101110011111 1(39)78 39 1 4
 8.1101110001111 1(39)91 52 1 3
 9.0000011110111 1(13)13 0 0 1
 10.1111111111111 1(1)1925 1898 2 1899
```

214
intentionally blank

λ ratio = .625 $Z = .625$ 01101000100000001-10000001000010110-rule   1753317654
= 5-code   18 -010010   Length=3 -16



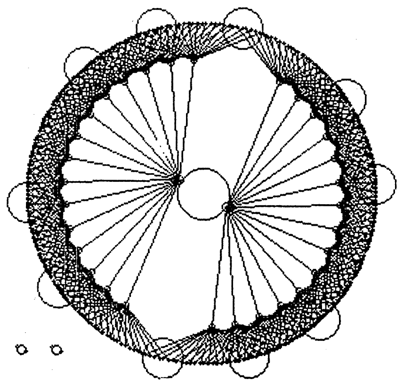




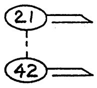

```
5-code  18 =010010

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)2 1 1 2
 2.001 3(1)2 1 1 2
 total NATs = 4
L= 4
 1.0000 1(1)8 7 1 8
 2.1011 2(2)4 2 1 2
 total NATs = 3
L= 5
 1.00000 1(1)32 30 2 22
 total NATs = 1
L= 6
 1.000000 1(1)22 21 1 22
 2.110111 3(2)4 2 1 2
 3.000011 3(2)4 2 1 2
 4.000101 3(2)4 2 1 2
 5.001001 3(1)2 1 1 2
 total NATs = 13
L= 7
 1.0000000 1(1)16 15 1 16
 2.1000010 7(3)16 10 4 6
 total NATs = 8
L= 8
 1.00000000 1(1)40 35 2 32
 2.00000001 4(8)32 22 2 4
 3.01001000 4(4)20 14 2 4
 4.10111011 2(2)4 2 1 2
 total NATs = 11
L= 9
 1.000000000 1(1)362 262 7 56
 2.000000111 9(3)8 4 2 2
 3.000001011 1(9)36 27 1 4
 4.000001101 1(9)36 27 1 4
 5.001001001 3(1)2 1 1 2
 total NATs = 15
L= 10
 1.0000000000 1(1)864 707 5 124
 2.0000001011 10(8)16 8 1 2
 total NATs = 11
L= 11
 1.00000000000 1(1)1300 1013 11 156
 2.01101100000 11(3)68 45 7 8
 total NATs = 12
L= 12
 1.000000000000 1(1)422 337 4 206
 2.111011101110 2(2)856 612 12 34
 3.010000001000 6(4)156 122 9 18
 4.000000000101 6(4)76 56 4 14
 5.111101111101 3(2)76 60 4 12
 6.001001001001 3(1)70 53 5 6
 7.000000100111 1(12)24 12 1 2
 8.000000111001 1(12)24 12 1 2
 9.011000011000 3(2)24 20 2 10
 10.000101000101 3(2)4 2 1 2
 total NATs = 29
L= 13
 1.0000000000000 1(1)366 339 3 314
 2.1101000010111 13(3)440 331 21 52
 3.0000000000101 13(4)74 56 3 22
 4.0010001000000 13(3)88 64 6 14
 total NATs = 40
L= 14
 1.00000000000000 1(1)634 591 3 550
 2.00011101110000 7(4)692 550 16 70
 3.11100000011100 7(4)732 546 12 40
 4.00010001000000 14(3)86 68 6 26
 5.00001010000101 7(3)654 496 16 32
 total NATs = 36
L= 15
 1.000000000000000 1(1)1082 1050 3 952
 2.111100000011110 15(12)1542 1253 16 104
 3.111100001111101 15(5)322 253 9 46
 4.000000000010111 1(90)1860 1380 6 16
 5.000000000011101 1(90)1860 1380 6 16
 6.001001001001001 3(1)2 1 1 2
 total NATs = 36
L= 16
 1.0000000000000000 1(1)10008 8139 16 1424
 2.0100000000000100 8(12)2856 2220 19 180
 3.0100000000001000 16(6)500 437 7 90
 4.0111110111110000 16(5)1190 905 17 68
 5.1011101110111011 2(2)692 536 7 20
 6.0001001000010010 4(4)960 778 12 36
 7.0000000100000001 4(8)104 90 3 24
 total NATs = 51
```

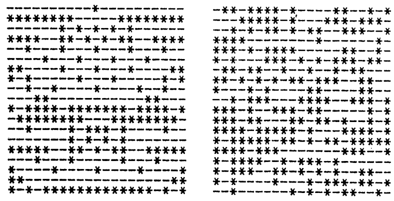
λ ratio = 1 $Z = 1$

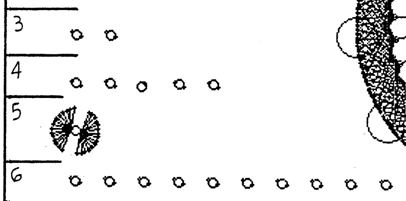
0110100110010110-1001011001101001-rule 1771476585
= 5-code 21 -010101   Length=3 -16

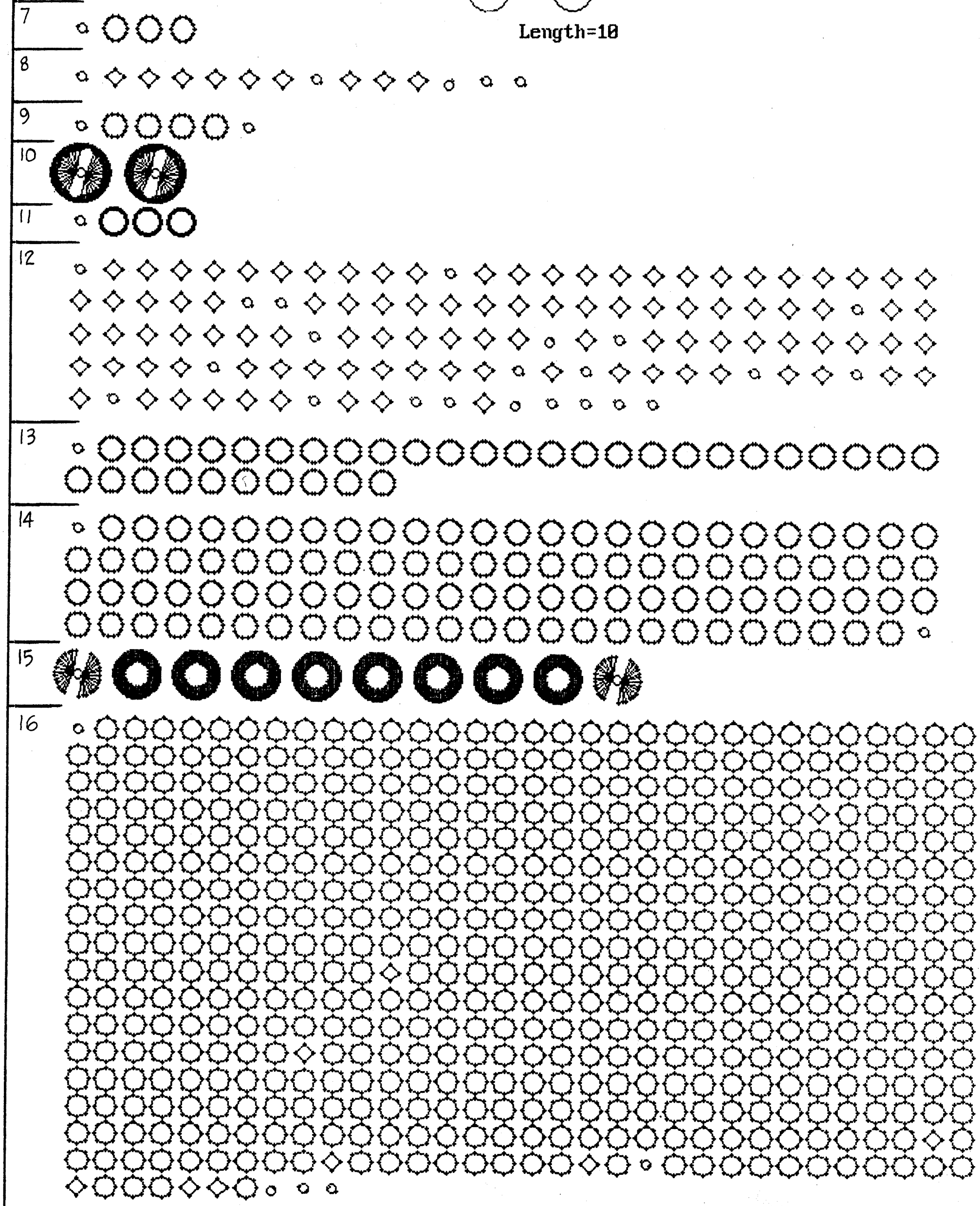


Length=10



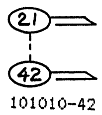


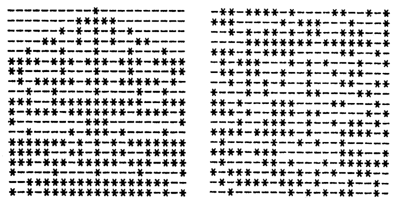


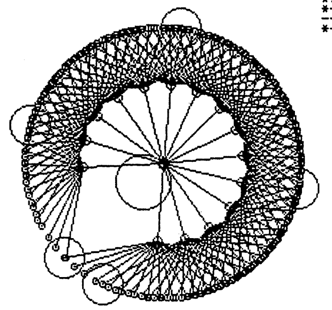



```
5-code  21 =010101

ty. at no(p)s g ml mp
L= 3
 1.000 1(2)2 0 0 1
 2.001 3(2)2 0 0 1
 total NATs = 4
L= 4
 1.0000 1(2)2 0 0 1
 2.0001 2(2)2 0 0 1
 3.0011 4(1)1 0 0 1
 4.0101 1(2)2 0 0 1
 5.0111 2(2)2 0 0 1
 total NATs = 10
L= 5
 1.00000 1(2)32 30 1 16
 total NATs = 1
L= 6
 1.000000 1(2)2 0 0 1
 2.000001 3(2)2 0 0 1
 3.000011 6(2)2 0 0 1
 4.000101 6(2)2 0 0 1
 5.000111 3(2)2 0 0 1
 6.001001 3(2)2 0 0 1
 7.001011 3(2)2 0 0 1
 8.001101 3(2)2 0 0 1
 9.010101 1(2)2 0 0 1
 10.011111 3(2)2 0 0 1
 total NATs = 32
L= 7
 1.0000000 1(2)2 0 0 1
 2.0000001 7(14)14 0 0 1
 3.0001011 1(14)14 0 0 1
 4.0001101 1(14)14 0 0 1
 total NATs = 10
L= 8
 1.00000000 1(2)2 0 0 1
 2.00000001 8(4)4 0 0 1
 3.00000011 8(4)4 0 0 1
 4.00000101 4(4)4 0 0 1
 5.00001011 8(4)4 0 0 1
 6.00001101 8(4)4 0 0 1
 7.00001111 4(4)4 0 0 1
 8.00010001 2(2)2 0 0 1
 9.00010111 8(4)4 0 0 1
 10.00011011 4(4)4 0 0 1
 11.00011111 8(4)4 0 0 1
 12.00110011 4(1)1 0 0 1
 13.01010101 1(2)2 0 0 1
 14.01110111 2(2)2 0 0 1
 total NATs = 70
L= 9
 1.000000000 1(2)2 0 0 1
 2.000000001 9(14)14 0 0 1
 3.000000111 9(14)14 0 0 1
 4.000001011 9(14)14 0 0 1
 5.000001101 9(14)14 0 0 1
 6.001001001 3(2)2 0 0 1
 total NATs = 40
L= 10
 1.0000000000 1(2)512 480 2 16
 2.0101010101 1(2)512 480 2 16
 total NATs = 2
L= 11
 1.00000000000 1(2)2 0 0 1
 2.00000000001 11(62)62 0 0 1
 3.00000001011 11(62)62 0 0 1
 4.00000001101 11(62)62 0 0 1
 total NATs = 34
L= 12
 1.000000000000 1(2)2 0 0 1
 2.000000000001 12(4)4 0 0 1
 3.000000000011 12(4)4 0 0 1
 4.000000000101 6(4)4 0 0 1
 5.000000000111 12(4)4 0 0 1
 6.000000001001 12(4)4 0 0 1
 7.000000001011 12(4)4 0 0 1
 8.000000001101 12(4)4 0 0 1
 9.000000001111 6(4)4 0 0 1
 10.000000010001 6(4)4 0 0 1
 11.000000010011 12(4)4 0 0 1
 12.000000010101 12(2)2 0 0 1
 13.000000010111 12(4)4 0 0 1
 14.000000011001 12(4)4 0 0 1
 15.000000011011 6(4)4 0 0 1
 16.000000011101 12(4)4 0 0 1
 17.000000011111 12(4)4 0 0 1
 18.000000100011 12(4)4 0 0 1
 19.000000100101 12(4)4 0 0 1
 20.000000100111 6(4)4 0 0 1

110.000111101111 6(4)4 0 0 1
111.000111111011 6(4)4 0 0 1
112.001001001001 3(2)2 0 0 1
113.001001011101 6(4)4 0 0 1
114.001010011011 6(4)4 0 0 1
115.001010011111 6(2)2 0 0 1
116.001011001011 3(2)2 0 0 1
117.001011111101 6(4)4 0 0 1
118.001100110011 4(1)1 0 0 1
119.001101001101 3(2)2 0 0 1
120.010101010101 1(2)2 0 0 1
121.011101110111 2(2)2 0 0 1
122.011111011111 3(2)2 0 0 1
total NATs = 1096
```

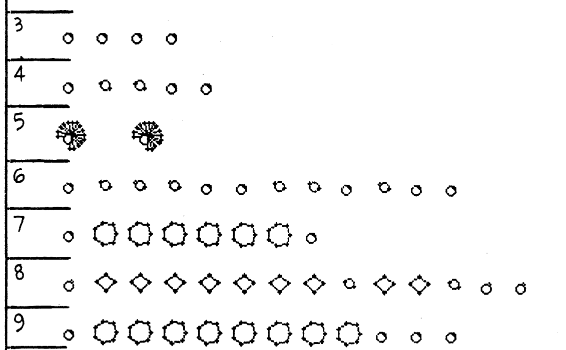
λ ratio = 1 $Z = 1$ 1001011001101001-0110100110010110-rule 2523490710
= 5-code 42 -101010   Length=3 -16

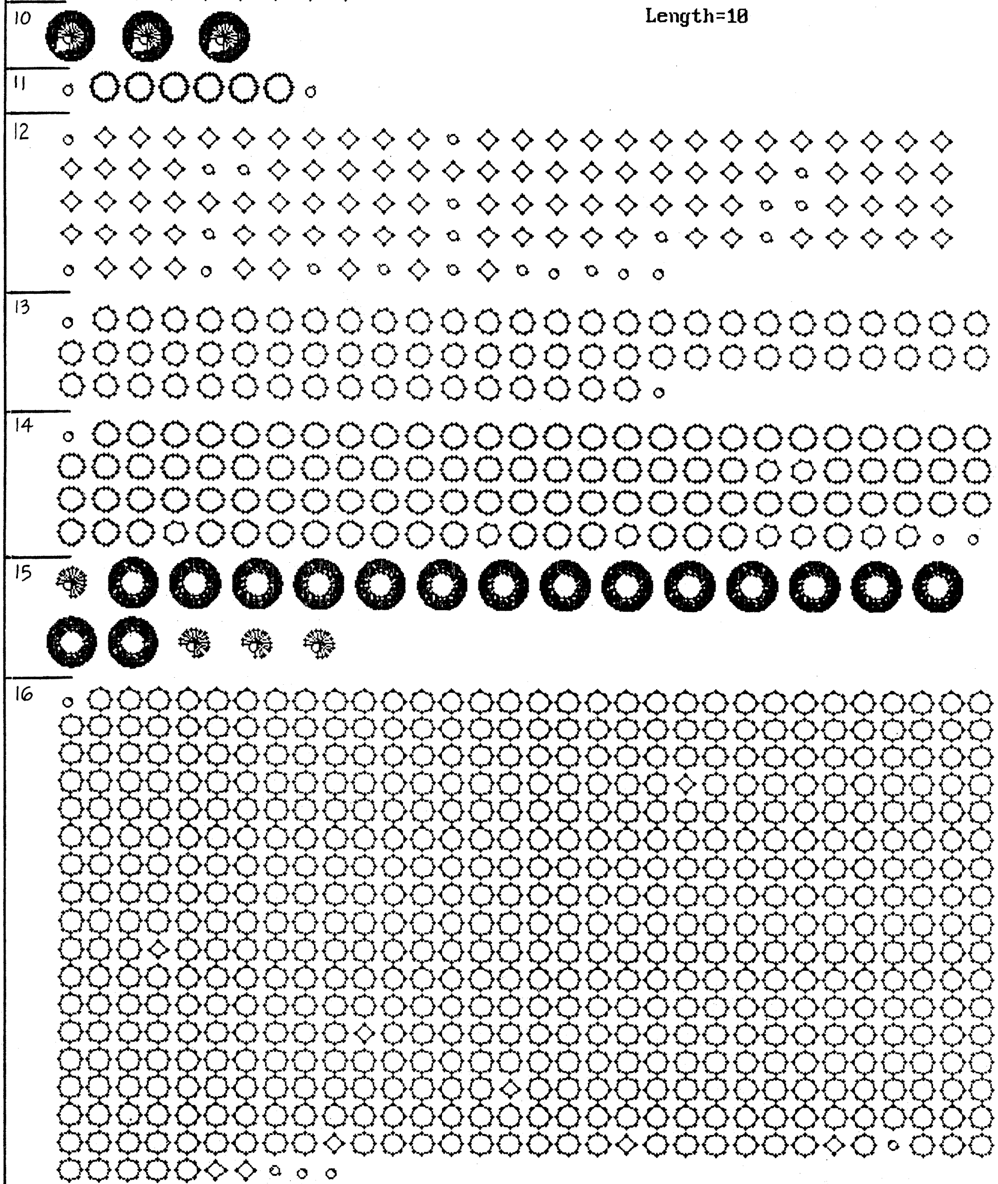






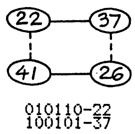
Length=10

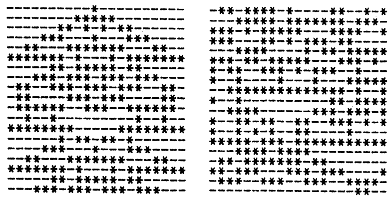


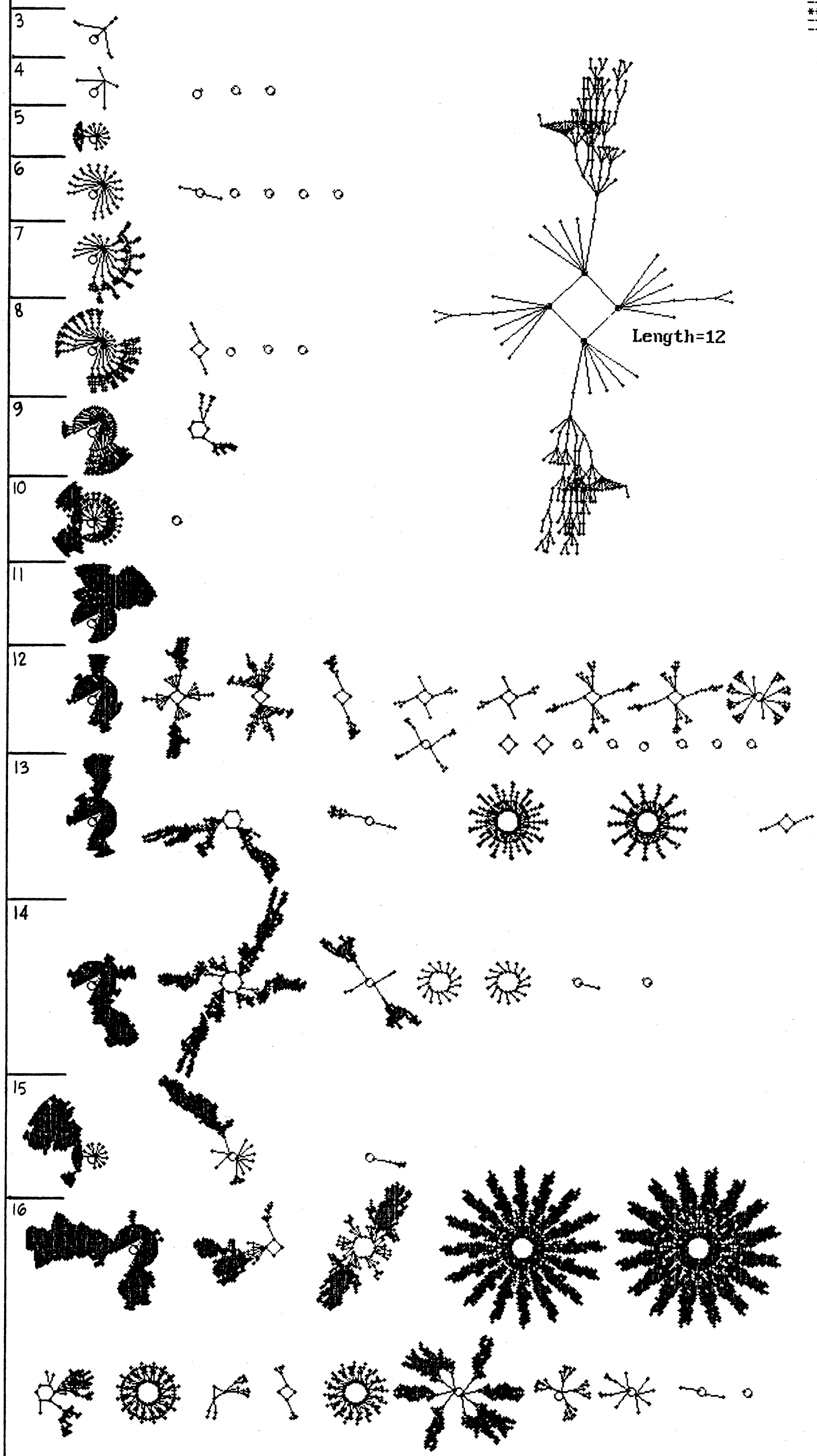



```
5-code  42 =101010

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)1 0 0 1
 2.001 3(1)1 0 0 1
 3.011 3(1)1 0 0 1
 4.111 1(1)1 0 0 1
 total NATs = 8
L= 4
 1.0000 1(1)1 0 0 1
 2.0001 4(2)2 0 0 1
 3.0011 2(2)2 0 0 1
 4.0101 2(1)1 0 0 1
 5.1111 1(1)1 0 0 1
 total NATs = 10
L= 5
 1.00000 1(1)16 15 1 16
 2.11111 1(1)16 15 1 16
 total NATs = 2
L= 6
 1.000000 1(1)1 0 0 1
 2.000001 6(2)2 0 0 1
 3.000011 3(2)2 0 0 1
 4.000101 3(2)2 0 0 1
 5.000111 6(1)1 0 0 1
 6.001001 3(1)1 0 0 1
 7.001011 6(2)2 0 0 1
 8.001111 3(2)2 0 0 1
 9.010101 2(1)1 0 0 1
 10.010111 3(2)2 0 0 1
 11.011011 3(1)1 0 0 1
 12.111111 1(1)1 0 0 1
 total NATs = 40
L= 7
 1.0000000 1(1)1 0 0 1
 2.0000001 7(7)7 0 0 1
 3.0000011 7(7)7 0 0 1
 4.0001011 1(7)7 0 0 1
 5.0001101 1(7)7 0 0 1
 6.0010111 1(7)7 0 0 1
 7.0011101 1(7)7 0 0 1
 8.1111111 1(1)1 0 0 1
 total NATs = 20
L= 8
 1.00000000 1(1)1 0 0 1
 2.00000001 8(4)4 0 0 1
 3.00000011 8(4)4 0 0 1
 4.00000101 4(4)4 0 0 1
 5.00000111 8(4)4 0 0 1
 6.00001011 8(4)4 0 0 1
 7.00001101 8(4)4 0 0 1
 8.00001111 4(4)4 0 0 1
 9.00010001 4(2)2 0 0 1
 10.00010111 8(4)4 0 0 1
 11.00100111 4(4)4 0 0 1
 12.00110011 2(2)2 0 0 1
 13.01010101 2(1)1 0 0 1
 14.11111111 1(1)1 0 0 1
 total NATs = 70
L= 9
 1.000000000 1(1)1 0 0 1
 2.000000001 9(7)7 0 0 1
 3.000000011 9(7)7 0 0 1
 4.000000111 9(7)7 0 0 1
 5.000001001 9(7)7 0 0 1
 6.000001011 9(7)7 0 0 1
 7.000001101 9(7)7 0 0 1
 8.000010111 9(7)7 0 0 1
 9.000011101 9(7)7 0 0 1
 10.001001001 3(1)1 0 0 1
 11.011011011 3(1)1 0 0 1
 12.111111111 1(1)1 0 0 1
 total NATs = 80
L= 10
 1.0000000000 1(1)256 240 2 16
 2.0101010101 2(1)256 240 2 16
 3.1111111111 1(1)256 240 2 16
 total NATs = 4
L= 11
 1.00000000000 1(1)1 0 0 1
 2.00000000001 11(31)31 0 0 1
 3.00000000011 11(31)31 0 0 1
 4.00000001011 11(31)31 0 0 1
 5.00000001101 11(31)31 0 0 1
 6.00000010111 11(31)31 0 0 1
 7.00000011101 11(31)31 0 0 1
 8.11111111111 1(1)1 0 0 1
 total NATs = 68
L= 12
 1.000000000000 1(1)1 0 0 1
 2.000000000001 12(4)4 0 0 1
 3.000000000011 12(4)4 0 0 1
 4.000000000101 6(4)4 0 0 1
 5.000000000111 12(4)4 0 0 1
 6.000000001001 12(4)4 0 0 1
 7.000000001011 12(4)4 0 0 1
 8.000000001101 12(4)4 0 0 1
 9.000000001111 6(4)4 0 0 1
 10.000000010001 6(4)4 0 0 1
 11.000000010011 12(4)4 0 0 1
 12.000000010101 12(2)2 0 0 1
 13.000000010111 12(4)4 0 0 1
 14.000000011001 12(4)4 0 0 1
 15.000000011011 6(4)4 0 0 1
 16.000000011101 12(4)4 0 0 1
 17.000000100011 12(4)4 0 0 1
 18.000000100101 12(4)4 0 0 1
 19.000000100111 6(4)4 0 0 1
 20.000000101001 12(4)4 0 0 1

110.001001101011 6(4)4 0 0 1
111.001001111111 6(4)4 0 0 1
112.001011001011 6(2)2 0 0 1
113.001011001111 12(4)4 0 0 1
114.001100110011 2(2)2 0 0 1
115.001100111111 6(4)4 0 0 1
116.001101100111 12(2)2 0 0 1
117.001110011111 6(4)4 0 0 1
118.001111001111 3(2)2 0 0 1
119.010101010101 2(1)1 0 0 1
120.010111010111 3(2)2 0 0 1
121.011011011011 3(1)1 0 0 1
122.111111111111 1(1)1 0 0 1
total NATs = 1096
```

λ ratio = .75 Z = .75   0110100110010111-1001011101111110-rule   1771542398
= 5-code 22 -010110   Length=3 -16



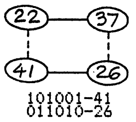


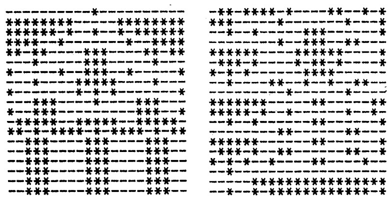


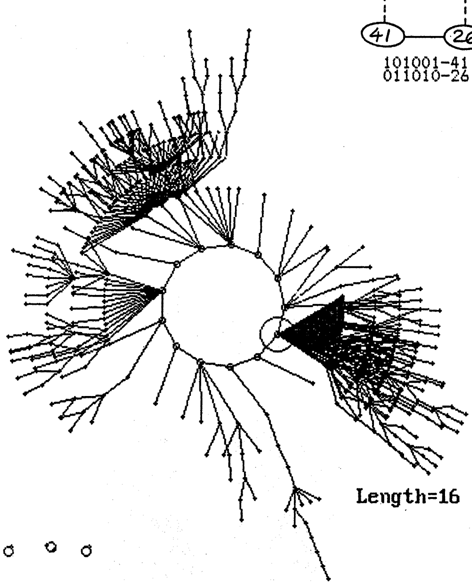

```
5-code  22 =010110

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)8 3 3 3
 total NATs = 1
L= 4
 1.0000 1(1)6 4 2 4
 2.0011 4(1)1 0 0 1
 3.0101 1(2)2 0 0 1
 4.0111 2(2)2 0 0 1
 total NATs = 8
L= 5
 1.00000 1(1)32 30 2 20
 total NATs = 1
L= 6
 1.000000 1(1)32 15 3 15
 2.110111 3(2)4 2 1 2
 3.000111 3(2)2 0 0 1
 4.001011 3(2)2 0 0 1
 5.001101 3(2)2 0 0 1
 6.010101 1(2)2 0 0 1
 total NATs = 14
L= 7
 1.0000000 1(1)128 56 9 14
 total NATs = 1
L= 8
 1.00000000 1(1)214 76 8 20
 2.11011000 4(4)8 2 2 2
 3.00110011 4(1)1 0 0 1
 4.01010101 1(2)2 0 0 1
 5.01110111 2(2)2 0 0 1
 total NATs = 12
L= 9
 1.000000000 1(1)188 90 8 39
 2.101111010 9(6)36 14 6 5
 total NATs = 10
L= 10
 1.0000000000 1(1)1022 710 8 80
 2.0101010101 1(2)2 0 0 1
 total NATs = 2
L= 11
 1.00000000000 1(1)2048 979 17 99
 total NATs = 1
L= 12
 1.000000000000 1(1)288 157 7 139
 2.111011101111 6(4)304 148 13 14
 3.111111111001 6(4)104 54 7 14
 4.101001011111 6(4)42 22 6 6
 5.110001110100 6(4)14 6 3 2
 6.110001100101 6(4)14 6 3 2
 7.110001101111 6(4)48 20 8 4
 8.100011111101 6(4)48 20 8 4
 9.110111110111 3(2)68 42 3 6
 10.111000111000 3(2)18 8 3 3
 11.000110110111 12(4)4 0 0 1
 12.001010011011 6(4)4 0 0 1
 13.001010011111 6(2)2 0 0 1
 14.001011001011 3(2)2 0 0 1
 15.001100110011 4(1)1 0 0 1
 16.001101001101 3(2)2 0 0 1
 17.010101010101 1(2)2 0 0 1
 18.011101110111 2(2)2 0 0 1
 total NATs = 86
L= 13
 1.0000000000000 1(1)1042 494 16 195
 2.1111100111111 13(6)488 224 28 24
 3.1001001001111 13(2)22 10 6 4
 4.1100111011110 1(52)208 78 5 3
 5.1100110111101 1(52)208 78 5 3
 6.0000010111011 13(4)8 2 2 2
 total NATs = 42
L= 14
 1.00000000000000 1(1)4664 2352 20 308
 2.00011101110001 7(10)1386 672 37 37
 3.10010010011111 7(2)274 162 15 23
 4.00001101011111 1(14)28 14 1 2
 5.00001111101011 1(14)28 14 1 2
 6.00010011111001 14(2)3 1 1 2
 7.01010101010101 1(2)2 0 0 1
 total NATs = 32
L= 15
 1.000000000000000 1(1)8963 7170 20 518
 2.111001010011111 15(2)1579 1034 37 16
 3.000100111111001 15(2)8 3 4 2
 total NATs = 31
L= 16
 1.0000000000000000 1(1)28598 15596 41 756
 2.0000000000000011 16(4)850 501 22 51
 3.1100000000011111 8(14)1090 586 19 24
 4.0111111110110111 1(48)4384 2192 24 11
 5.0111111110111011 1(48)4384 2192 24 11
 6.1100000111110111 16(6)164 84 9 18
 7.0111011011000101 1(48)176 80 3 3
 8.1011011011100011 16(3)20 9 3 4
 9.0011011000110110 4(4)12 6 2 3
 10.1011011101010001 1(48)176 80 3 3
 11.1011101110111011 2(2)1042 576 15 8
 12.1100110011001100 4(1)33 20 3 5
 13.0000100111111001 16(2)16 10 3 5
 14.1111111001000100 8(2)4 2 1 2
 15.0101010101010101 1(2)2 0 0 1
 total NATs = 96
```

λ ratio = .75 $Z = .75$

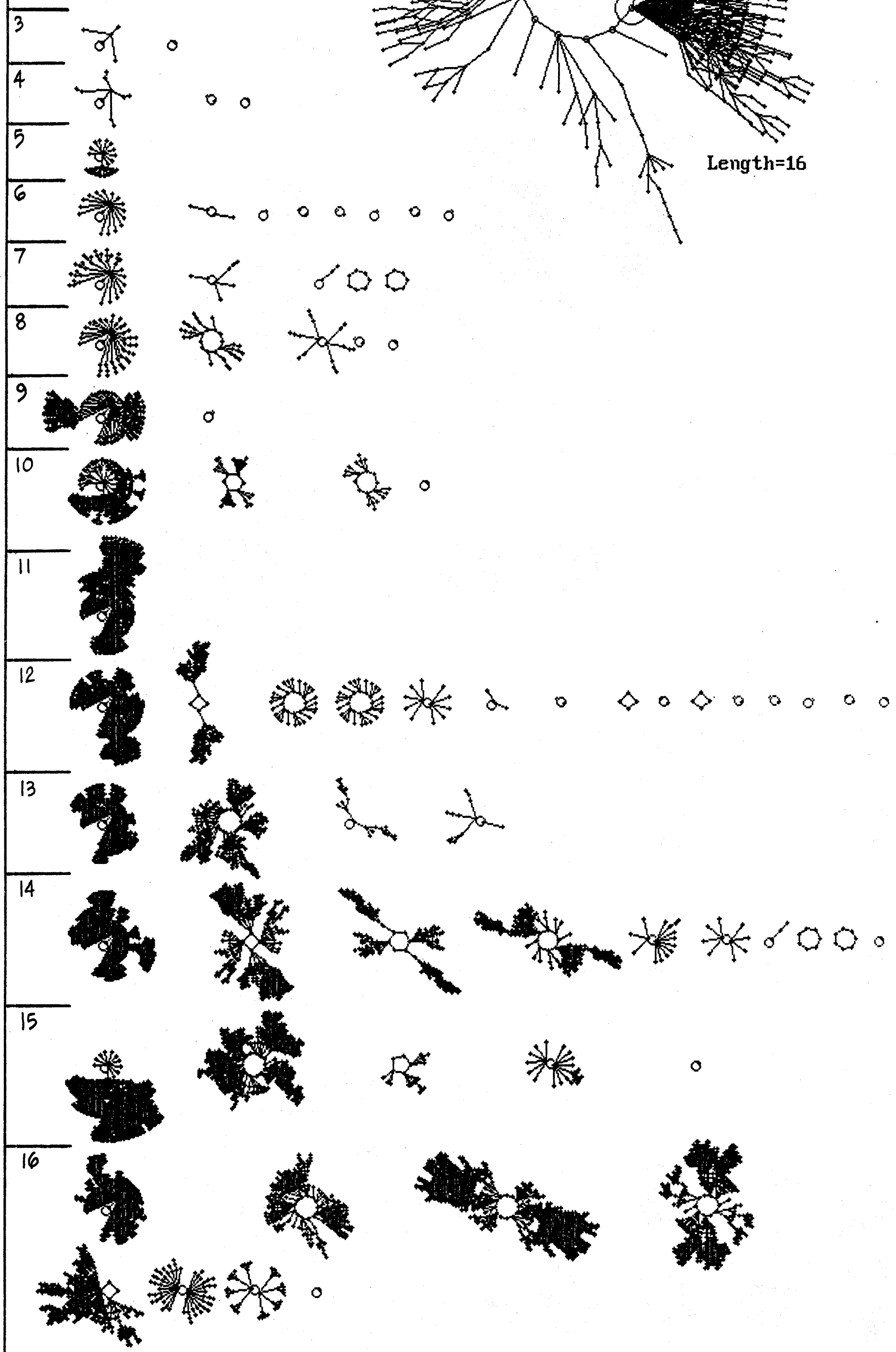
1001011001101000-0110100010000001-rule   2523424897
= 5-code 41 -101001   Length=3 -16





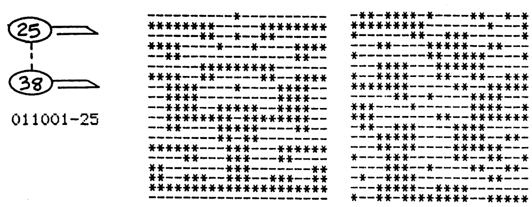


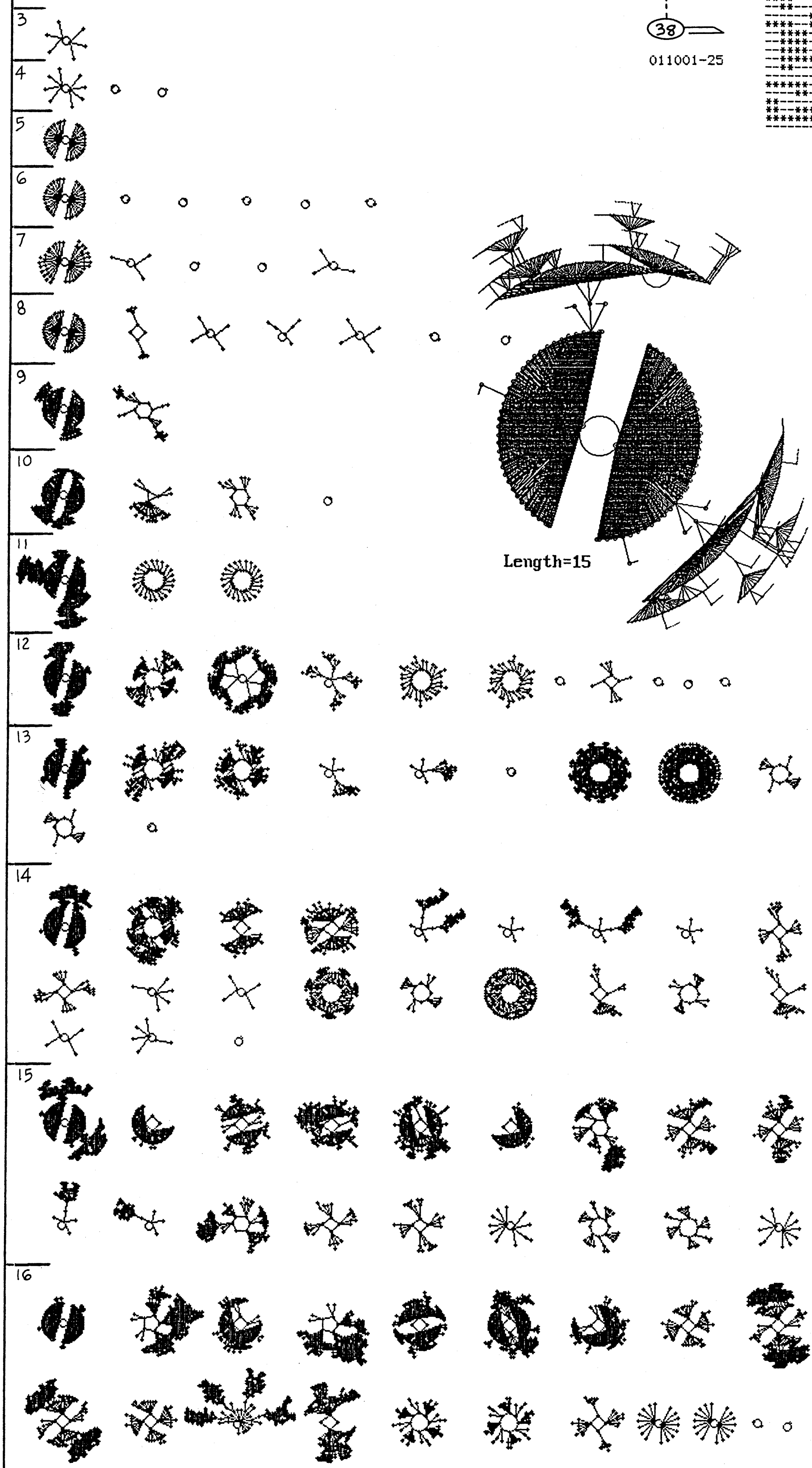
Length=16




```
5-code  41 =101001

ty. at no(p)s g ml mp
L= 3
 1.111 1(1)5 3 2 3
 2.011 3(1)1 0 0 1
 total NATs = 4
L= 4
 1.1111 1(1)10 4 3 4
 2.0011 2(2)2 0 0 1
 3.0101 2(1)1 0 0 1
 total NATs = 5
L= 5
 1.11111 1(1)32 30 2 20
 total NATs = 1
L= 6
 1.111111 1(1)17 15 2 15
 2.000001 3(2)4 2 1 2
 3.000111 6(1)1 0 0 1
 4.001011 6(2)2 0 0 1
 5.001111 3(2)2 0 0 1
 6.010101 2(1)1 0 0 1
 7.010111 3(2)2 0 0 1
 8.011011 3(1)1 0 0 1
 total NATs = 27
L= 7
 1.1111111 1(1)37 21 4 14
 2.0000001 7(2)8 4 3 4
 3.0000111 7(1)3 1 2 2
 4.0010111 1(7)7 0 0 1
 5.0011101 1(7)7 0 0 1
 total NATs = 17
L= 8
 1.11111111 1(1)42 20 4 20
 2.00000001 4(10)34 10 3 4
 3.00000011 4(2)18 6 3 4
 4.00110011 2(2)2 0 0 1
 5.01010101 2(1)1 0 0 1
 total NATs = 13
L= 9
 1.111111111 1(1)509 216 12 39
 2.011011011 3(1)1 0 0 1
 total NATs = 4
L= 10
 1.1111111111 1(1)712 500 6 80
 2.0000000001 5(6)40 30 2 12
 3.0000000011 5(8)22 12 2 4
 4.0101010101 2(1)1 0 0 1
 total NATs = 13
L= 11
 1.11111111111 1(1)2048 979 20 99
 total NATs = 1
L= 12
 1.111111111111 1(1)1837 889 14 139
 2.100111110011 6(4)322 156 12 14
 3.000000100111 2(12)36 24 1 4
 4.000000111001 2(12)36 24 1 4
 5.000001000001 3(2)12 10 1 6
 6.000111000111 6(1)3 2 1 3
 7.001011001011 6(2)2 0 0 1
 8.001011001111 12(4)4 0 0 1
 9.001100110011 2(2)2 0 0 1
 10.001100111111 6(4)4 0 0 1
 11.001101100111 12(2)2 0 0 1
 12.001111001111 3(2)2 0 0 1
 13.010101010101 2(1)1 0 0 1
 14.010111010111 3(2)2 0 0 1
 15.011011011011 3(1)1 0 0 1
 total NATs = 69
L= 13
 1.1111111111111 1(1)717 325 6 195
 2.0000000000001 13(18)533 241 13 24
 3.0011100001110 13(1)27 14 7 3
 4.0000001000001 13(2)15 6 4 4
 total NATs = 40
L= 14
 1.11111111111111 1(1)3019 1505 9 308
 2.11000010000111 7(4)1026 518 13 37
 3.10010101001111 7(6)356 162 19 11
 4.00011110000000 7(8)486 252 18 22
 5.00000001000001 7(2)16 12 3 10
 6.00000010000011 14(2)10 8 1 5
 7.00001110000111 7(1)3 1 2 2
 8.00101110010111 1(7)7 0 0 1
 9.00111010011101 1(7)7 0 0 1
 10.01010101010101 2(1)1 0 0 1
 total NATs = 54
L= 15
 1.111111111111111 1(1)11390 7620 20 518
 2.000000000000001 15(14)1371 966 11 46
 3.000000000001001 15(5)28 20 3 11
 4.000000100000011 15(2)26 21 3 9
 5.011011011011011 3(1)1 0 0 1
 total NATs = 49
L= 16
 1.1111111111111111 1(1)2874 1652 10 756
 2.0000000000000001 16(13)717 383 10 77
 3.0000010010000000 8(12)3950 2140 26 51
 4.1000111110001111 4(10)1654 946 14 40
 5.1000000000100100 16(4)791 432 16 37
 6.0000001100000011 4(2)58 26 3 14
 7.0011001100110011 2(2)42 32 2 5
 8.0101010101010101 2(1)1 0 0 1
 total NATs = 53
```

λ ratio = 1 $Z = .5$ 0111111011101000-1110100010000001-rule   2129193089
= 5-code   25 -011001   Length=3 -16



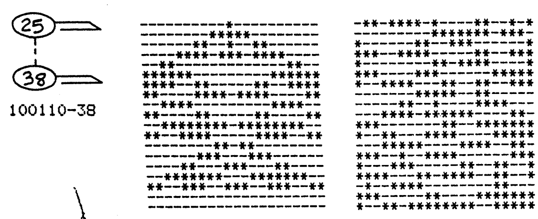


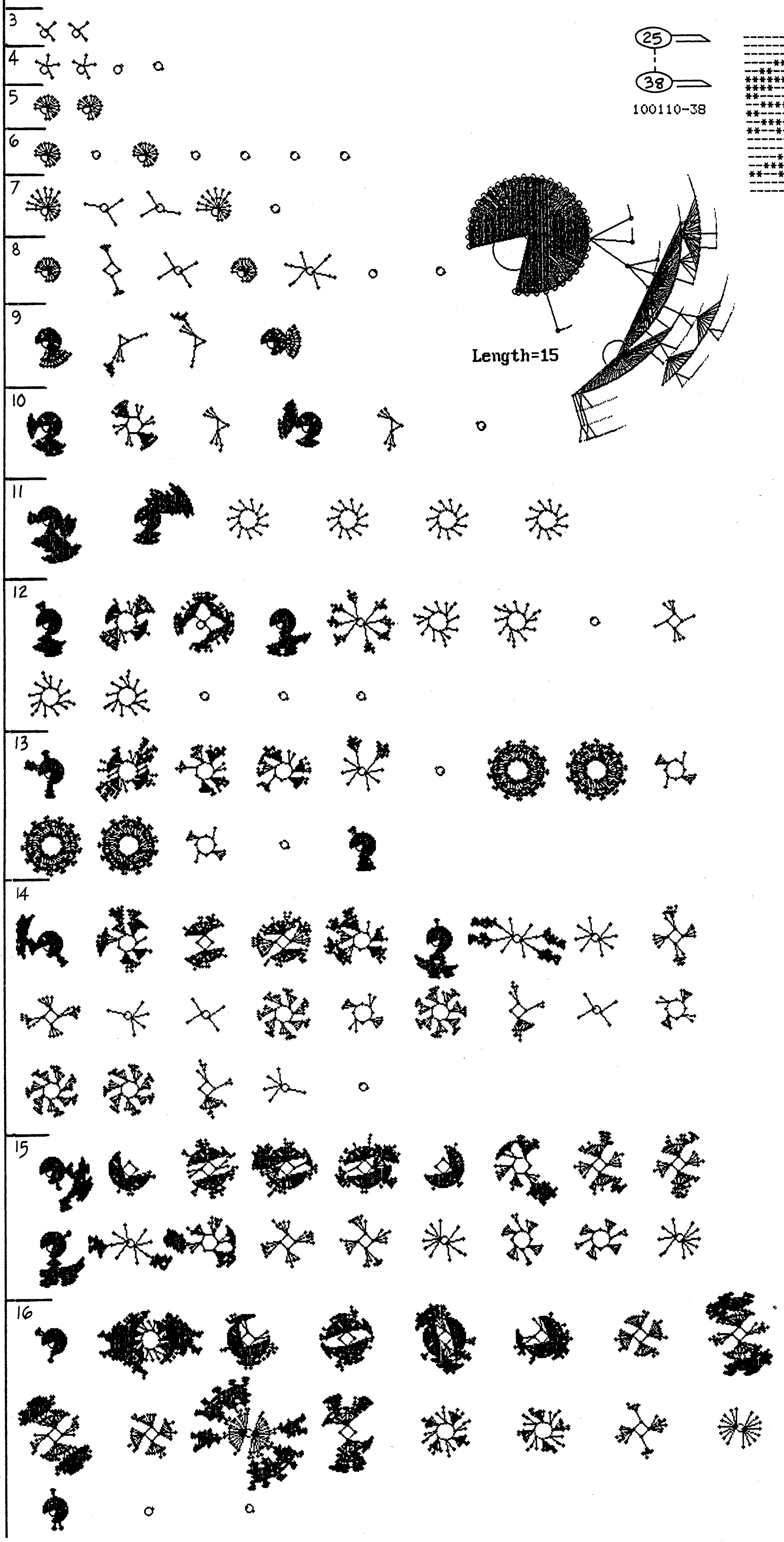
5-code 25 =011001

ty. at no(p)s g ml mp

L= 3
1.000 1(2)8 6 1 4
total NATs = 1
L= 4
1.0000 1(2)10 8 1 5
2.0011 2(2)2 0 0 1
3.0101 2(1)1 0 0 1
total NATs = 5
L= 5
1.00000 1(2)32 30 1 16
total NATs = 1
L= 6
1.000000 1(2)32 30 1 16
2.000001 3(2)2 0 0 1
3.000111 6(1)1 0 0 1
4.001011 6(2)2 0 0 1
5.010101 2(1)1 0 0 1
6.011111 3(2)2 0 0 1
total NATs = 21
L= 7
1.0000000 1(2)44 28 2 15
2.0000001 7(2)5 3 1 3
3.0000111 7(1)1 0 0 1
4.0001111 7(1)1 0 0 1
5.1001111 7(2)5 3 1 3
total NATs = 29
L= 8
1.00000000 1(2)42 40 1 21
2.00111000 8(4)14 8 2 4
3.00000011 4(2)6 4 1 3
4.00001111 8(1)6 3 2 4
5.10011111 4(2)6 4 1 3
6.00110011 2(2)2 0 0 1
7.01010101 2(1)1 0 0 1
total NATs = 29
L= 9
1.000000000 1(2)152 96 3 40
2.011111100 9(6)40 24 5 6
total NATs = 10
L= 10
1.0000000000 1(2)432 310 4 76
2.0011111000 10(3)39 30 2 21
3.0000000011 10(6)20 12 2 4
4.0101010101 2(1)1 0 0 1
total NATs = 23
L= 11
1.00000000000 1(2)1960 1342 9 100
2.00100011111 1(22)44 22 1 2
3.00000111011 1(22)44 22 1 2
total NATs = 3
L= 12
1.000000000000 1(2)928 770 5 140
2.000000000001 12(12)120 90 3 14
3.001100110011 2(2)464 324 4 24
4.001110001110 6(1)41 28 4 5
5.001100011111 4(18)48 24 2 3
6.000000111011 4(18)48 24 2 3
7.000001000001 3(2)2 0 0 1
8.000001011111 12(4)12 8 1 4
9.001011001011 6(2)2 0 0 1
10.010101010101 2(1)1 0 0 1
11.011111011111 3(2)2 0 0 1
total NATs = 55
L= 13
1.0000000000000 1(2)522 390 3 196
2.0000000000001 13(12)230 176 5 27
3.0000000000011 13(14)194 148 3 21
4.0001110001111 13(1)29 19 5 6
5.0011100001110 13(1)29 19 5 6
6.0000001000001 13(2)2 0 0 1
7.0011000000111 1(78)442 260 3 5
8.0011000011111 1(78)442 260 3 5
9.0000001011111 13(8)18 10 1 5
10.0000001111101 13(8)18 10 1 5
11.0011111011111 13(2)2 0 0 1
total NATs = 107
L= 14
1.00000000000000 1(2)3432 2968 7 309
2.00000000000011 14(14)452 356 5 37
3.00111111100000 7(4)82 62 3 24
4.00000000000111 14(4)112 74 3 22
5.00011100001110 7(1)97 60 9 6
6.00111100001110 14(1)5 4 1 5
7.00111100011110 7(1)97 60 9 6
8.00111100011100 14(1)5 4 1 5
9.00111110000111 14(4)32 20 3 5
10.00000001001111 14(4)32 20 3 5
11.00000010000001 7(2)7 5 1 5
12.00000010000011 14(2)6 4 1 3
13.00110000001111 2(14)126 98 2 5
14.00000010011111 14(8)22 14 1 6
15.00110000111111 2(14)126 98 2 5
16.00000010111111 14(4)25 15 3 10
17.00000011011111 14(8)22 14 1 6
18.00000011111101 14(4)25 15 3 10
19.10011111011111 14(2)6 4 1 3
20.10011111001111 7(2)7 5 1 5
21.01010101010101 2(1)1 0 0 1
total NATs = 210
L= 15
1.000000000000000 1(2)7928 6906 8 514
2.000000000000011 15(4)91 83 2 51
3.001111111100000 15(4)118 92 3 31
4.000000000000111 15(4)239 175 5 47
5.001100011000000 15(4)239 175 5 47
6.001111110000000 15(4)91 83 2 51
7.000011000011111 15(6)226 164 8 12
8.001111000001100 15(4)86 60 4 11
9.000011110000110 15(4)86 60 4 11
10.000111100011110 15(1)52 40 7 9
11.000111000011110 15(1)52 40 7 9
12.001111100011111 15(6)226 164 8 12
13.001111000000110 15(4)32 22 2 5
14.001111000000011 15(4)32 22 2 5
15.000000100000011 15(2)13 10 2 7
16.001100000011111 15(8)30 20 2 5
17.001100001111011 15(8)30 20 2 5
18.001111110011111 15(2)13 10 2 7
total NATs = 256

λ ratio = 1   Z = .5   1000000100010111-0001011101111110-rule 2165774206
= 5-code 38 -100110   Length=3 -16





5-code 38 =100110

ty. at no(p)s g mi mp

L= 3
1.000 1(1)4 3 1 4
2.111 1(1)4 3 1 4
total NATs = 2

L= 4
1.0000 1(1)5 4 1 5
2.1111 1(1)5 4 1 5
3.0011 4(1)1 0 0 1
4.0101 1(2)2 0 0 1
total NATs = 7

L= 5
1.00000 1(1)16 15 1 16
2.11111 1(1)16 15 1 16
total NATs = 2

L= 6
1.000000 1(1)16 15 1 16
2.000001 6(2)2 0 0 1
3.111111 1(1)16 15 1 16
4.000111 3(2)2 0 0 1
5.001011 3(2)2 0 0 1
6.001101 3(2)2 0 0 1
7.010101 1(2)2 0 0 1
total NATs = 18

L= 7
1.0000000 1(1)22 14 2 15
2.0000001 7(2)5 3 1 3
3.0000011 7(2)5 3 1 3
4.1111111 1(1)22 14 2 15
5.0000111 7(2)2 0 0 1
total NATs = 23

L= 8
1.00000000 1(1)21 20 1 21
2.11000111 8(4)14 8 2 4
3.00000011 8(2)6 4 1 3
4.11111111 1(1)21 20 1 21
5.00001111 4(2)12 6 2 4
6.00110011 4(1)1 0 0 1
7.01010101 1(2)2 0 0 1
total NATs = 27

L= 9
1.000000000 1(1)76 48 3 40
2.100000011 9(3)20 12 5 6
3.000000011 9(3)20 12 5 6
4.111111111 1(1)76 48 3 40
total NATs = 20

L= 10
1.0000000000 1(1)216 155 4 76
2.1100000111 5(6)78 60 2 21
3.0000000011 10(3)10 6 2 4
4.1111111111 1(1)216 155 4 76
5.1100111111 10(3)10 6 2 4
6.0101010101 1(2)2 0 0 1
total NATs = 28

L= 11
1.00000000000 1(1)980 671 9 100
2.11111111111 1(1)980 671 9 100
3.11011100000 1(11)22 11 1 2
4.00000111011 1(11)22 11 1 2
5.11111000100 1(11)22 11 1 2
6.11110010001 1(11)22 11 1 2
total NATs = 6

L= 12
1.000000000000 1(1)464 385 5 140
2.000000000001 12(12)120 90 3 14
3.001100110011 4(1)232 162 4 24
4.111111111111 1(1)464 385 5 140
5.110001110001 3(2)82 56 4 5
6.110011100000 4(9)24 12 2 3
7.000000111011 4(9)24 12 2 3
8.000001000001 6(2)2 0 0 1
9.000001011111 12(4)12 8 1 4
10.000010011111 4(9)24 12 2 3
11.000011111001 4(9)24 12 2 3
12.001011001011 3(2)2 0 0 1
13.001101001101 3(2)2 0 0 1
14.010101010101 1(2)2 0 0 1
total NATs = 62

L= 13
1.0000000000000 1(1)261 195 3 196
2.0000000000001 13(12)230 176 5 27
3.0000000000011 13(7)97 74 3 21
4.1100000011111 13(7)97 74 3 21
5.1110001110000 13(2)58 38 5 6
6.0000001000001 13(2)2 0 0 1
7.1100111111000 1(39)221 130 3 5
8.1100111100000 1(39)221 130 3 5
9.0000001011111 13(8)18 10 1 5
10.1100110001111 1(39)221 130 3 5
11.0000001110011 1(39)221 130 3 5
12.0000001111101 13(8)18 10 1 5
13.0000010000011 13(2)2 0 0 1
14.1111111111111 1(1)261 195 3 196
total NATs = 110

L= 14
1.00000000000000 1(1)1716 1484 7 309
2.00000000000011 14(7)226 178 5 37
3.11000000011111 7(4)82 62 3 24
4.00000000000111 14(4)112 74 3 22
5.11000000111111 14(7)226 178 5 37
6.11111111111111 1(1)1716 1484 7 309
7.11100011110001 7(2)194 120 9 6
8.11000011110001 14(2)10 8 1 5
9.11000001111000 14(4)32 20 3 5
10.00000001001111 14(4)32 20 3 5
11.00000010000001 7(2)7 5 1 5
12.00000010000011 14(2)6 4 1 3
13.11001111110000 2(7)63 49 2 5
14.00000010011111 14(8)22 14 1 6
15.11001111000000 2(7)63 49 2 5
16.00000010111111 14(4)25 15 3 10
17.00000011000001 14(2)6 4 1 3
18.00000011011111 14(8)22 14 1 6
19.11001100001111 2(7)63 49 2 5
20.00000011110011 2(7)63 49 2 5
21.00000011111101 14(4)25 15 3 10
22.00000110000011 7(2)7 5 1 5
23.01010101010101 1(2)2 0 0 1
total NATs = 207

222

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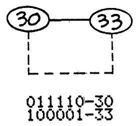
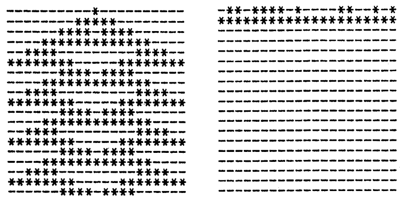
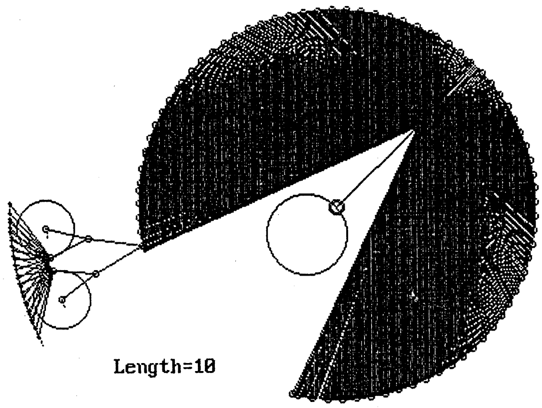
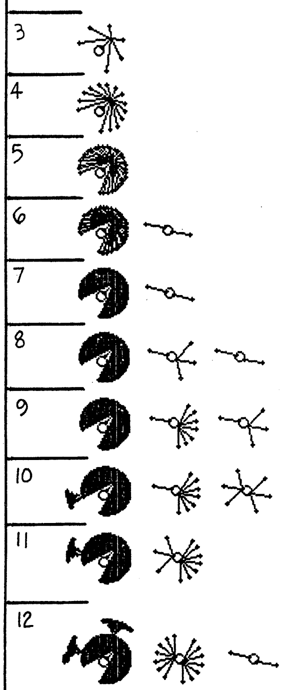
λ ratio = .125 $Z = .125$

0111111111111111-1111111111111110-rule   2147483646
= 5-code 30 -011110   Length=3 -12










```
5-code  30 =011110

ty. at no(p)s g ml mp

L= 3
 1.000 1(1)8 6 2 6
 total NATs = 1
L= 4
 1.0000 1(1)16 14 2 14
 total NATs = 1
L= 5
 1.00000 1(1)32 30 2 30
 total NATs = 1
L= 6
 1.000000 1(1)52 50 2 50
 2.110111 3(2)4 2 1 2
 total NATs = 4
L= 7
 1.0000000 1(1)100 98 2 98
 2.1100111 7(2)4 2 1 2
 total NATs = 8
L= 8
 1.00000000 1(1)192 190 2 190
 2.11000111 8(2)6 4 1 4
 3.11001111 4(2)4 2 1 2
 total NATs = 13
L= 9
 1.000000000 1(1)368 366 2 366
 2.110000111 9(2)10 8 1 8
 3.110001111 9(2)6 4 1 4
 total NATs = 19
L= 10
 1.0000000000 1(1)884 857 5 712
 2.1100001111 10(2)10 8 1 8
 3.1100011111 5(2)8 6 1 4
 total NATs = 16
L= 11
 1.00000000000 1(1)1916 1859 5 1364
 2.11000011111 11(2)12 10 1 8
 total NATs = 12
L= 12
 1.000000000000 1(1)3988 3884 6 2630
 2.110000111111 6(2)16 14 1 8
 3.110111110111 3(2)4 2 1 2
 total NATs = 10
```

224
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APPENDIX 3

Mutants

A3.1 The Effect of Mutating a Rule on the Basin of Attraction (see section 4.2)

Diagrams A3.1-16 show a sequence of basins of attraction (or basin fields) corresponding to a sequence of mutated rules.

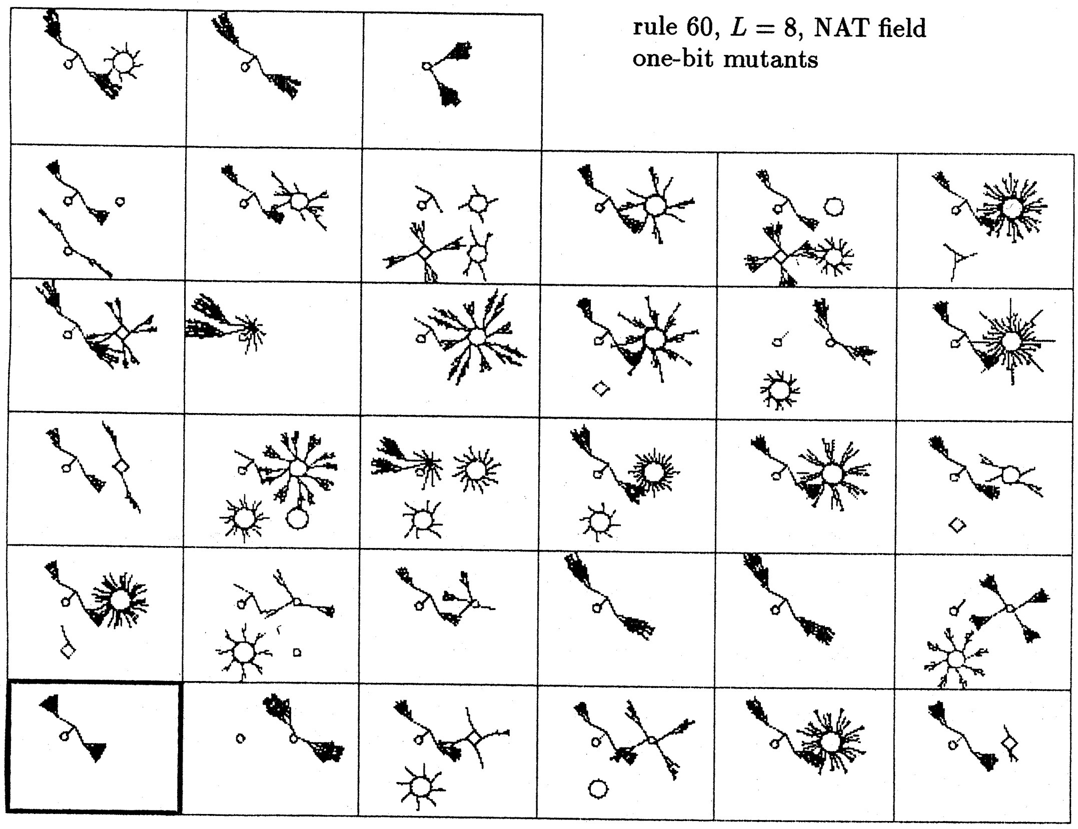
The mutations are made to the rule's $n = 5$ rule table, even if the rule is an $n = 3$ rule ($n = 3$ rules can be expressed as $n = 5$ rules; see 3.3.10). The rule table consists of a 32-bit string arranged in conventional order (see 3.3.9). The mutations are made to successive bits in the rule table from left to right. The series of mutated rules consists of a source rule, and 32 one-bit mutants. In diagrams A3.2 and A3.10 only, the one-bit mutations are cumulative, so that the source rule is progressively changed to its complement.

In the diagrams, the source rule is located in the botton left-hand corner frame (bold outline); successive mutants are located to the right of the source rule, and then from left to right in ascending rows.

Diagrams A3.1–2 show the basin of attraction field. Diagrams A3.3–16 show single basins.

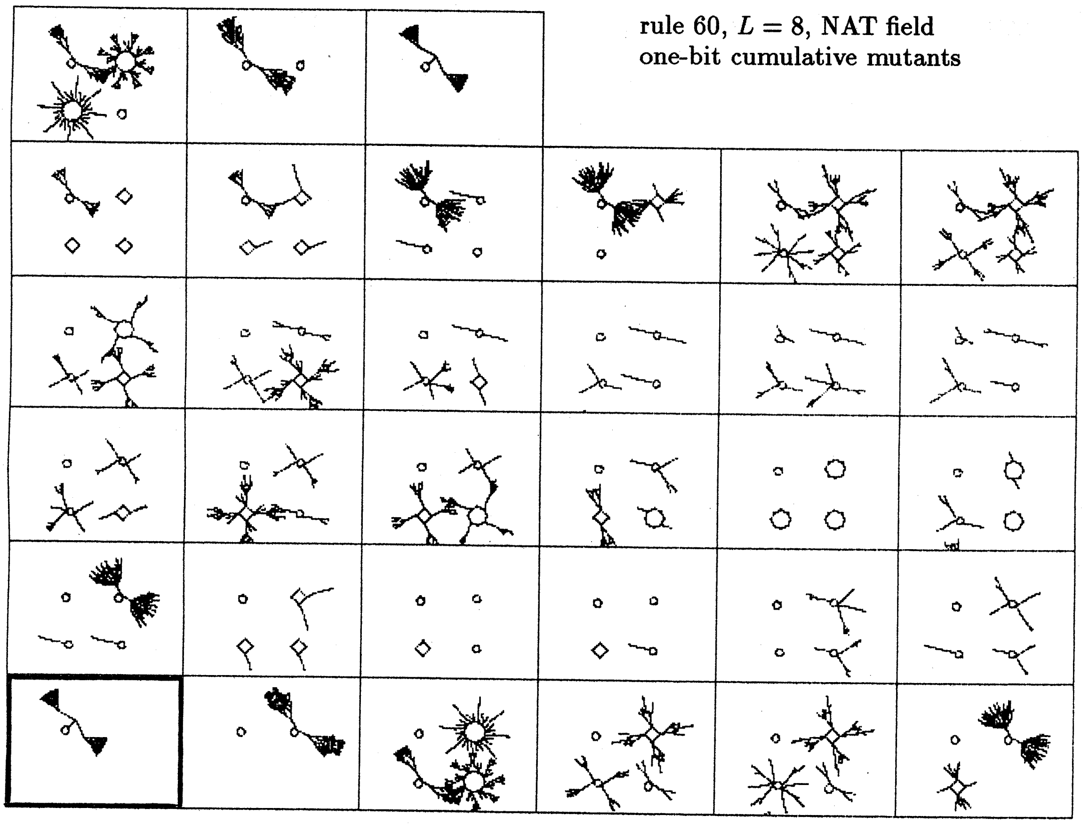
The rule table may be thought of as analogous to a genetic code or ***genotype***, and the basin structure as the resulting ***phenotype***. Small mutations to the genotype result, in general, to small changes in the phenotype.

A3.1



rule 60, $L = 8$, NAT field
one-bit mutants

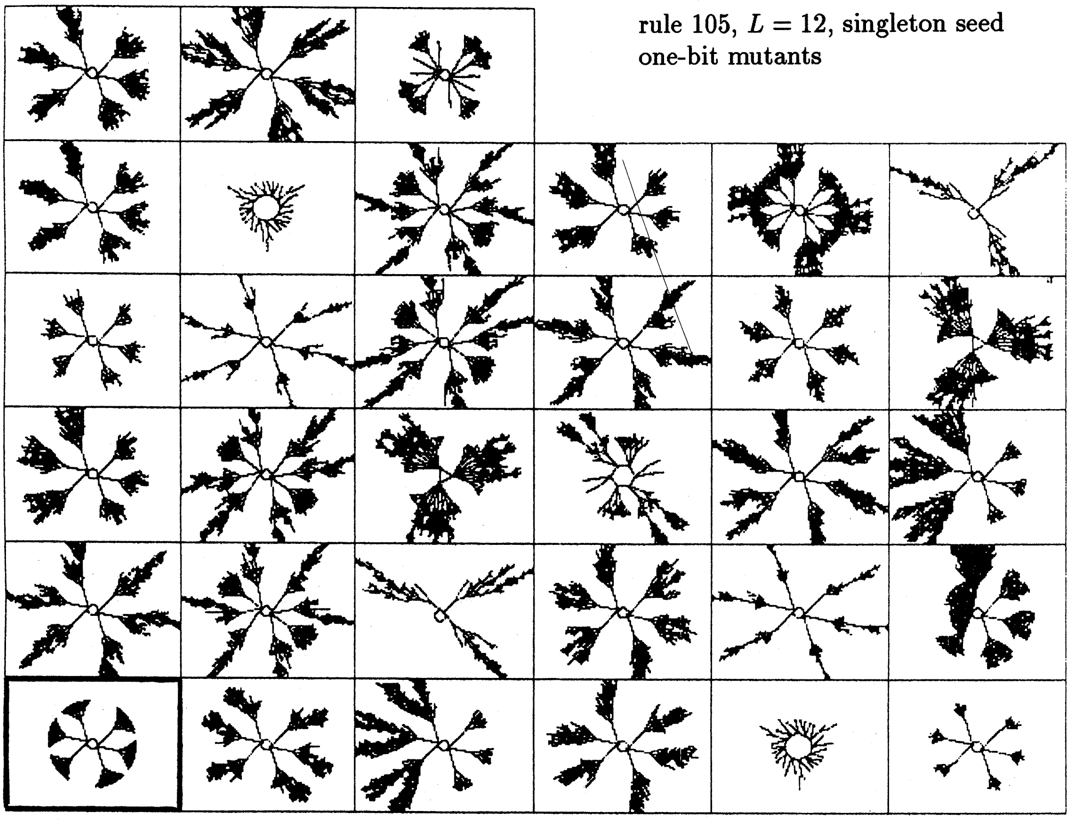
A3.2



rule 60, $L = 8$, NAT field
one-bit cumulative mutants

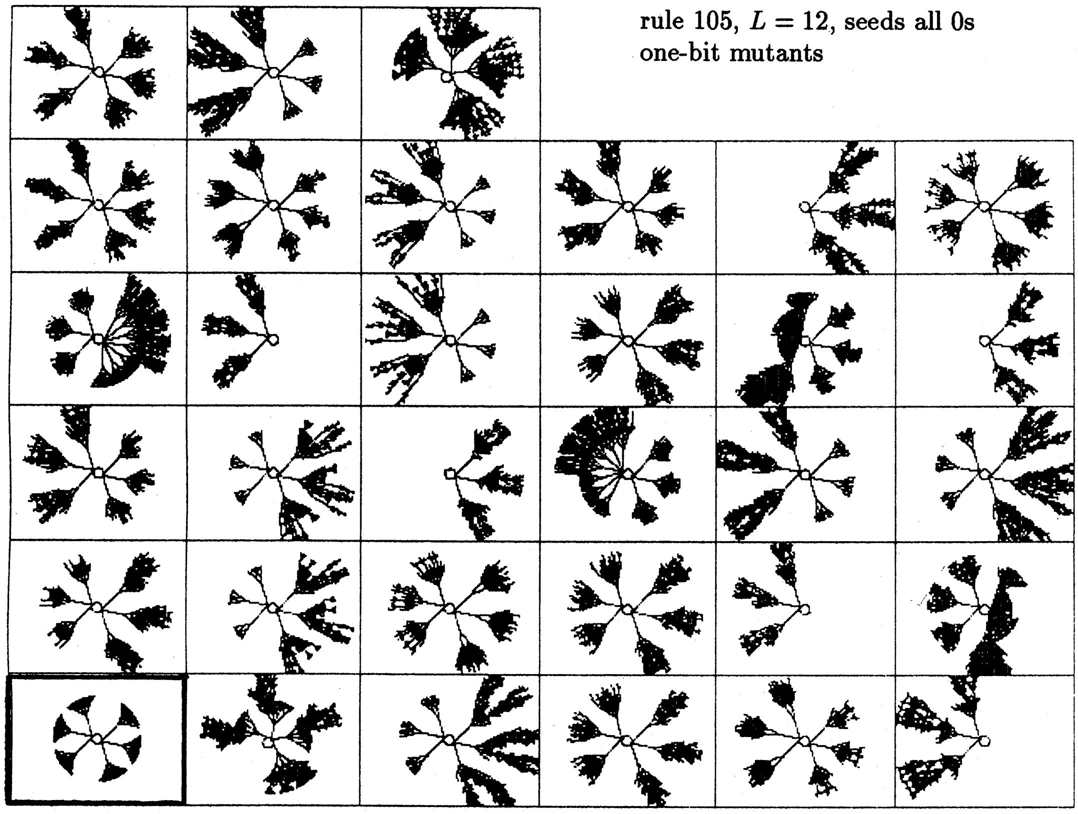
A3.3

rule 105, $L = 12$, singleton seed
one-bit mutants

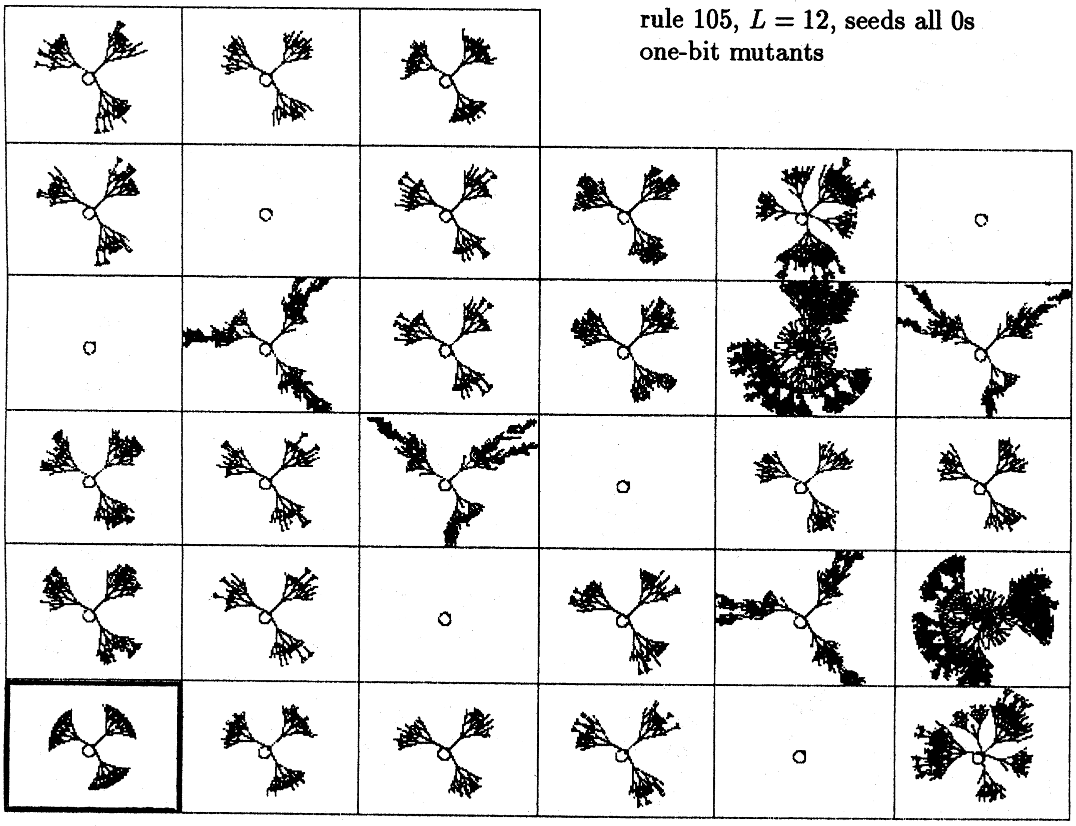


A3.4

rule 105, $L = 12$, seeds all 0s
one-bit mutants

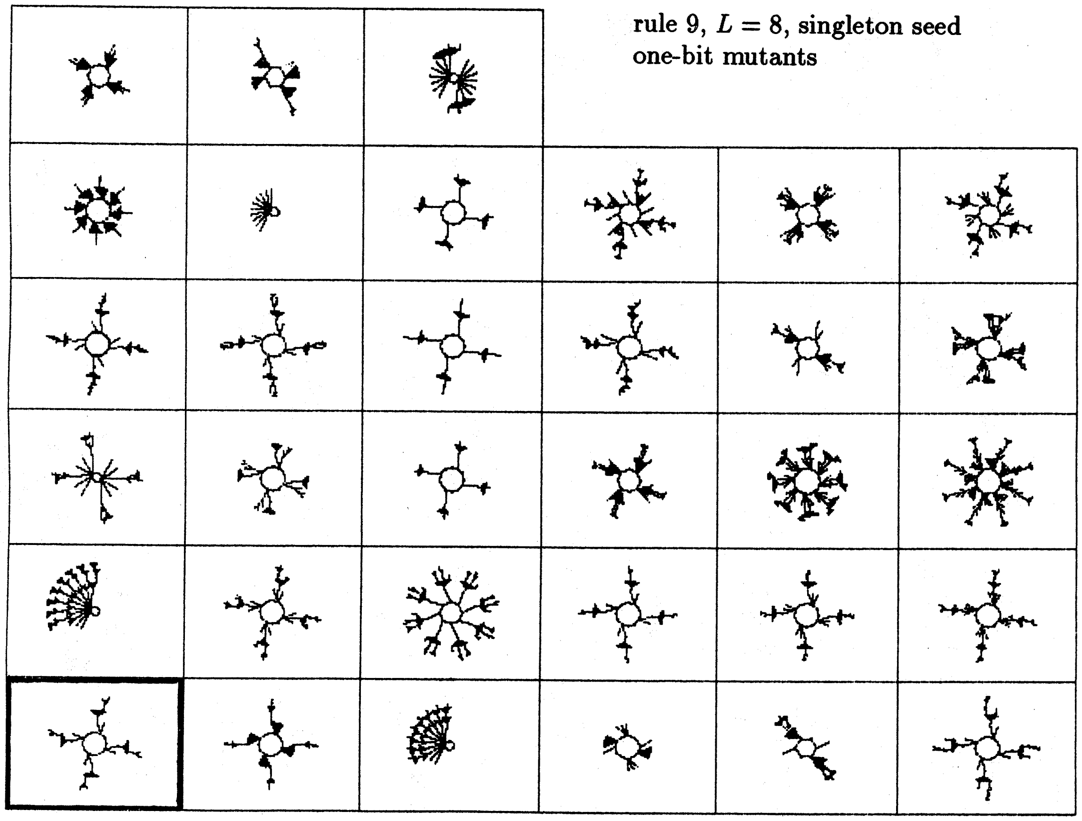


A3.5



rule 105, $L = 12$, seeds all 0s
one-bit mutants

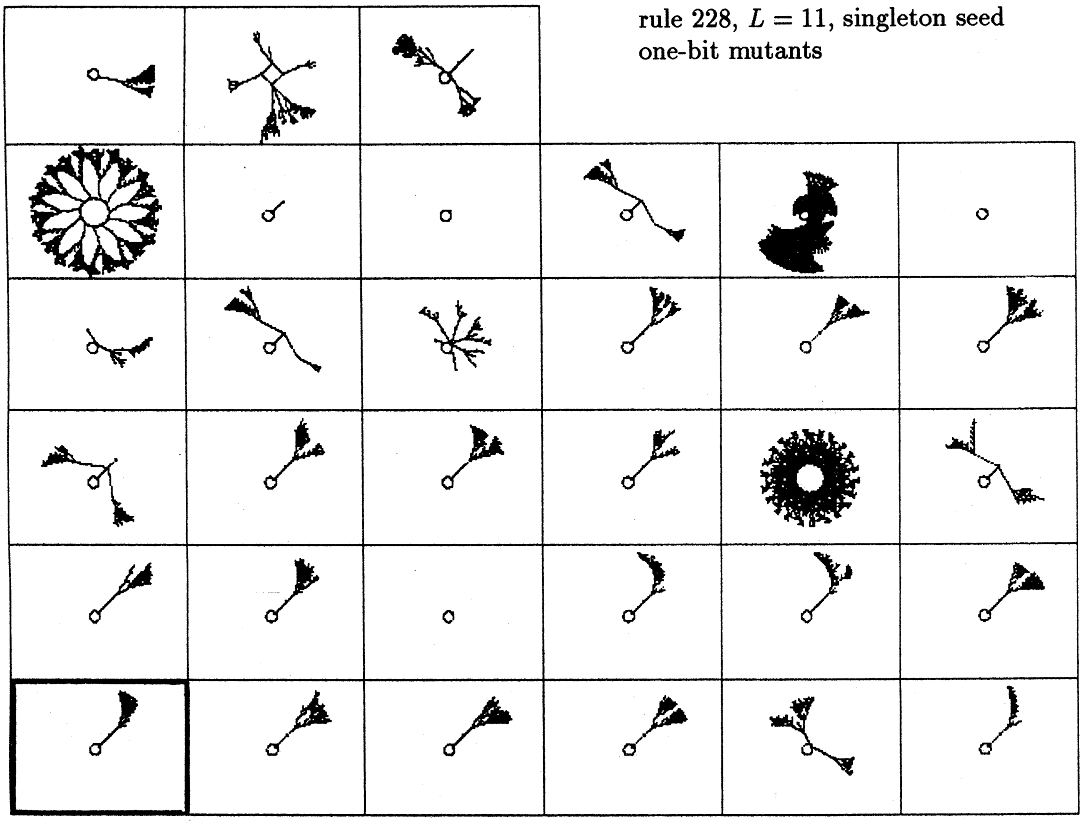
A3.6



rule 9, $L = 8$, singleton seed
one-bit mutants

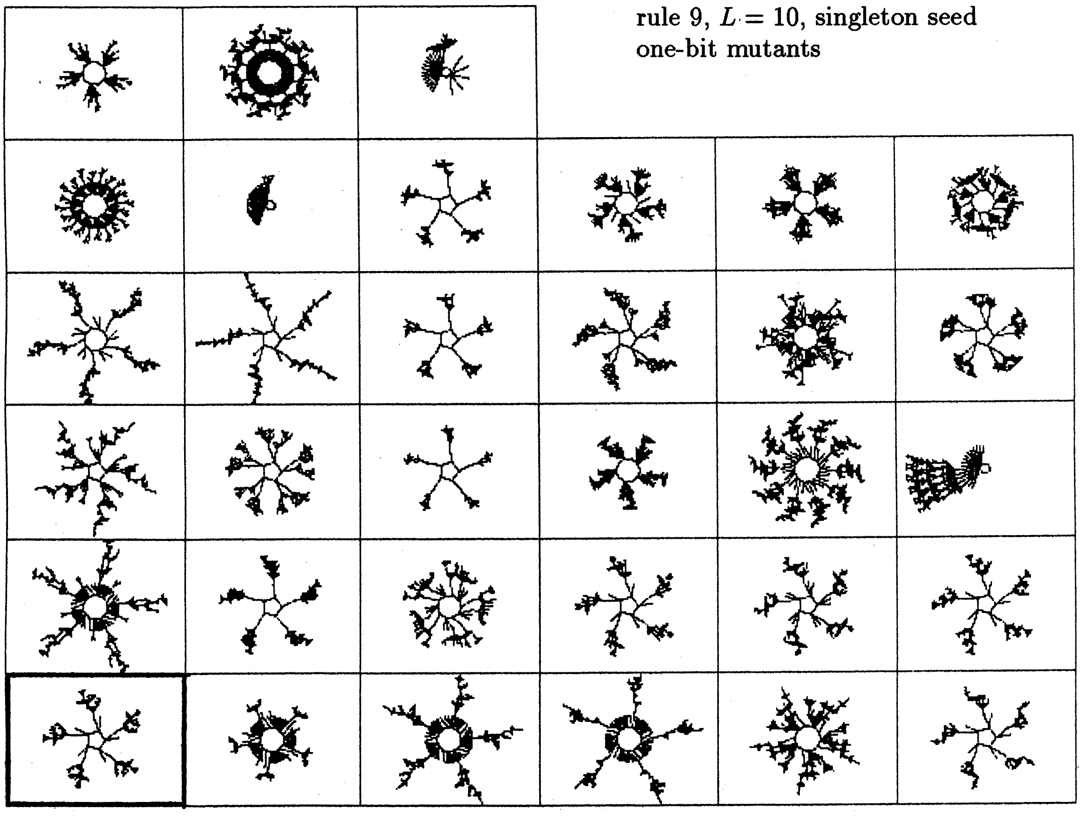
A3.7

rule 228, $L = 11$, singleton seed
one-bit mutants

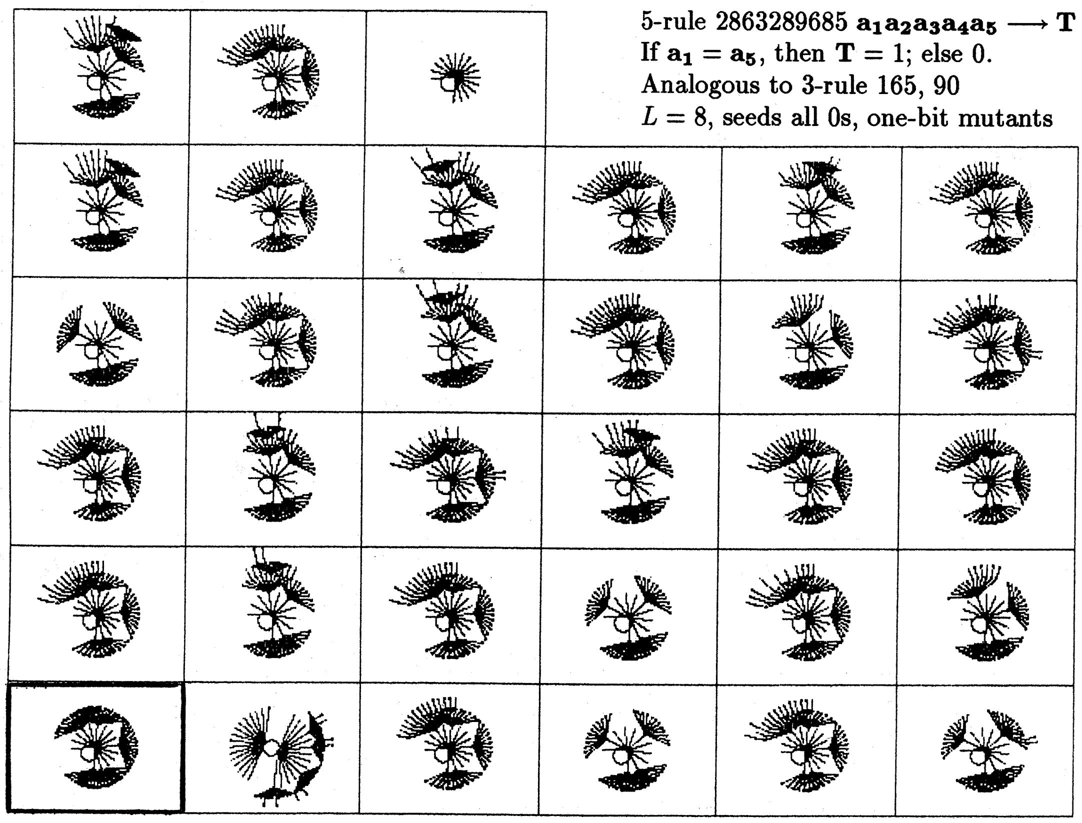


A3.8

rule 9, $L = 10$, singleton seed
one-bit mutants

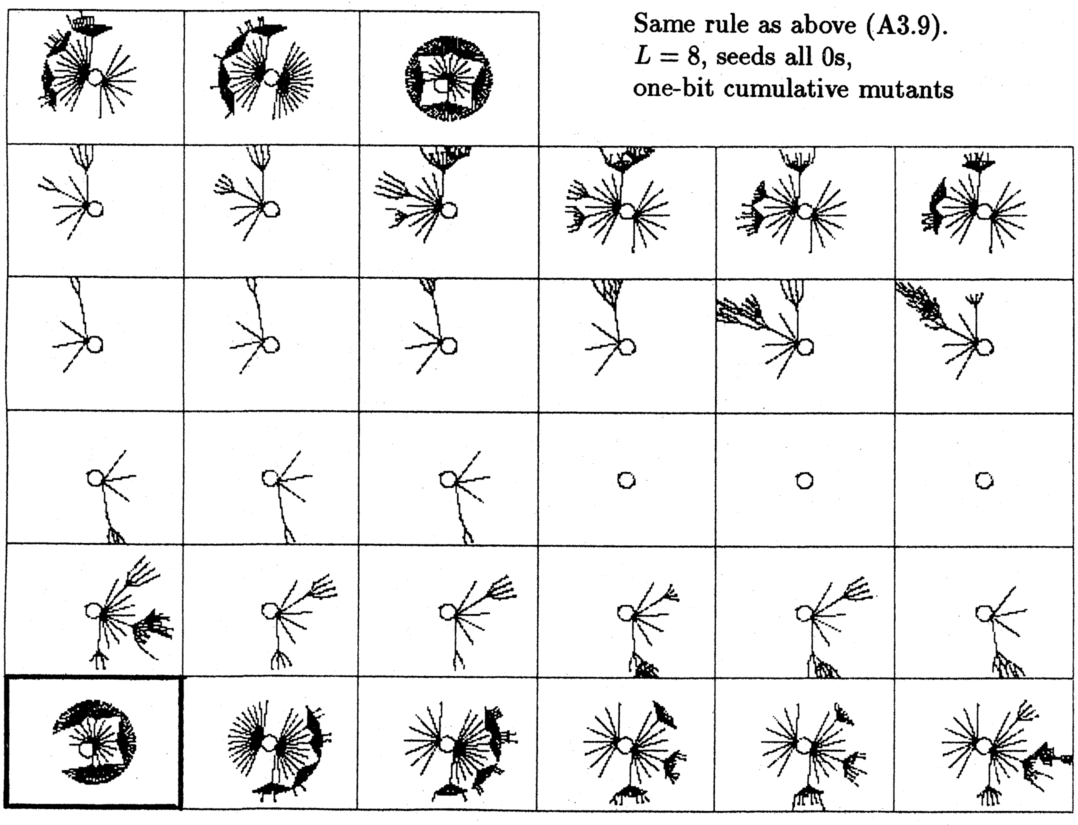


A3.9



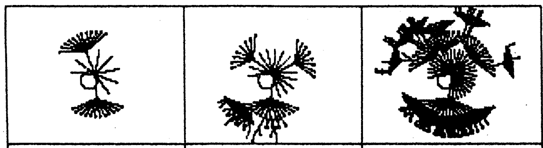
5-rule 2863289685 $\mathbf{a_1a_2a_3a_4a_5} \longrightarrow \mathbf{T}$
If $\mathbf{a_1} = \mathbf{a_5}$, then $\mathbf{T} = 1$; else 0.
Analogous to 3-rule 165, 90
$L = 8$, seeds all 0s, one-bit mutants

A3.10

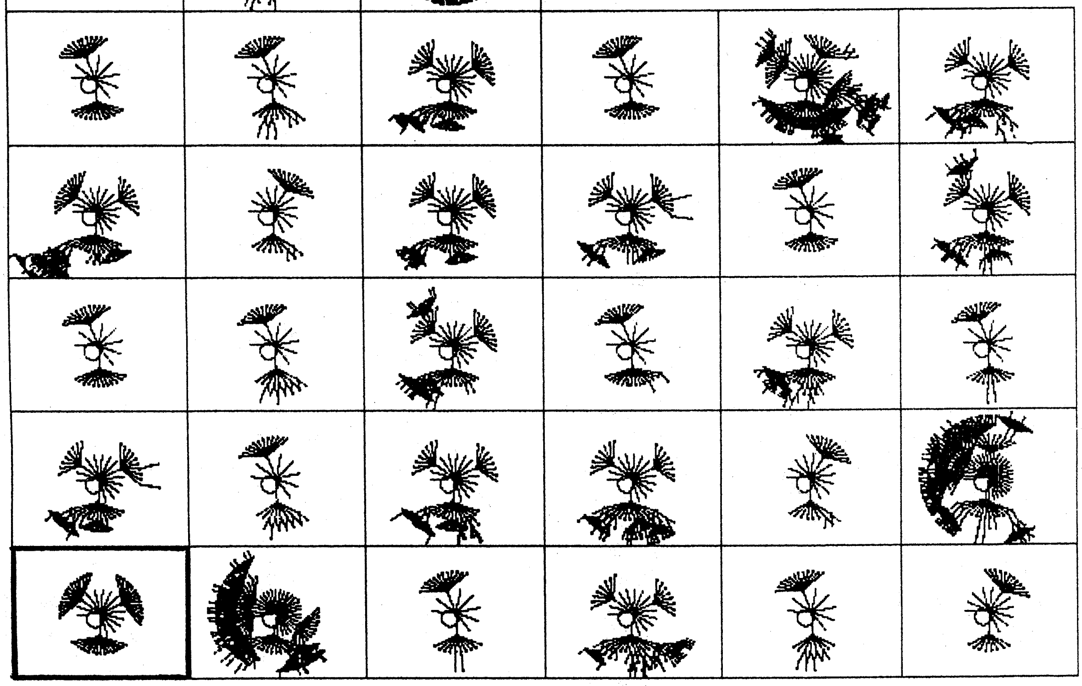


Same rule as above (A3.9).
$L = 8$, seeds all 0s,
one-bit cumulative mutants

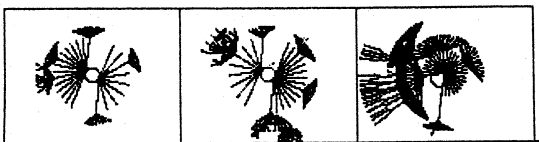
A3.11



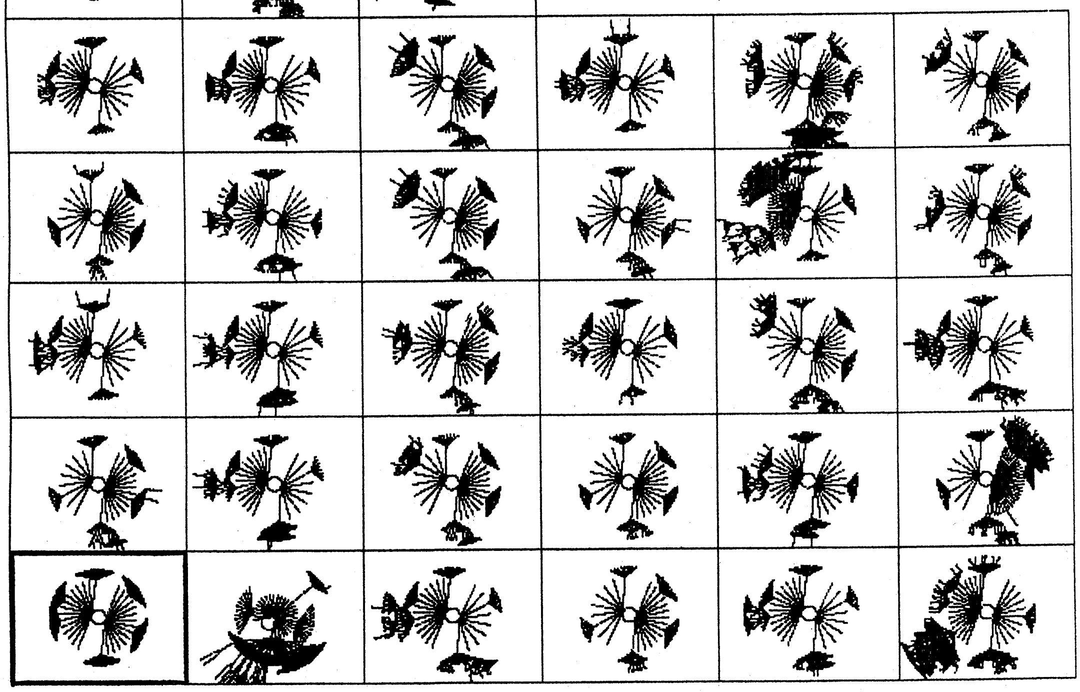
5-rule 2779077210 $\mathbf{a_1a_2a_3a_4a_5} \longrightarrow \mathbf{T}$
if $\mathbf{a_1} = \mathbf{a_5}$, then $\mathbf{T} = \mathbf{a_3}$; else $\overline{\mathbf{a}}_\mathbf{3}$;
analogous to 3-rule 150
$L = 12$, seed all 0s, one-bit mutants



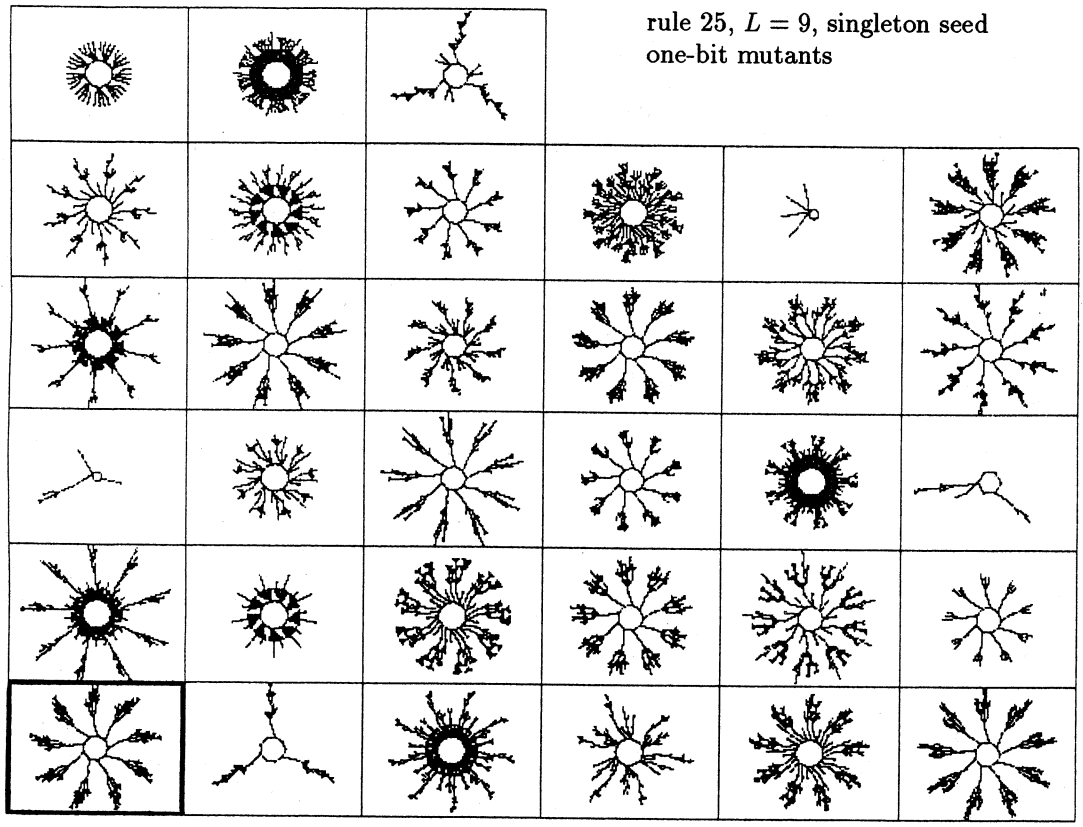
A3.12



5-rule 1515890085 $\mathbf{a_1a_2a_3a_4a_5} \longrightarrow \mathbf{T}$
if $\mathbf{a_1} = \mathbf{a_5}$, then $\mathbf{T} = \overline{\mathbf{a}}_\mathbf{3}$; else $\mathbf{a_3}$;
analogous to 3-rule 105
$L = 12$, seed all 0s, one-bit mutants

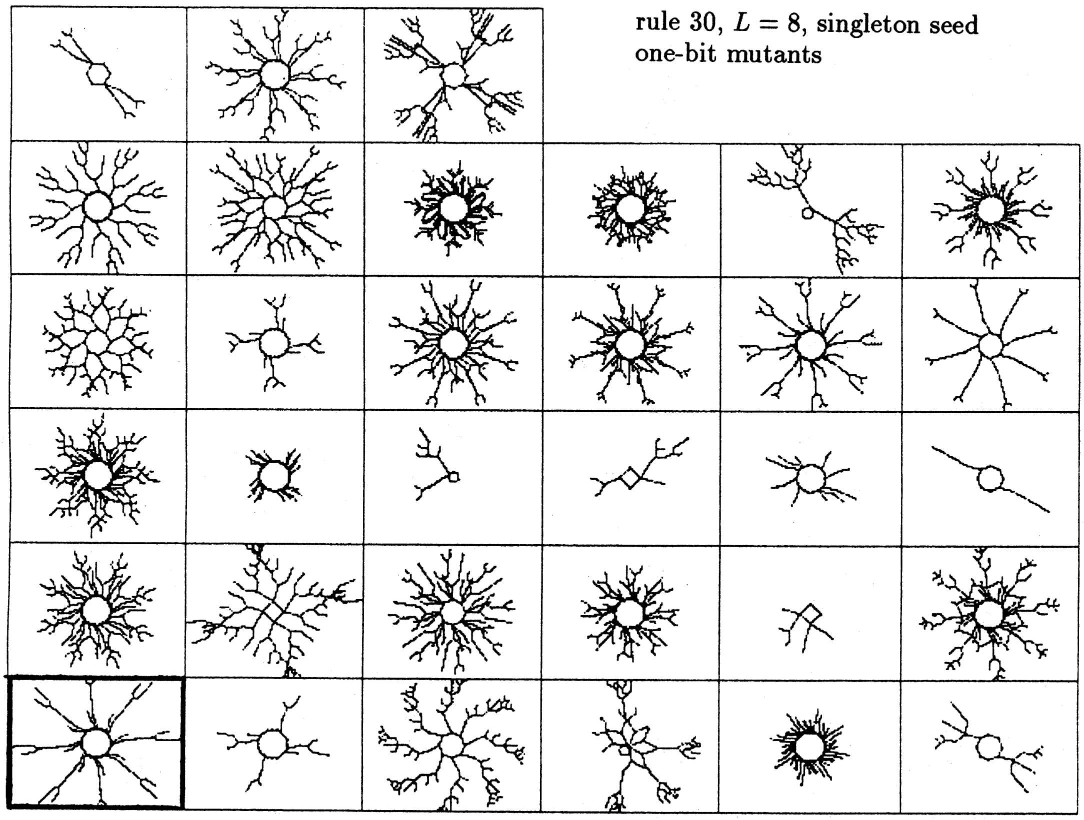


A3.13



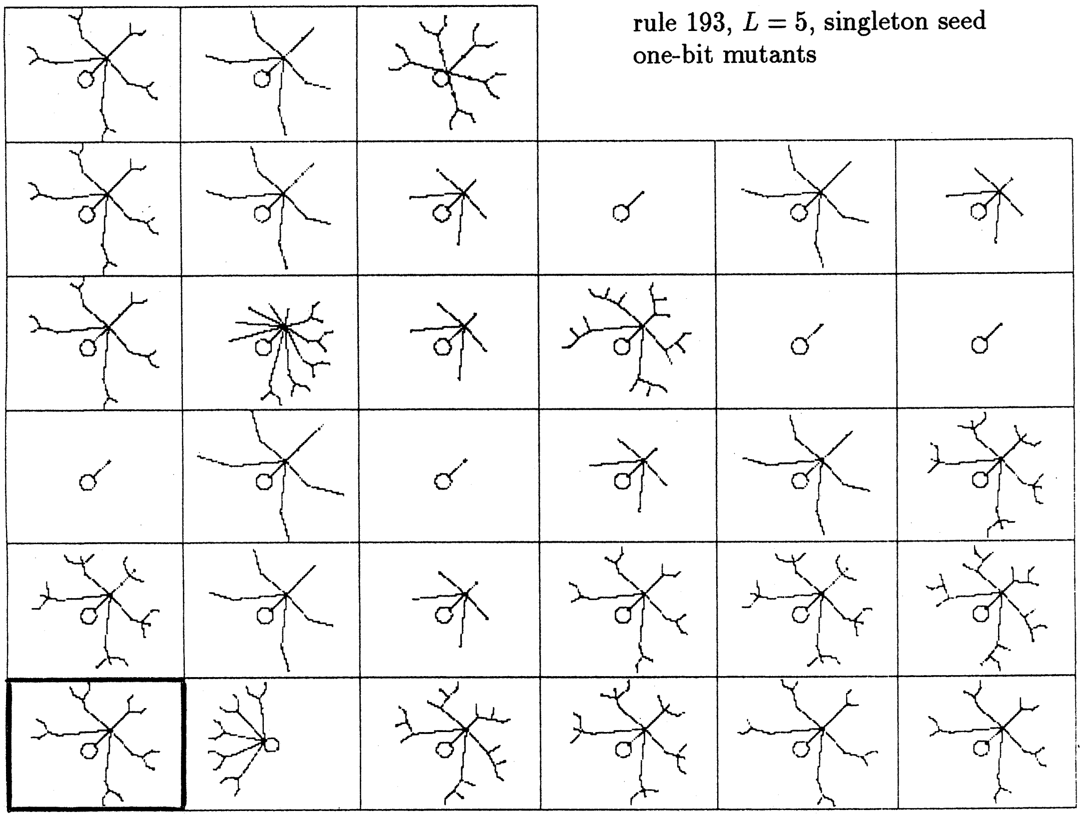
rule 25, $L = 9$, singleton seed
one-bit mutants

A3.14



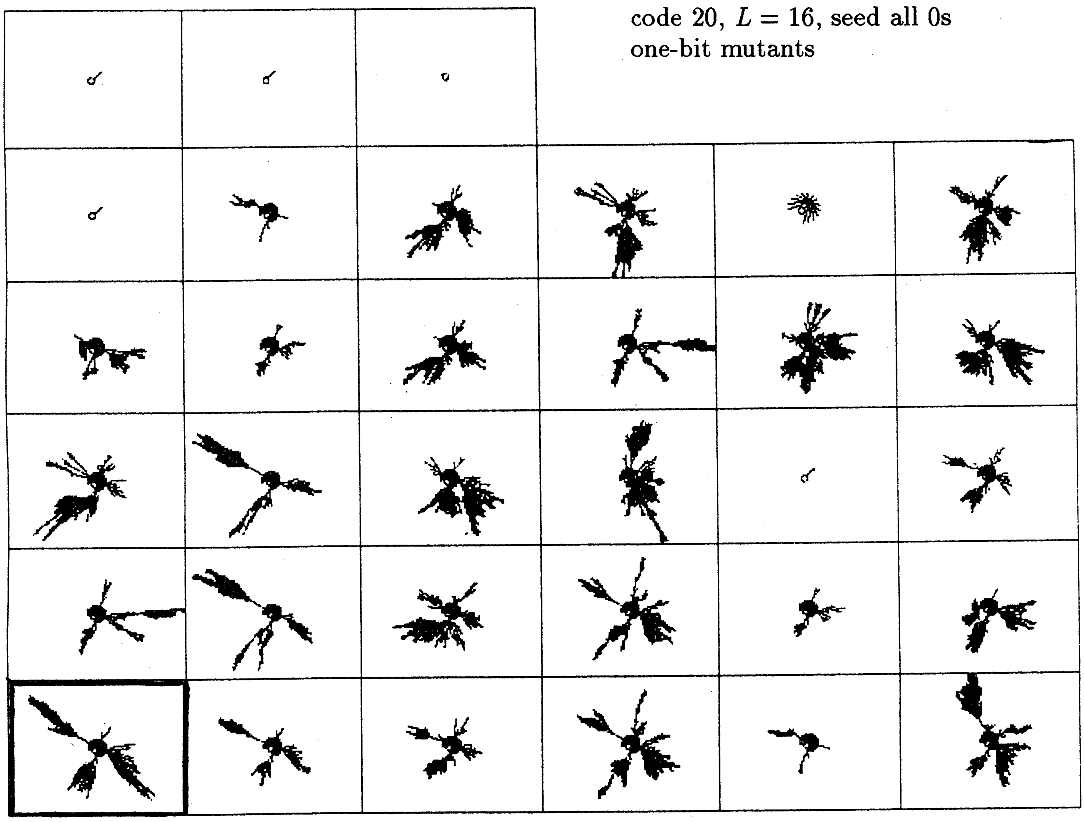
rule 30, $L = 8$, singleton seed
one-bit mutants

A3.15



rule 193, $L = 5$, singleton seed
one-bit mutants

A3.16



code 20, $L = 16$, seed all 0s
one-bit mutants

234

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APPENDIX 4

The Rule-Space Matrix, n=3 Rules

The 256 $n = 3$ rules (0 to 255) may be set out on a 16×16 matrix. Rows i and columns j are numbered 0 to 15 as shown on the next page, and equivalently by the 4-bit binary numbers 0000 to 1111. Each entry in the matrix, a_{ij}, is a function of its position, and is assigned the decimal equivalent of the 8-bit binary number formed by the concatenation (denoted by the symbol +) of its 4-bit row and column binary expressions. If bin\$ (x) = the 4-bit binary string equivalent of x, and dec(x\$) = the decimal equivalent of the binary string x\$, then

$$a_{ij} = \mathrm{dec}(\mathrm{bin\$}(i) + \mathrm{bin\$}(j));$$

for example, to establish the entry $a_{5,6}$

$$\mathrm{bin\$}(5) = \text{“0101”},\ \text{and}\ \mathrm{bin\$}(6) = \text{“0110”}$$
$$\mathrm{bin\$}(5) + \mathrm{bin\$}(6) = \text{“0101”} + \text{“0110”} = \text{“01010110”},\ \mathrm{dec}(\text{“01010110”}) = 86$$

Conversely, given a rule number, 0 to 255, its position on the matrix is found by the separating its 8-bit binary expression into two equal parts. The left 4 bits denotes the row i and the right 4 bits the column j. The resulting matrix is set out on the next page.

i \ *j*	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	21
2	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
3	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
4	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
5	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
6	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
7	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127
8	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
9	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159
10	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175
11	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191
12	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207
13	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223
14	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
15	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255

The position of rules on the matrix depends on the rule numbering convention implicit in the sequence of the rule table entries. The conventional sequence as described in section 3.3 is as follows:

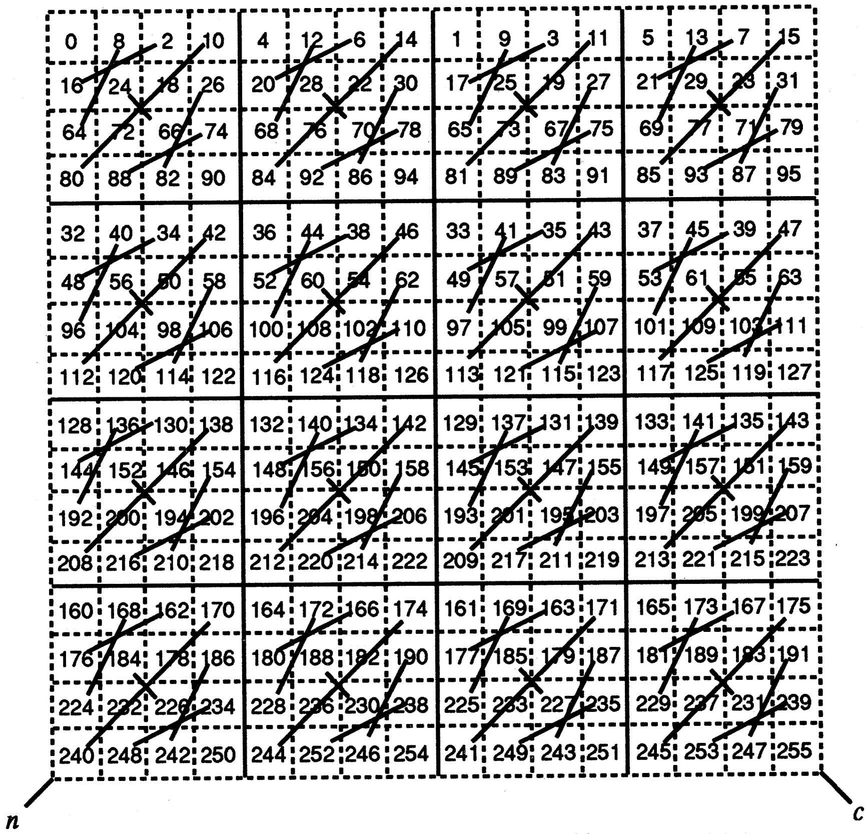
	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

Thus $T_7T_6T_5T_4T_3T_2T_1T_0$ is the conventional binary expression of the rule. Other sequences of the rule table would be equally valid; indeed, there are $8! = 13440$ permutations, and thus the same number of possible alternative numbering conventions.

If the equivalence relationships are indicated by drawing lines between equivalent rules on the matrix, a systematic pattern is apparent, however the clarity of this pattern varies for different numbering conventions. A limited search of alternatives has turned up a permutation that results in a pattern of exceptional clarity; this is the rearranged sequence:

	111	101	110	100	000	010	001	011	neighbourhoods
Rule table..	T_7	T_5	T_6	T_4	T_0	T_2	T_1	T_3	outputs

giving the alternative binary expression of the rule $T_7T_5T_6T_4T_0T_2T_1T_3$. In the matrix shown on the next page, rules are *positioned* according to the alternative numbering system, but are still numbered according to the conventional system. The diagonals, labeled c (complement) and n (negative), are indicated.

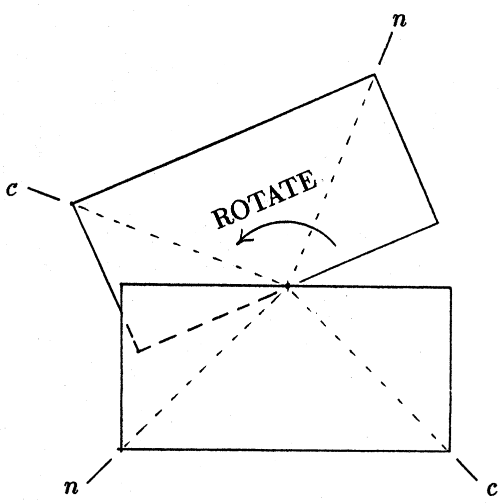


The matrix shows reflection equivalent rules linked by lines, and provides a graphic demonstration of rule categories and relationships as follows:

1. Simulates all the rule cluster transformations.
2. Distinguishes between symmetric, semi-asymmetric, and fully asymmetric rules.
3. Identifies special status rules resulting in collapsed clusters.
4. Accounts for the numbers of rules in equivalence classes and symmetry categories.

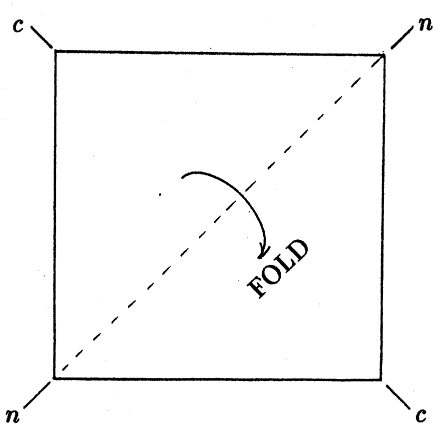
A4.1 Complimentary Transformations

R and R_c will be superimposed if any half division of the matrix is rotated over the other half (this is true of *all* numbering conventions).



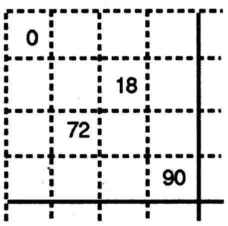
A4.2 Negative Transformations

Given a rule R, R_n is its reflection across the n diagonal. Alternatively, R and R_n will be superimposed if the matrix is *folded* across the n diagonal.

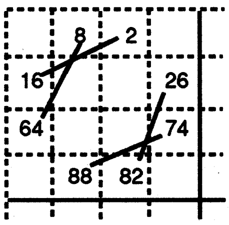


A4.3 Reflection Transformations

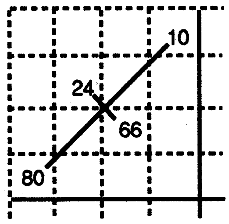
The matrix is shown subdivided into sixteen 4 x 4 segments, containing 16 rules. R and R_r pairs are contained within a segment, and are shown linked. Rules in each segment have the same characteristic layout according to their symmetry categories as illustrated below:



4 symmetric rules



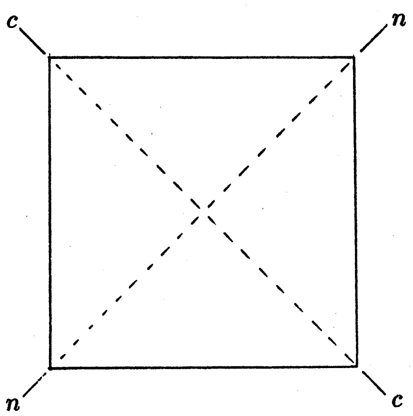
8 semi-asymmetric rules



4 fully asymmetric rules

A4.4 Collapsed Clusters

The two diagonals across the matrix, labelled n (negative) and c (compliment), have special significance, because when the complimentary and negative manipulations of the matrix are carried out, rules related to the diagonals exhibit additional relationships, resulting in collapsed clusters as described in section 3.3.8.



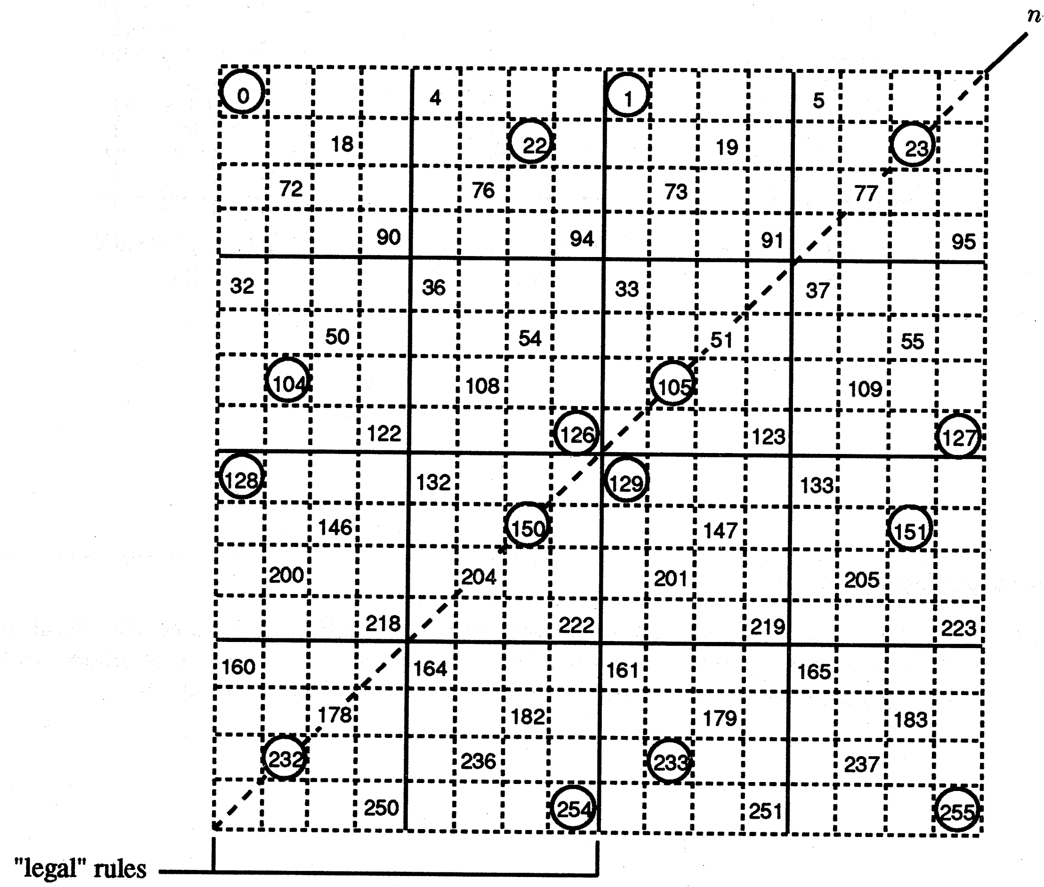
Rules that lie on the c diagonal have the property, for a given rule R, $R_c = R_n$. Fully asymmetric rules whose reflection link is bisected by the c diagonal, have the property, for a given rule R, $R_c = R_{nr}$. As stated earlier, R and R_n will be superimposed if the matrix is folded across the n diagonal; rules that lie on this diagonal (on the fold) have the property, $R = R_n$. Fully asymmetric rules whose reflection link is bisected by the n diagonal have the property that, for a given rule R, $R_n = R_r$.

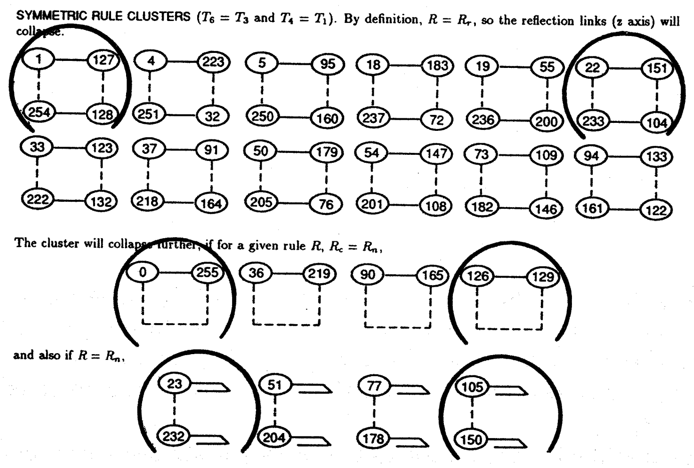
Semi-asymmetric rules are unrelated to the diagonals in the ways described above, and therefore have no collapsed clusters.

A4.5 Symmetric Rules

The equivalence classes among the symmetric rules (total 36) consist of the rules which are superimposed when the matrix is folded across the n diagonal.

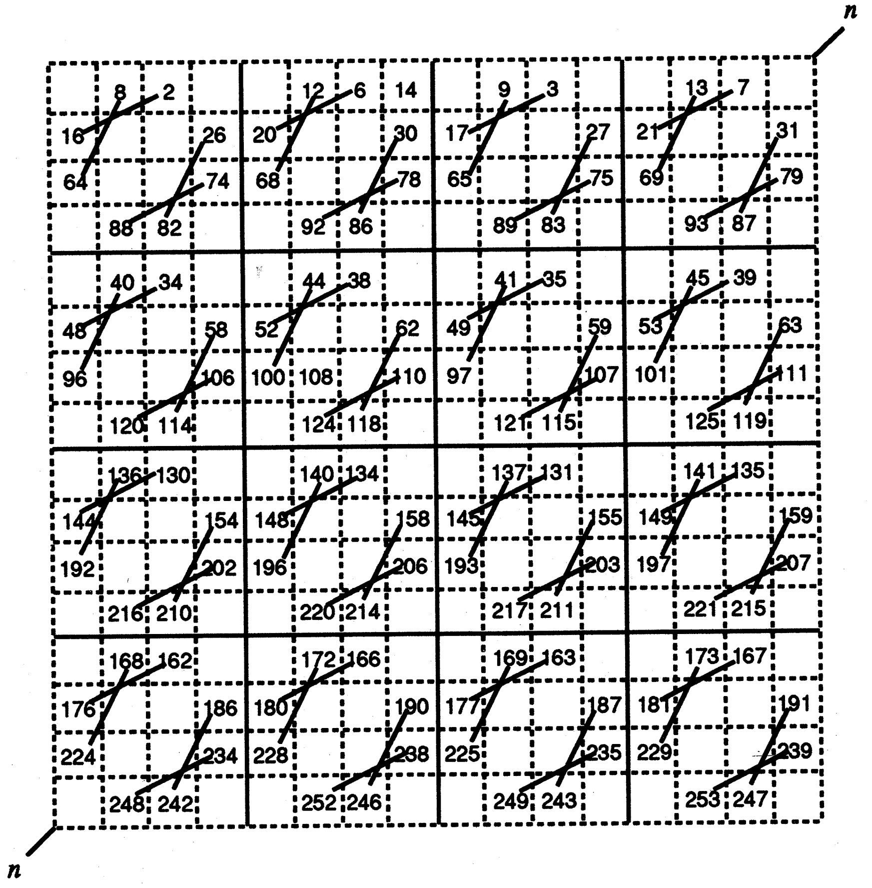
The 32 rules that have sometimes been designated as "legal"[33] (both symmetric and even) are located in the left half of the matrix. The 16 totalistic rules among the $n = 3$ rules are circled.



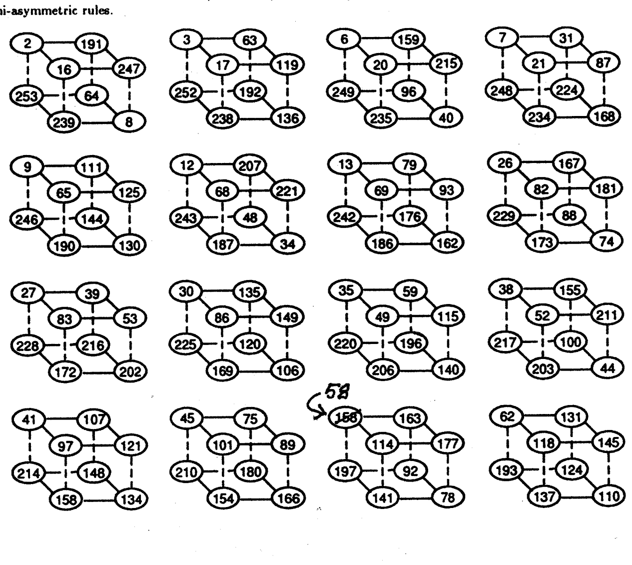


A4.6 Semi-Asymmetric Rules

The equivalence classes among the fully asymmetric rules (total 32) consist of the linked rules which are superimposed when the matrix is folded across the n diagonal.

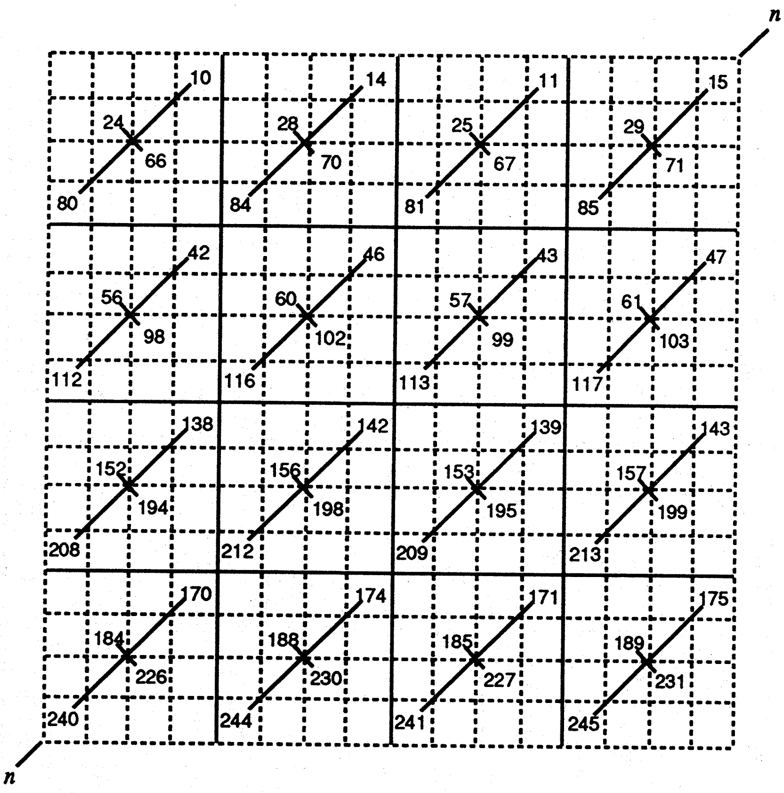


SEMI-ASYMMETRIC RULE CLUSTERS (either $T_6 \neq T_3$ or $T_4 \neq T_1$). There are no collapsed clusters among the semi-asymmetric rules.

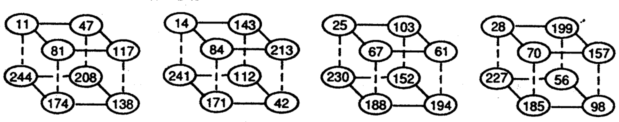


A4.7 Fully Asymmetric Rules

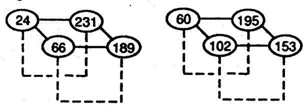
The equivalence classes among the fully asymmetric rules (total 20) consist of the linked rules which are superimposed when the matrix is folded across the n diagonal.



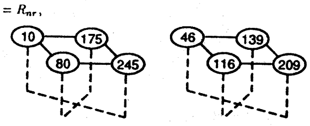
FULLY ASYMMETRIC RULE CLUSTERS



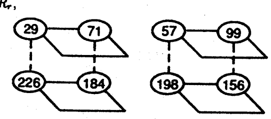
The cluster will collapse if, for a given rule R, $R_c = R_n$,



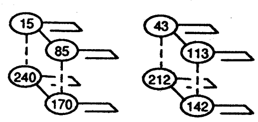
And if, for a given rule R, $R_c = R_{nr}$,



And if, for a given rule R, $R_n = R_r$,



And also if $R = R_n$,



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Index

U

V

W

Z